



Why Model Complex Dynamic Systems Using Fuzzy Cognitive Maps?



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Submission: September 01, 2017; **Published:** September 21, 2017

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Abstract

Today's systems have become more and more complex and dynamic. The concept of complex dynamic systems (CDS) arises in many scientific fields, technological areas and everyday's problems. The fundamental basics of complex dynamic systems are briefly presented and discussed. Modelling and controlling complex dynamic systems is a very difficult and challenging task. Different modelling approaches basically focus on the interaction between (microscopic) subsystems and the emergence of new qualities at the (macroscopic) system level. However, these models are not sufficient to describe the dynamic behavior of these systems.

The need for new advanced and innovative approaches is justified. Theories of Fuzzy Cognitive Maps are proposed and mathematically formulated as a new way to model and control complex dynamic systems. Examples for modelling CDS are given and the obtained results are promising in provided a useful solution to many problems of CDS. Certain drawbacks and inefficiencies of FCM are described and certain solutions to overcome these drawbacks are given. Conclusions are given and future research directions are provided.

Keywords: Modelling; Complex dynamic systems; Fuzzy logic; Intelligent systems; Fuzzy Cognitive maps

Introduction

Throughout the natural and artificial world one observes phenomena of great complexity. Yet research in physics and to some extent biology and other fields has shown that the basic components of many systems are quite simple. It is now a crucial problem for many areas of science to elucidate the mathematical mechanisms by which large numbers of such simple components, acting together, can produce behavior of the great complexity observed. Therefore today's systems have become more and more complex and dynamic. The concept of complex dynamic systems (CDS) arises in many scientific fields and technological areas. Modelling and controlling complex dynamic systems is a very difficult and challenging task. As a result "complex systems theory" cuts across the boundaries between conventional scientific disciplines. It makes use of ideas, methods and examples from many different fields.

Today, one of the most critical scientific challenges of accepting the "operation" of any complex dynamic system (CDS) is the ability to make Decisions, so the system runs efficiently, and cost effectively. However making Decisions within CDS operations often strains our cognitive capabilities.

Uncertainty, risk and ambiguity are prominent in the research and accompanied literature on Decision-Making. The complex dynamic systems (CDS) are no longer static but most of its subsystems are dynamic and highly nonlinear. Uncertainty and fuzziness are common terms been used in subtly different ways in a number of scientific fields, including: energy generation and distribution, eco systems, statistics, economics, finance, physics, psychology, engineering, health delivery, environment, biology, safety and security systems, sociology, philosophy, insurance, geology, military systems and Information and Communication Technologies (ICT).

Therefore one scientific practice that all of us must be doing constantly is to listen to others and raise serious and challenging questions. Here are some. What is a Complex Dynamic System (CDS)? What are its main characteristics? [1-4]. What is the best models for studying them? Do all models have detailed software tools that can adequately simulate their behavior? How do we model Multilevel Hierarchical systems? [5]. Do we have a clear and sound scientific understanding of the concepts of chaos, complexity and uncertainty? And

how these three concepts are taken into consideration when studying, modeling, analyzing and designing a CDS?? How theories of Large Scale Systems (LSS) as well as for Multilevel Hierarchical have taken into consideration the concepts of chaos, complexity and uncertainty? We can continue raising one question after the other and then try to understand the provided solutions and then raising more questions.

Do all these models and associated solutions provide satisfactory and working conditions to the everyday behavior of the complex dynamic systems? We can say that for a good number of real cases the provided models and solutions meet the objectives and goals of the complex dynamic system [6-9]. However there are also a good and large number of situations where today's models and solutions fail to give satisfactory answers to a number of problems associated with Complex Dynamic Systems. Can we search and identify the sources for this failure? May be! The main reasons are our inability to comprehend and understand well and precisely the actual dynamic and chaotic behavior of complex dynamic systems in the presence of uncertainty, fuzziness and structural complexity [7].

This is also due to the fact all these concepts have different interpretations and mathematical explanations by different people. Another important factor here is the solid knowledge and experience of the scientists been involved in the process and on making decisions subsequently [7-10]. Some scientist combine all above factors into one term: UNCERTAINTY and try to explain everything using theories and techniques that have been developed to model, understand, analyze and finally arrive in taking decisions [10]. Most of these are based on classical models, Probability theory, statistical methods and Artificial Intelligence (AI). However all these efforts have still failed to provide satisfactory answers to the real problems faced by the behavior of complex dynamic systems? Therefore new advanced and innovative approaches are needed. The aim of this paper is to carefully address the generic term of uncertainty. Furthermore using Fuzzy Cognitive Maps (FCM) [11-12] an effort is made to provide some answers to many of the above raised questions and the ones that will presented in details on the next two sections. In section 2 two definitions and the basics of CDS are briefly outlined.

In section 3 the different and challenging problems in modelling and controlling CDS are explained in details and the need for new approaches and new models for CDS become obvious. In section 4 the basics and mathematical theories of FCMs are briefly provided while two illustrative examples from real problems are model using FCMs are given in section 5. The obtained simulation results are discussed on the same section 5. In section 6 major as well as minor drawbacks and deficiencies of today's FCM theories and algorithms are identify and challenging. For the first time from an overall point of view for FCMs proposed solutions to overcome some

of these drawbacks are briefly provided in the same paper and the appropriate references are given. Finally section 7 draws interesting conclusions and also provides very interesting and challenging future research directions which will further explore ways and approaches in order to formulate more accurate FCMs models and new more effective algorithms.

Basics of Complex Dynamic System

Nowadays, practical dynamic systems have become more and more complex. The concept of complex dynamic systems (CDS) arises in many scientific fields, technological areas and everyday's problems. Examples of these systems are: energy networks, energy storage and distribution, hybrid power systems with different renewable energy sources, robotics, health, artificial intelligence systems, gene regulation and health delivery, safety and security systems, telecommunications, transportation networks, environmental systems, swarm of software agent, traffic patterns, ecosystems, biological systems, social and economic systems, and many other scientific areas can be considered to fall into the realm of complex dynamical systems.

