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Gyroscope Torques Acting on Rolling Disc

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Abstract

Recent investigations in gyroscope effects have demonstrated that their origin has more complex nature that represented in known publications. On a gyroscope are acting simultaneously and interdependently eight inertial torques around two axes. These torques are generated by the centrifugal, common inertial and Coriolis forces as well as the change in the angular momentum of the masses of gyroscope's spinning rotor. The action of these forces manifests the internal inertial resistance and precession torques of gyroscopic devices. New mathematical models for the inertial torques demonstrate fundamentally different approaches for solving of gyroscope problems in engineering. This is very important because the stubborn tendency in engineering the increasing a velocity of rotating parts with different designs like turbines, rotors, discs and other rotating components leads to the proportional increase of acting forces that are expressed on their motions in space. This work considers a typical example of computing the action of internal inertial torques acting on the running disc.

Keywords : Gyroscope theory; Torques; Motions; Forces

Introduction

Most of the textbooks of machine dynamics and books that dedicated to gyroscope theory content typical examples with solving of gyroscope effects [1-3]. Practice demonstrates the known mathematical models do not match the actual forces and motions in these devices [4,5]. Recent investigations into the physical principles of gyroscopic motions have presented new mathematical models of forces acting on a gyroscope [6,7]. The action of the external load on a gyroscope generates several internal resistance and precession torques based on the action of the inertial forces. Resistance torque is generated by the action of the centrifugal and Coriolis forces of the gyroscope's mass elements. The precession torque is generated by the action of the common inertial forces of the gyroscope's mass elements and by the well-known torque that is generated by the change in the angular momentum of the spinning rotor. These resistance and precession torques act simultaneously and interdependently and are strictly perpendicular to each other around their axes. Equations of internal torques are shown in Table 1 [6].

Table 1 contains the following symbols: J is the rotor's mass moment of inertia around the spinning axle; ω_i is the angular velocity of the precession of a spinning rotor around axis i and ω is the angular velocity of a spinning rotor. The following analysis of the actions of several torques and motions around the two axes has used the system of subscripts signs. All components of the equations are marked by subscript signs that indicating the axis of action. For example, $T_{r.x}$ is the resistance torque acting around axis ox, ω_y is the angular velocity of precession around axis oy, etc.

Table 1: Equations of the gyroscope's internal torques.

Type of Torque Generated By	Equation, (N.m)
Centrifugal forces, $T_{ct.i}$	$T_{ct.i} = T_{in.i} = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_i$
Common inertial forces, $T_{in.i}$	
Coriolis forces, $T_{in.i}$	$T_{cr} = (8/9)J\omega\omega_i$
Change in angular momentum, $T_{am.i}$	$T_{am.i} = J\omega\omega_i$
Resistance torque $T_{r.i} = T_{ct.i} + T_{cr.i}$	$T_{r.i} = \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_i$
Precession torque $T_{p.i} = T_{in.i} + T_{am.i}$	$T_{p.i} = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right]J\omega\omega_i$

A different type of the free rolling flat cylindrical type objects as a bicycle wheel, rims, hoops, discs, etc., possess gyroscope properties. Generally, this rolling motion is considered as a flat motion that more complex than simple spinning of the gyroscope, and its mathematical treatment is considerably more complicated. However, free rolling of the inclined cylindrical disc on the flat surface, possess inherent the gyroscope effects that influence on the motion of the rolling object. This work presents the mathematical model for motions of the tilted rolling disc on the flat surface.

Methodology

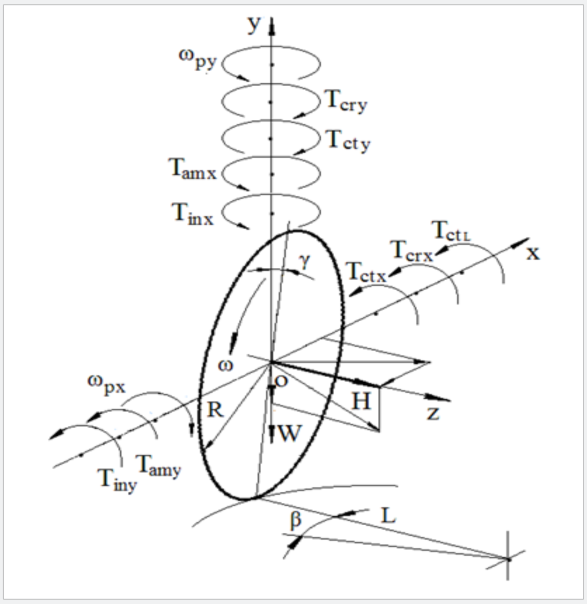

Figure 1: Free rolling of the disc on a flat surface.

The simple bicycle wheel or thin disc is unstable on the vertical plane, but rolling motion demonstrates its stability and steering itself in case of the disc tilts. This tilts leads to turn the rolling disc in the direction in which it started tofall. This motion of the thin disc is the demonstration of the gyroscope effects which presented by the action of centrifugal, common inertial, and Coriolis forces and the rate change of the angular momentum of the rolling disc. These forces enable to bring the disc back to a vertical position and allow it to roll onin right way or the new direction that resulting of the disc new tilt.

