



Research Article

Volume 5 Issue 1 - November 2019
DOI: 10.19080/RAEJ.2019.05.555652

Robot Autom Eng J

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Mathematical Models for Precession of a Gyroscope with One Side Support



Albina Omorova and Ryspek Usubamatov*

Department of Automation and Robotics, Kyrgyz State Technical University, I. Razzakov, Kyrgyzstan

Submission: October 26, 2019; Published: November 14, 2019

*Corresponding author: Ryspek Usubamatov, Department of Automation and Robotics, Kyrgyz State Technical University, I. Razzakov, Kyrgyzstan

Abstract

New research in the area of spinning objects has demonstrated a system of the eight inertial torques generated by the rotating masses acts on a gyroscope. The action of this system of interrelated inertial torques manifests all gyroscopic effects. New mathematical models for the inertial torques are used for solving of gyroscopic problems in engineering and presented in several publications. The analytical solution of these publications contains obscurity and vagueness in the mathematical model for the gyroscope precession motion. This model cannot be used for the direct computing of the precession motion. The latter one should be presented by the clear logic and physical principles of the interrelated action of the inertial and external torques. The expression of the precession torque should be presented by a component of the load torque acting on a gyroscope. The novelty of this work is presented by the modified mathematical model for the precession motion of a gyroscope with one side support that enables for the direct computing of the angular velocity. The new model well-matched with practical tests.

Keywords: Gyroscope theory; Precession; Test; Inertial torque

Nomenclature

- g – gravity acceleration
- e - base of natural logarithm
- M – mass of a gyroscope
- J - mass moment of inertia of a rotor's disc
- J_y - mass moment of inertia of a gyroscope around axis y
- n – ratio of load and precession torque
- $T_{p,x}$ – precession torque acting around axis oy
- $T_{r,x}$ - resulting resistance torque acting around axis ox
- t - time
- γ - angle of inclination of a rotor's axle
- ω - angular velocity of a rotor
- ω_i - angular velocity of precession around axis i

Introduction

In engineering, rotating objects are presented by numerous mechanisms that manifest gyroscopic effects in which physics is not well described [1-4]. Gyroscopic effects play an important role in the dynamics of rotating objects and this is the reason textbooks and manuals of engineering mechanics necessarily contain chapters of gyroscope theory [5-9]. Numerous publications dedicated

to gyroscopic effects and present different mathematical models for the gyroscope properties [10-13]. Nevertheless, known publications contain simplified analytical models for gyroscopic effects based on the principle of the change in the angular momentum that do not match practical tests [14,15].

Recent investigations of the physical principles of gyroscopic effects demonstrate the action of the system of interrelated inertial torques generated by a rotating mass of spinning objects [16-19]. The new mathematical models for the system of inertial torques produced by rotating mass constitute the fundamental principles of the gyroscope theory. The new publications related to inertial torques acting on gyroscopic devices contain the expression of the precession torque, in which a mathematical model cannot be used for direct computing of the precession motion. The latter is defined by the intermediate equation of the ratio of the angular velocities around two axes. The proposed manuscript presents modified mathematical models for the direct computing of the precession torque and motions around axes for a gyroscope with one side support.

Methodology

New mathematical models for the system of inertial torques generated by rotating mass and acted on the gyroscope are presented in several publications [16-19]. This system was used to formulate the mathematical models for motions of a gyroscope

where all components are as specified above. Equation (9) contains the component of the load torque and enables for direct computing of the angular velocity of gyroscope around axis oy .

Case Study

The mathematical model for the motion around axis oy of the gyroscope suspended from the flexible cord is considered for its horizontal location ($\cos 0^\circ = 1.0$). All defined technical data and parameters of the gyroscope are used for the case study [17]. Substituting the gyroscope data [17] into Eq. (9) and transformation yields the following expression:

$$1.9974649 \times 10^{-4} \frac{d\omega_y}{dt} = \frac{(32\pi^2 + 136) \times 0.146 \times 9.81 \times 0.0325}{38\pi^2 + 161} - \frac{8}{9} \times 0.5726674 \times 10^{-4} \omega_y \quad (10)$$

The following solution is the same as presented in [17] which comments are omitted.

$$3.924001981 \frac{d\omega_y}{dt} = 770.773136698 - \omega \omega_y \quad (11)$$

$$\frac{d\omega_y}{770.773136698 - \omega \omega_y} = 0.254841869 \omega dt \quad (12)$$

$$\int_0^{\omega_y} \frac{d\omega_y}{770.773136698 - \omega \omega_y} = 0.1254841869 \int_0^t \omega dt \quad (13)$$

$$-\ln \left(\frac{770.773136698}{\omega} - \omega_y \right) \Big|_0^{\omega_y} = 0.254841869 \omega t \Big|_0^t \text{ giving rise to the following}$$

$$1 - \frac{\omega_y}{\frac{770.773136698}{\omega}} = e^{-0.254841869 \omega t} \quad (14)$$

$$\omega_y = \frac{770.773136698}{\omega} (1 - e^{-0.254841869 \omega t}) \quad (15)$$

$$\omega_y = \frac{70.773136698}{\omega} = \frac{70.773136698}{2\pi \times 10000/60} = 0.736034128 \text{ rad/s} = 0.171^\circ/\text{s} \quad (16)$$

The obtained result is the same as presented in [17] and well-matched despite some minor differences that can be explained by the accuracy in computing of the parameters for equations. Both equations yield the result of the action of inertial torques generated by the external one. This result is the validation of the correctness of Eq. (9) that can be used for the direct computing of the angular velocity of precession around axis oy .

Results and Discussion

The new analytical study of acting inertial forces on the gyroscope presented by mathematical models of inertial torques and new equations for gyroscope motions around axes. The mathematical model for the gyroscope precession motion does not allow the direct computing of the angular velocity around axis. It can be obtained by the intermediate equation presented by the ratio of the angular velocities around two axes. Practically, it is better to conduct the direct computing of gyroscope precession motion by the modified mathematical model for the precession torque that

yields the same result as the known method. The new mathematical model of the precession motion is accepted as correct.

Conclusion

New studies in the area of gyroscope theory generated the new system of mathematical models for the inertial torques and motions of the gyroscope that can raise questions for clarifications. The equation for the gyroscope precession motion cannot be used for computing because contains only expressions of inertial torques. The uncertainties and vagueness in an analytical approach for the precession torque and motion of the gyroscope were resolved. The new model for the precession torque acting on the gyroscope contains the component of the load torque that generates all inertial torques. The modified mathematical model for the precession torque enables for direct computing the angular velocity of the gyroscope motion, clearly describes the physics of acting forces and validated by the practical tests.

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DOI: [10.19080/RAEJ.2019.05.555652](https://doi.org/10.19080/RAEJ.2019.05.555652)

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