Such systems are often concurrent and distributed, because they have to react to various kinds of events, signals, and conditions. They are characterized by a system with uncertainties, time delays, stochastic perturbations, fuzziness, complexity, hybrid dynamics, distributed dynamics and a large number of algebraic loops. The science of complex dynamical systems (CDS) is a multidisciplinary field aiming at understanding the complex real world that surrounds us [1-9]. Although the last 20 years many efforts have been made, there is no commonly accepted definition of a complex dynamic system (CDS). Heuristic approaches basically focus on the interaction between (microscopic) subsystems and the emergence of new qualities at the (macroscopic) system level. However these models are not sufficient to describe the dynamic behavior of these systems. Two definitions could be given here for the sake of this paper. (The term Complex system is kept as is given, however it should be taken as CDS).

I. Complex systems are systems with multiple interacting components whose behavior cannot be simply inferred from the behavior of the components –New England Complex Systems Institute.

II. By Complex system, it system, it is meant a system comprised of a (usually large) number of (usually strongly) interacting entities, processes, regents, the understanding of which requires the development, or the use of, new scientific tools, non linear models, out of equilibrium descriptions and computer simulations. –Journal Advances in Complex Systemssystem, it is meant a system comprised of a (usually large) number of (usually strongly) interacting entities, processes, regents, the understanding of which requires the development, or the use of, new scientific tools, non

linear models, out of equilibrium descriptions and computer simulations.-Journal Advances in Complex Systems

Never the less, whatever definition one relies on, any complex system is a system with numerous components and interconnections, interactions or interdependencies which are difficult to describe, understand, predict, manage, design, and/or change. For this reason, computer simulations [10-11], play a crucial role in studying complex dynamic systems (CDS) and in understanding of how these systems function and work, and how they could be efficiently controlled. Nowadays, information technologies and computer simulations have evolved into an essential tool for modelling, assessment and support in any domain requiring decision making. The complexity and uncertainty of the nature of complex systems, and the heterogeneity of related information, require a complex approach for their study, based on and consisting of data and knowledge management, modelling, simulation and, lastly, decision making support systems, [12-13]. So, these arches for the ways for formalization and automation of processes of modeling, control, and decision support in complex dynamic systems (CDS) continue to attract much attention.

Complex dynamic systems contain a large number of mutually interacting entities (components, agents, processes, etc.) whose aggregate activity is nonlinear, not derivable from the summations of the activity of individual entities, and typically exhibit hierarchical self-organization. Another important characteristic of complex systems is that they are in some sense purposive. The description of complex dynamic systems requires the notion of purpose, since the systems are generally purposive. This means that the dynamics of the system has a definable objective, activity or function.

Complex dynamic systems (CDS) are more often understood as dynamical systems with complex and unpredictable behavior. Multidimensional systems, nonlinear systems or systems with chaotic behavior, adaptive systems, modern control systems, and also the systems, which dynamics depend on, or determined by human being(s) are the formal examples of complex dynamic systems [7-9]. Thus complex dynamic systems is a rather broad research field, whose researches are motivated by a variety of practical engineering systems or health, social, economic, and ecological concerns. Today modeling, control, and optimization are major research issues for complex dynamic systems.

Furthermore in all dynamic processes and on our everyday activities decisions must be made. One of the challenges of accepting the "operation" of any complex dynamic system is the ability to make Decisions so the system runs efficiently and cost effectively. However making Decisions concerning complex dynamic systems often strains our cognitive capabilities. Uncertainty, fuzziness and related concepts such as risk and ambiguity are prominent in the research and accompanied literature on Decision-Making [14]. Uncertainty

is a term used in subtly different ways in a number of scientific fields, including statistics, economics, finance, physics, psychology, engineering, medicine, energy, environment, biology, sociology, philosophy, insurance, geology, military systems and Information and Communication Technologies (ICT). More on the characteristics of CDS will be presented on the next section when the challenging problem of modelling and controlling them is addressed.

Challenging Issues in Modelling and Controlling complex Dynamic Systems

Modeling is a fundamental work which is always the starting point for the control, optimization, and implementation of any physical and/or human made system. This is also the case for complex dynamic systems (CDS). However the last 20-30years complex dynamic systems present many difficulties and problems, both in mathematical modelling, control implementation and philosophical foundations. Complex dynamic systems (CDS) comprise of collections of many heterogeneous entities which interact with other entities and their environment which usually are having a lot of uncertainties, fuzziness, ambiguities and structural complexities. Interactions among subsystems are localized, often seeking autonomy and self-organizing, while most of the times are nonlinear, dynamic, fuzzy and possibly chaotic.

The study of CDS represents a new approach to science that investigates how relationships between parts give rise to the collective behaviors of a system and how the system interacts and forms relationships with its environment. CDS have some specific characteristics, among which are: uniqueness, weak structuredness of knowledge about the system, incompleteness of its dynamic behavior, antagonism among different agents, the composite nature of system, heterogeneity of elements composing the system. Furthermore decisions must be made ensuring the smooth, reliable, stable and cost effective operation of each of the subsystem as well the whole CDS. Another important feature of CDS is that a network structure, including hierarchical one, self organization can amount to:

- I. Disconnecting certain constituent nodes from the system.
- II. Connecting previously disconnected nodes to the same or to other nodes.
- III. Acquiring new nodes.
- IV. Discarding existing nodes.
- V. Acquiring new links.
- VI. Discarding existing links.
- VII. Removing or modifying existing links.