To study the motion of a rolling thin disc on the flat surface, it is assumed that thin disc is rolling with constant angular velocity. Figure 1 represents rolling disc that slightly tilted in its path on the flat surface. This slight tilt causes it to travel temporarily in a curved path. Analysis of the motion of the free rolling thin disc on the flat surface is considered with starting condition when the disc is perpendicular to the surface. The thin disc of the mass of 0.5kg with the radius of 0.2m, rolling with the linear velocity 1.0m/s and getting the tilt of 10^0 from the vertical line as shown in Figure 1. It is necessary to find the internal torques, exerted on the rolling disc and its precessions. The inclination of the rolling disc generates the following torques:

- The resistance torques based on the action of the centrifugal T_{ctx} and Coriolis forces T_{crx} acting around axis ox .
- The precession torques based on the action the change in the angular momentum T_{amx} of the rolling disc and the inertial forces T_{inx} acting around axis oy .
- The resistance torques based on the action of the centrifugal T_{ctx} and Coriolis forces T_{crx} acting around axis oy .
- The precession torques based on the action the change in the angular momentum T_{amy} and the inertial forces T_{iny} around axis ox .

The precession torques turn the rolling disc to the direction of the tilting and the disc rolls by the curve path with the radius L .The curvilinear motion of the disc generates the action of the centrifugal force acting horizontally and creating the torque about the contact point of the disc with the surface. This torque acts in the same direction as the resistance torques and altogether bring the disc back to a vertical position. From the data presented above is defined as the angular precession of the disc around axis at starting condition. The equations of the torques balance around axis and are represented as follow:

$$WR\sin\gamma = T_{ctx} + T_{crx} + T_{ctL} + T_{iny} + T_{amy} \quad (1)$$
$$WR\sin\gamma = mgR\sin\gamma$$

where $WR\sin\gamma = mgR\sin\gamma$ is the action of the disc's weight to its incline; is the mass of the disc; g is the gravity acceleration; R is the radius of the disc; T_{ctx} and T_{cty} are the resistance torques generated by the centrifugal forces; T_{crx} and T_{cry} are the resistance torques of the Coriolis forces and acting on the rolling disc around axis and respectively; T_{amx} and T_{amy} are the precession torques generated by the inertial forces and and are the torque generated by the change in the angular momentum of the rolling disc acting about the axis ox and oy respectively (Table 1); T_{ctL} is the torque generated by the centrifugal forces of the rolling disc by the curve pass with the radius L . From Eq. (1) is removed the torques of centrifugal and inertial forces acting about the axis oy as self-compensated [7].

The disc is rolling on the curve pass generates the centrifugal force and hence the torque acting about the point of the disc contact with the surface. This torque is defined by the following equation:

$$T_{ctL} = F_{ctL} \times R = m\frac{V^2}{L} \times R$$

Where is the centrifugal force generated by the rolling disc, other parameters are as specified above.

Substituting defined parameters into Eq. (1) and transforming yield the following equation:

$$WR\sin\gamma - \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_x - m\frac{V^2}{L} \times R + J\omega\omega_y = 0 \quad (2)$$

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Where $J = {mR^2}/{2}$ is the mass moment of inertia of the disc, $\omega = {V}/{R}$ is the angular velocity of the disc, V is the linear velocity one, other parameters are as specified above.

The angular velocity of the disc is defined by the following equation:

$$\omega = \frac{V}{R} = \frac{1.0m/s}{0.2m} = 5.0rad/s$$

The angular velocity of precession about the axis oy is defined by the following equation:

$$\omega_y = \frac{V}{L} = \frac{1.0m/s}{L} rad/s$$

Where L is the variable radius of the disc motion by the curvilinear path.

Substituting defined numerical data of the initial parameters into the first equation of Eq. (2) and transforming gives the following equation of the angular precession about the axis ox

$$\omega_x = \frac{mR\left(g\sin\gamma - \frac{V^2}{L} + \frac{R}{2}\omega\omega_y\right)}{\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]\frac{mR^2}{2}\omega} =$$

$$\frac{\left(9.81\sin 10^0 - \frac{1.0^2}{L} + \frac{0.2}{2}\times 5.0\times\frac{1.0}{L}\right)}{\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]\frac{0.2}{2}\times 5.0} = \left(1.105395 - \frac{0.324450}{L}\right) rad/s$$

The velocity of the angular precession ω_x about the axis ox is variable and depends on the radius of the curve pass L. When the $L = (0.324450/1.105395) = 0.293m$ then $\omega_x = 0$, i.e., the torques acting around axis ox in balance and the disc is rolling by this curve pass without fall. Change of the radius L leads to change of the magnitudes of the centrifugal force and precession torques acting on the disc rolling. Increasing of the linear velocity for the rolling the disc leads to its vertical location and decreasing velocity leads to falling of the disc.

Results and Conclusion

New analytical approach to the gyroscopic devices enables developing the equations for the forces and motions of any rotating objects moving in the space. The mathematical models for the rolling disc motions based on the action of the centrifugal, common inertial and Coriolis forces, as well as by the change in the angular momentum. The action of these forces is interrelated and occurs at one time in the gyroscopic devices. The new analytical approach to gyroscope problems demonstrates and explain the physical principles of acting forces on a gyroscopic device and its motions. The mathematical model of the rolling disc motion on the flat surface are validated the gyroscope properties by practical observation and represent a good example of the educational process.

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