In addition today's society's challenging problems demand CDS to have a number of Properties-Abilities (P-A) such as;

co-evolution, anticipation, adaptation, cooperation such as swarming, intelligence, consciousness, genetic regulation-homeostasis, development, disease, cascading failures in electrical grid, invasiveness in plants, hurricanes and self-repairing materials, cognition, emergence, self-evaluation and organization, robustness and wisdom. All these collective dynamics of a CDS give rise to 'Emergent Evolution Properties-Abilities' (E.E.P-A) at higher scales in space and/or time. Under such conditions, the key problem of complex dynamic systems and control theory consists in the development of methods of qualitative analysis of the dynamics and behavior of such systems and in the construction of efficient control algorithms for their efficient operation. In a general case, the purpose of control is to bring the system to a point of its phase space which corresponds to maximal or minimal value of the chosen efficiency criterion [15-18]. Another one of the main and actual problems in the theory of complex dynamical systems (CDS) and control sciences is a solution of "ill-posed, weakly- and poorly-structured and weakly- formalizable complex problems" associated with complex technical, organizational, social, economic, cognitive and many other objects, and with the perspectives of their evolution.

One more critical aspect that must be seriously taken into consideration is that the human presence in all CDS is inevitable. This problem is very critical in studying CDS because we are actually dealing with Dynamic Systems and want to understand their long-term qualitative behavior. However the focus is not on finding precise solutions to the equations, which most of the times are not well mathematically defining the complex dynamic system. Such a search is often hopeless. The solutions been sought would rather answer questions like "Will the CDS settle down to a steady state in the long term, and if so, what are the possible steady states?" "Are the steady states, precise or are fuzzy and ambiguous?," or "Does the long-term behavior of the system depend on its initial condition?" or "how important is to depend on the knowledge of experts and if so, how many experts should be consulted?" or "does the past history of the behavior of the CDS influences its long-term behavior" or " what is more important: correlation or causality between the states?" or "how sufficient are the mathematical models to predict the long term-behavior of the CDS?" Or "if not what other alternatives do we have?"

Therefore the modeling ,analysis and design of complex dynamic systems (CDS) needs to be readdressed, in the presence of uncertainty, fuzziness, ambiguity and of principally non- formalizable problems and probably not having strict and precise mathematical formulation of the system, on environments that decisions are semi-structured or unstructured. All above characteristics must be taken into consideration. Construction of models of CDS must be based on the use of experts and their extensive knowledge about the

CDS. This knowledge should be wisely used. Thus modeling CDSs is indeed a real challenge. It is not so a straightforward and an easy task.

Indeed it is a Difficult Exercise and Cannot Be Addressed Completely and Sufficiently Using Today's Formal Methods:

Therefore the need for seeking new advanced conceptual modelling methods is a must.

For all the above reasons the approach in modeling Complex Dynamic Systems (CDS) using Fuzzy Cognitive Maps (FCMs), a new mathematical modelling approach for CDS, seem promising as will be demonstrated in the next sections. However a number of drawbacks of today's FCM theories have emerged and will be addressed.

Basics of Fuzzy Cognitive Maps (FCM)

Introductory remarks

Fuzzy Cognitive Maps (FCMs) is a new methodology for modeling complex dynamic systems (CDS) and has been around only for the last 25-30years. FCMs basically exploit the knowledge and experience of "people". Fuzzy Cognitive Maps came as a combination of the methods of fuzzy logic and neural networks. They constitute a computational method that is able to examine situations during which the human thinking process involves fuzzy and/or uncertain descriptions. They are the evolution of the Cognitive Maps which were introduced by Axelrod [18]. Fuzzy Cognitive Maps were introduced by Kosko [19]. They are a soft computing methodology which gives users the ability to encounter problems in the same way the human mind does using a conceptual procedure which can include ambiguous or fuzzy descriptions [18-20].

Therefore FCMs offer a simple, fast, flexible, economical, and versatile approach to a variety of problems(engineering, health, environmental, economic, mechanical, business, tourism, political, social) which are extremely complex and a purely mathematical approach would be time consuming, laborious and require wasting many resources without at the end to find meaningful and realistic solutions. Kosko [19] introduced FCMs a same method to represent the causal relationship between concepts, variables, states, nodes and constraints. Their goal is to represent knowledge in a symbolic way and model the behavior of systems containing elements with complex relationships, which sometimes can be hidden or illegible.

Why use fuzzy cognitive maps?

Studying very carefully and at the same time wisely the above two sections the need for searching for new innovative and advance mathematical models is obvious.

There are five main reasons that suggest and require? The utilization of Fuzzy Cognitive Maps (FCMs) in modelling and controlling complex dynamic systems (CDS):

- a. Complexity
- b. Nonlinearities
- c. Uncertainty
- d. Ambiguity and
- e. Fuzziness

The majority of the real world systems include these five properties-parameters. The conventional control methods for such systems cannot confront these properties-parameters as the FCMs can do. Thus, FCMs are about to play a major role in the future regarding the modeling, analysis, design and control of complex dynamic systems (CDS). However, this must be done in a careful and systematic way and at the same time without an alazonic approach [21].

Brief historical remarks

A historical link of FCM theories are connected to graph theory and goes back to the 18th century. As said FCMs are directed graphs, or digraphs, and thus they have their historical origins in graph theory. Informally a graph is a set of nodes joined by a set of lines or arrows. Graph theory is the study of graphs, mathematical structures used to model pair wise relations objects from a certain collection [22]. A graph is thus context refers to a collection of vertices or nodes and a collection of edges that connect pairs of vertices. The paper written by Leonard Euler on the Seven Bridges of Konigsberg and published in 1736 [23] is regarded as the first paper in the graph theory. This paper, as well as the one written by Wandermonde on the knight problem, carried on with the analysis sites initiated by Leibniz. Eulers formula relating the number of edges, vertices, and faces of a convex polyhedron was studied and generalized by Cauchy [24,25] and is at the origin of topology. Graphs are among the most ubiquitous models of both natural and human-made structures.

For many centuries ideas now embodied in graph theory have been implicit in lay discussions of networks. The explicit linking of graph theory and network analysis began only in 1953 and has been rediscovered many times since. Analysts have taken from graph theory mainly concepts and terminology; its theorems, though potentially valuable for the analysis of real data, are generally neglected. Network analysts thus make too little use of the graph theory. However till today, they have been used to model many types of relations and process dynamics in physical, networks, engineering, biological, health, energy and social systems. Surprisingly, graphs have not been used, almost at all, on economic and business systems.

Political scientist Axelrod [17] was the first to use digraphs to show causal relationship among variables as defined and

described by people, rather than by the researcher. Axelrod called these digraphs Cognitive Maps (CM). Many studies have used CM to look at decision-making as well as to examine people’s perceptions of complex social systems. Kosko [19] modified Axelrods [18] CMs, which were binary, by applying fuzzy causal functions with real numbers in (-1, 1) to the connections, thus the term Fuzzy Cognitive Maps (FCM). Kosko [18] was also the first to model FCMs and to compute the outcome of a FCM, or the FCM inference, as well as to model the effect of different policy options using a neural network computational method [19,20].

Mathematical foundations

A FCM presents a graphical representation used to describe the cause and effect relations between nodes, thus giving us the opportunity to describe the behavior of a complex dynamic system exhibiting on or more than one of the above mentioned properties-parameters a)-e) of CDSs, in a simple and symbolic way. In order to ensure the operation of the system, FCMs embody the accumulated knowledge and experience from experts who know how the system behaves in different circumstances and for long time. In other words they recommend a modeling process consisting of an array of interconnected and interdependent nodes C_i (variables), as well as the relationships between them W (weights). Concepts take values in the interval (0,1) and weights belong in the interval (-1,1). Figure 1 shows a representative diagram of a Fuzzy Cognitive Map (FCM). The sign of each weight represents the type of influence between concepts. There are three types of interconnections between two concepts C_i and C_j :

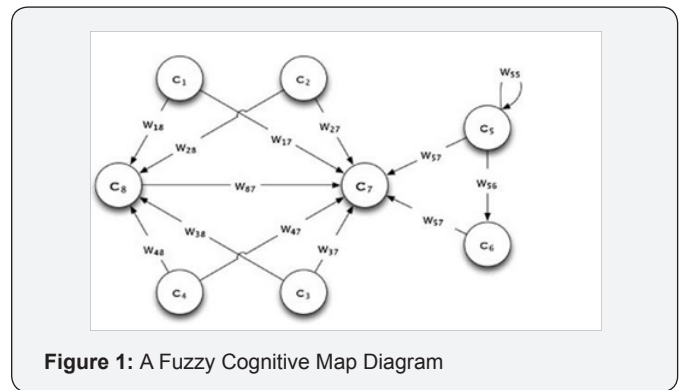


Figure 1: A Fuzzy Cognitive Map Diagram

- I. $w_{ij} > 0$, an increase or decrease in C_i causes the same result in concept C_j .
- II. $w_{ij} < 0$, an increase or decrease in C_i causes the opposite result in C_j .
- III. $w_{ij} = 0$, there is no interaction between concepts C_i and C_j .

The degree of influence between the two concepts is indicated by the absolute value of w_{ij} . The value of each concept at every simulation step is calculated, computing the influence of the interconnected concepts to the specific concept, by

applying the following calculation rule:

$$A_i(k+1) = f(k_1 A_i(k) + k_2 \sum_{j=1}^N A_j(k) W_{ji}) \dots\dots\dots(1)$$

where k represents time, N is the number of concepts and

$A_i(k+1)$: The value of the concept C_i at the iteration step

k+1

$A_i(k)$: The value of the concept C_i at the iteration step k

W_{ij} : The weight of interconnection from concept C_i to concept C_j

k_1 : the proportion of the contribution of the previous value of the concept in the computation of the new value

k_2 : the influence of the interconnected concepts in the configuration of the new value of the concept A_i

f : the sigmoid function

$$f = \frac{1}{1 + e^{-\lambda x}} \dots\dots\dots(2)$$

where $\lambda > 0$ determines the steepness of function f. The FCM's concepts are given some initial values which are then changed depending on the weights; the way the concepts affect each other. The calculations stop when a steady state is achieved, the concepts' values become stable. In most applications k_1 and k_2 are set equal to one (1).

One major drawback of the early FCM approach has been the convergence problem of the algorithms. Given the values of the initial values of the weights at least two problems have been observed:

- a. always the final values of the weights converge to the same value regardless the original conditions of the system and
- b. in some cases the algorithms do not converge at a final steady state value. In order to overcome these two convergence problems learning algorithms are used.

The main ideas stem from neural networks. Unsupervised methods such as Hebbian techniques are the most common used. More specifically Nonlinear Hebbian Learning (NHL) has been used to overcome partially this drawback [26].

In this learning algorithm the nodes are triggered simultaneously and interact in the same iteration step with their values to be updated through this process of interaction. The algorithm which modifies the initial weights defined by experts is described by the following relationship:

$$f = \frac{1}{1 + e^{-\lambda x}} \dots\dots\dots(3)$$

Where, the coefficient g called weight reduction learning parameter and the coefficient h is a very small positive scalar factor also called learning parameter. The "learning

parameters" g and h of the above equation are very important and they usually take values between $g \in [0.9, 1]$ and $h \in [0, 0.1]$. However these values are to be further investigated as to who and how are determined. The weights w_{ij} are updated for each iteration step and they are used in equation (1) in order to compute the new values of concepts. Two stopping criteria terminate the procedure. The first one concerns the minimization of function F1 which is the sum of the square differences between each Desired Output Concept i (DOC_i) and a target value T_i . T_i is defined as the mean value of the range of

$$DOC_i = [T_i^{min}, T_i^{max}]$$

$$F_1 = \sqrt{\sum_{i=1}^m (DOC_i - T_i)^2} \dots\dots\dots(4)$$

$$T_i = \frac{T_i^{min} + T_i^{max}}{2} \dots\dots\dots(5)$$

The second criterion is the minimization of the variation of two subsequent values of Desired Output Concepts:

$$F_2 = |DOC_i^{(k+1)} - DOC_i^{(k)}| \dots\dots\dots(6)$$

When the termination conditions are met the new final weight matrix w_{ij} with the DOCs are returned. More on other drawbacks and proposed solutions of the up today theories of FCMs are given in the next section. A more comprehensive mathematical presentation of FCMs theories, methods and algorithms is provided in [19-21] and [26-36]. The reader is encouraged to consult these references.

Two Illustrative Examples

Example 1 modelling hybrid energy systems using FCMs

The energy sector worldwide faces evidently significant challenges that everyday become even more acute. Innovative technologies, Renewable Energy Sources (RES) and energy efficiency measures are nowadays well known and widely spread. The main issue is to identify those solutions that will be proven to be the more effective and reliable in the long term. One problem that seeks solution is the optimal mix of RES to produce energy. In this example a Hybrid Energy System combining Wind and Photovoltaic is modeled using FCMs Figure 2.

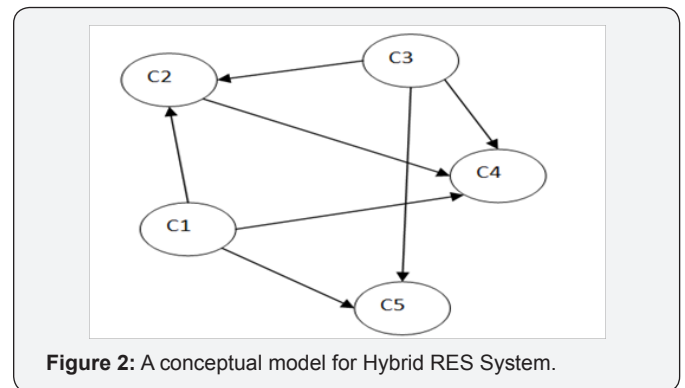


Figure 2: A conceptual model for Hybrid RES System.

In this example the following 5-concepts are considered:

- i. C1: sun insolation (kWp/m²)
- ii. C2: environment temperature
- iii. C3: wind
- iv. C4: PV-System
- v. C5: Wind-Turbine-System

In the proposed model there are two RES decision concepts (outputs), i.e. the two renewable energy sources are studied: concept 4 PV-System and concept 5 Wind-Turbine-System. The factor concepts are considered as measurements (via special sensors) that determine how each RES will function in this model and they are: Figure 2.

Table 1: Weights between concepts for FCM for Hybrid RES System.

	C1	C2	C3	C4	C5
C1	0	0.2	0	0.8	0.1
C2	0	0	0	-0.3	0
C3	0	-0.2	0	0.2	0.7
C4	0	0	0	0	0
C5	0	0	0	0	0

The connections between the concepts are determined from Table1. Detailed information for hybrid renewable energy systems are given in [11,14]. One case study from the literature is examined here concerning the decision making approach of hybrid renewable energy source system. In Table.2 the initial factors used by the model are presented. In addition, the degree of occurrence of each factor is denoted with qualitative degrees of very very high, very high, high, medium, low, very low, and 0 for insolation (C1), low, medium, high and very high for temperature (C2) and for wind (C3). Respectively for the output concepts C4, C5 the qualitative degrees are low, medium and high. C4 and C5 are considered a % percentage of the maximum performance at STC conditions.

Table 2: Initial factor-concepts fuzzy value

Factor-concepts	Case 1
C1	VVH
C2	M
C3	M

Table 3: Final decision-concepts

Decision-Concepts	Case 1
C4 (PV-System)	0.7749
C5 (Wind-Turbine-System)	0.7885

Case_1 (without training algorithm): The initial values of the outputs were set equal to zero while the final decision concepts are given in Table 3. The iterative procedure is being terminated when the values of Ci concepts has no difference between the latest two iterations. Considering $\lambda=1$ for the unipolar sigmoid function after N=10 iteration steps the

system reaches an equilibrium point.

We considered initial values for the concepts after COA defuzzyfication method:

$$\hat{A}((0)) = [0.90 \ 0.3334 \ 0.3334 \ 0.839 \ 0.459]$$

The fuzzy rule which is being considered for the calculation of the initial condition of the output concepts C4, C5 follows:

If C1 is VVH and C2 is M and C3 is M Then C4 is VVH and C5 is M;

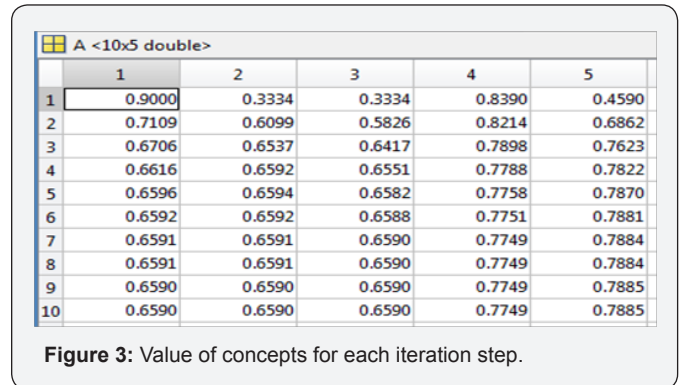


Figure 3: Value of concepts for each iteration step.

(Figure 3) It is observed that in the latest three iterations there is no difference between the values of concepts Ci. So after 11 iteration steps, the FCM reaches an equilibrium point where the values do not change any more from their previous ones, that is: (Figure 4)

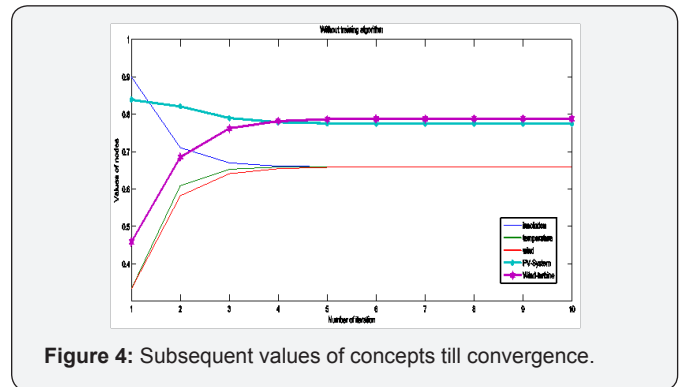


Figure 4: Subsequent values of concepts till convergence.

$$A^{(11)} = [0.6590 \ 0.6590 \ 0.6590 \ 0.7749 \ 0.7885]$$

Case_2 (with nonlinear hebbian training algorithm):

Firstly the experts suggested us a desired region where the decision output concepts (DOCs) should move. The desired regions for the output nodes reflect the prospered operation of the modeled system.

$$0.72 \leq [DOC]_4 \leq 0.83$$

$$0.72 \leq [DOC]_5 \leq 0.85$$

Basic factor of the NHL algorithm is the minimization of two basic criterion functions in order to have a convergence after a finite number of iteration steps [16,17]

$$F1 = J1 = \left\| [DOC_i - T_i] \right\|_2^2 < 0.005$$

$$F2 = J2 = \left| [DOC_i^{(k)} - DOC_i^{(k-1)}] \right| < 0.005$$

and

Where T_i is the hypothetical desired value of the output and it is usually the average of the defined range by the experts. Basic idea of the NHL algorithm is the adaption of the initial $w_{ij}^{initial}$ matrix weights defined by the experts in a way that the Decision Output Concepts converge inside the desired reason. Each non-zero element of the final weight matrix w_{ij}^{final} final has been improved and has been converted into an optimal value according to the specific criteria functions of the problem. We could define an acceptable range of change for these weights. If one weight takes a value out of the desired regions then we should consider it better whether experts' suggestion of the initial value of the weight is correct or not, i.e.

if $|w_{ij}^{final} - w_{ij}^{initial}| < l$ then that means that concept C_i has a different relationship (than the initial defined from the experts one) with concept C_j and their correlation should be reevaluated.

Defining the initial values of the concepts:

$$\hat{A}^{initial} = [0.90 \ 0.3334 \ 0.3334 \ 0.839 \ 0.459]$$

The following final values are obtained and after only 7 iterations:

$$\hat{A}^{final} = [0.6592 \ 0.6753 \ 0.6588 \ 0.78300.7923]$$

It is observed that the values of the concepts $C_4=0.7830$ and $C_5=0.7923$ in the final state are inside the suggested desired regions. Thus the proposed model for the Hybrid energy system using FCMs proves that is a very useful approach and a tool that can be used for more complex Hybrid energy system.

Example 2: Decision Making in Stability of an Enterprise in a crisis period using FCMs

A simple example of Decision Making for the Stability of an Enterprise in a Crisis Period using FCMs can show that the new approach of FCMs in modelling CDS is very promising.

In the current FCM model there is only one decision concept (output), i.e. the stability of an enterprise in a crisis period is studied: concept_8. The factor concepts are considered as measurements (via special statistic research) that determine how each measurement-concept will function in this model and they are: C1: sales, C2: turnover, C3: expenditures, C4: debts & loans, C5: research & innovation, C6: investments, C7: market share, C9: present capital, while C8: stability of enterprise is the output of the system. Studying this simple problem using classical business approaches is not very easy.

Most methods are heuristic not been able to give a concrete answer as is the case using FCM as it will be shown in the following paragraphs. Figure 5 shows a simple FCM model for the enterprise system. At this point it should be noted that in economic systems we can't talk about causality but only for correlation between the defined factor-concepts of this problem. Experts noted that the acceptable-desired region for the final value of concept C8 is:

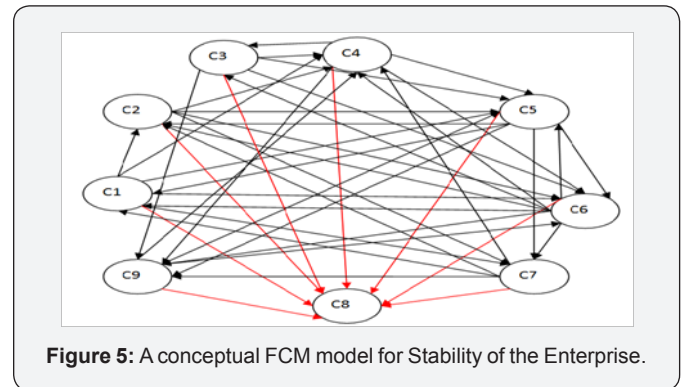


Figure 5: A conceptual FCM model for Stability of the Enterprise.

$$0.70 \leq C_8(\hat{final}) \leq 0.95$$

If $C_8^{(final)}$ is inside this region then we can say with great certainty that the enterprise is out of danger and the economic crisis period does not put at risk the stability and the smooth function of the enterprise. Weights in Table 4 are determined after defuzzifying (with COA method) the fuzzy values that were given from the experts (mostly economists).

Table 4: Weights between concepts for Enterprise Syste.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	0	0.6	0	-0.4	0.2	0.3	0.6	0.8	0
C2	0	0	0	-0.2	0.2	0.5	0.1	0.3	0
C3	0	0	0	0.4	-0.5	-0.4	0	-0.6	-0.5
C4	0	0	-0.4	0	-0.7	-0.8	0	-0.7	-0.4
C5	0.2	0.3	0	0	0	0.5	0.3	0.2	-0.2
C6	0.3	0.2	0.6	0.5	-0.3	0	0.3	0.3	-0.4
C7	0.4	0.3	0	-0.2	0	0	0	0.4	0.5
C8	0	0	0	0	0	0	0	0	0
C9	0	0	0	-0.3	0.2	0.4	0	0.2	0

(Figure 5) In addition, the degree of occurrence of each input-concept factor is denoted with qualitative degrees of high, medium, and low. Respectively for the output concept C8 the qualitative degrees are very low, low, medium, high and very high Table 5.

Table 5: Initial factor-concepts fuzzy value.

Factor-Concepts	Case 1
C1	H
C2	M
C3	L
C4	L
C5	M
C6	L
C7	L
C9	M

The iterative procedure is being terminated when the values of C_i concepts has no difference between the latest three iterations. Table 6 considering $\lambda = 1$ for the unipolar sigmoid function and after 11 iteration steps the FCM reaches an equilibrium point.

Table 6: Final decision-concepts.

Decision-concepts	Case 1
C8 (Stability of the Enterprise)	0.8391

We considered initial values for the concepts:

$$\hat{A}((0)) = [0.8867 \ 0.4667 \ 0.0967 \ 0.0967 \ 0.4667 \ 0.0967 \ 0.0967 \ 0.65 \ 0.4667]$$

It is observed that in the latest three iterations there is no difference between the values of concepts C_i . So after 11 iteration steps, Figure 6, the FCM reaches an equilibrium point where the values do not change any more from their previous ones, that is:

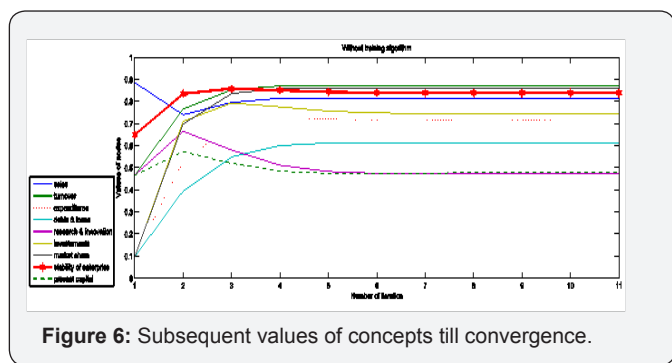


Figure 6: Subsequent values of concepts till convergence.

$$\hat{A}((0)) = [0.90 \ 0.3334 \ 0.3334 \ 0.839 \ 0.459]$$

Since the final value of $C_8^{(Final)}$ is inside the acceptable region, defined by the experts, then we could assume with great certainty that the enterprise can survive the crisis period. This simple business example shows clearly how concrete and effective answers can be obtained using FCMs when analyzing

complex dynamic systems (CDS). This is especially true when experts are consulted and provide some statistical information for the financial status of a company or a country. The number of concepts in these cases is rather large and classical heuristic models cannot predict the dynamic behavior of these systems. On the other hand the new models of FCMs, as been proposed in [37-40] provide useful solutions.

Some Drawbacks and New Challenges for Fuzzy Cognitive Maps

In the previous sections trying to answer all challenging problems and questions in modelling and controlling CDS, FCMs were proposed as a new alternative and innovative approach to deal with these fundamental issues. The mathematical presentation along with the two examples from real problems give the academic and scientific communities hopes for overcoming some of the problems been encountered in modelling and controlling CDS using the classical approaches. However, with the current modeling of FCMs and their extensive use in solving many real problems and applications, various interesting and challenging problems and drawbacks have emerged.

The early hope and enthusiasm that FCM would be a strong and effective approach to be able to solve the difficult problems of the complex dynamic systems. However FCMs theories are around only less than 30 years. We should not give up now. On the contrary these drawbacks should make us more determined to find ways to improve the theories of FCMs. We must re-address the basic fundamentals that drove the scientific and academic community to develop FCMs, while on the same time keeping the core of the “initial philosophy and methodology” intact. Artificial Intelligence (AI) has been around more than 70 years and despite the skepticism of some well known scientists AI still continues strong on funding and research efforts. Prof Stephen Hawking, one of Britain’s pre-eminent scientists, has said that efforts to create thinking machines pose a threat to our very existence.

He told in an interview the BBC: “The development of full artificial intelligence could spell the end of the human race.” His warning came in response to a question about a revamp of the technology he uses to communicate, which involves a basic form of AI. The theoretical physicist, who has the motorneurone disease amyotrophic lateral sclerosis (ALS), is using a new system developed by Intel to speak using AI. Prof Hawking is not alone in fearing for the future. In the short term, there are concerns that clever machines capable of undertaking tasks done by humans until now will swiftly destroy millions of jobs. In the longer term, the technology entrepreneur Elon Musk has warned that AI is “our biggest existential threat”. Many believe that AI is our biggest scientific mistake especially if it is left to scientists and politicians that do not have the human been as

the center of our activities and/or they do not respect human values.

Therefore re-addressing the FCMs we should be careful and wise. Let us restate what is a FCM and why we claim that is a promising and innovative method for studying CDS. A Fuzzy Cognitive Maps (FCM) provided a way to identify the most important structural elements in modeling and controlling complex dynamic systems (CDS). Complete, efficient and practical mechanisms to analyze and predict the evolution of data been fuzzy, incomplete, and vague in CDS were not available for years due to several reasons. Numerical data for years have been considered as crisp and exact values. However today most data may be fuzzy, uncertain or hard to come by, and the formulation of a precise mathematical model may be difficult, costly or even impossible.

Then efforts to introduce knowledge on these systems should rely on natural language arguments and the human intervention in the absence of formal models. However, although very efficient and simple to use, FCM are causal maps (a subset of cognitive maps that only allow basic symmetric and monotonic causal relations between concepts), and, in most applications, avoiding the need to use extensive and time consuming differential equation models, while obtaining very interesting and encouraging results. By using true qualitative modelling techniques, FCM obtained results that look more realistic (plausible) than those obtained using quantitative approaches. Where results almost never show the short term uncertainties that are so characteristic of qualitative real-world dynamic systems.

In the end, the results of the FCM model and all related applications, that was developed more than 25 years ago, are surprisingly realistic and could have been used to predict and avoid the current world economic crisis, even if one considers its necessary incompleteness. An FCM is a qualitative mathematical tool rather than a quantitative tool. It provides a simple, flexible and straightforward approach to model the dynamic behavior of a complex system, which is composed of various components or subsystems. An FCM can always describe any CDS using a mathematical model with the following six (6) characteristics or attributes:

- a. Defined causality indicating positive or negative relationship between all components.
- b. The strength of the causal relationships always takes fuzzy values.
- c. The causal links are always dynamic and never static.
- d. Past knowledge of the CDS dynamic behavior is available and reliable.
- e. Human-like reasoning.

- f. Always availability of experts knowing the dynamic behavior of the CDS.

Given that the above hold and the FCM methodologies, so far been developed, we can model any given CDS. However we discussed the problem of convergence in section 4.4. Solutions to this drawback were also provided there. There also other drawbacks of FCM that need to be addressed and been solved. These solutions are going to overcome some of these limitations and offer more accurate results and a better view and knowledge of the CDS. One major drawback that has been raised by the author of this article is that concepts of an FCM include everything: states, inputs, outputs, constraints and all other parameters which are going to be examined regardless their nature [37]. However this is not mathematically correct and logical in any scientific approach. Why, for example in classic FCM theories the fact that some concepts are not being affected by others thus they have to stay static through the whole iteration process.

However due to the current approach, Equation 1 and Equation 2 their value changes after the first iteration which is not correct. In addition having all variables in one "concept vector" the iteration step k in Equation. 1, is the same for all concepts which is also not truth in the real problems and mathematically not correct. Why the inputs and outputs of the CDS must change at every iteration step k ? For example on a health treatment of a patient, why the inputs (concepts) (e.g. the medication dose of a drug that is given every morning) and the outputs (e.g. the blood test results (concepts) that are monitored every two or three days) must be change every time the "health conditions" (concepts) of the patient is monitored every second or every hour? However this is the case using classic FCM theories especially using equations 1 and 2. Even the calculation method of the values of the concepts, (Equation 1) has a serious problem-drawback. The calculation equation takes into consideration the change that each concept causes separately instead of the total change which is caused to the concept C_i .

This results in a large increase to the value of the concept C_i that goes far beyond the interval $(0, 1)$. This is the reason why the sigmoid function (Equation 2) is needed; to suppress the result to the interval $(0, 1)$. However due to the shape of the sigmoid curve any concept value beyond 3 leads the sigmoid function to correspond it to the value 1 which is greatly problematic as the final output is corresponded to the linguistic variable "high" even if this is not always the expected or correct result.

Continuing with the NHL learning method (Equation3), while running several simulations we have observed that due to the way weights are being calculated if the number of iterations of the algorithm is increased, in order to reach a steady state, the causality reverses and all or some of the W_{ij}

become positive. This is a very serious drawback as it changes the causality between concepts and in several occasions instead of having a lower we are going to have a larger result which can cause serious problems not only in the interpretation of the obtained results but also on stability issues to a number of systems. Another drawback with equation (3) is that the weights W_{ij} are modified for all concepts. This is also not mathematically correct and need further investigation after the separation of the total N concepts into states, inputs and outputs as is shown below

There are a number of other drawbacks that need to be addressed but this should be the work of future papers. However for the above raised drawbacks some solutions and explanations must be given. The research teams of the Laboratory for Automation and Robotics under the supervision of the author of this paper have provided some interesting and valuable solutions [37-40].

As it was mentioned, above in this section, in the classic FCM representation ALL the concepts are ALL the parameters which are going to be examined regardless their nature. However, in a CDS, even when it is described in a fuzzy way through an FCM the main concept is the same. Each system has its states, inputs, outputs and other parameters and constraints. However since an FCM is a representation of such a system, this fundamental characteristic should be taken into consideration. For this reason, as in the classic control theory methods [15-17], the concepts of a Fuzzy Cognitive Map are separated into the following three categories:

- I. Fuzzy State Concepts: The concepts describing the dynamic operation of the system, x
- II. Fuzzy Input Concepts: The inputs of the system, u
- III. Fuzzy Output Concepts: The concepts describing the outputs of the system, y

In this way a better knowledge of the dynamic behavior of the CDS is gained. The proposed separation facilitates not only the understanding of the system's operation but also the calculation of the concepts' values in their physical nature as the states, inputs and outputs of the real system. The conceptual formulation was first proposed by the author in [37]. Then in a second effort in [38] a revised approach in modelling Fuzzy Cognitive Maps was presented in details. The new model was composed by the following equations:

$$x_{k+1} = Ax_k + Bu_k \dots\dots\dots(7)$$

$$y_k = Cx_k + Du_k \dots\dots\dots(8)$$

They were used to calculate the variation caused by the change in the input and state concepts to the state and output concepts at each time step (k). In this representation A, B, C and D are individual weight matrices derived from the initial

weights defined by the experts [38]. Each weight matrix has the appropriate dimensions depending on the (A-C) categories of the total number N of concepts [20]. The elements of A depend on the states weights and the elements of B show how each input concept affects the state concepts of the system. C shows how the output concepts are related to the state concepts and D shows how the input concepts directly affect the output concepts. In the same paper a new sigmoid function f is proposed [38] and is:

$$R(x) = \left\{ \begin{array}{l} 0, x < 0.5 \\ \frac{x-0,5}{0,5}, x > 0.5 \end{array} \right\} \dots\dots\dots(9)$$

Another attempt to address some drawbacks of today s FCMs theories and methods has been made in [39]. After the implementation of the classical FCM method on various applications and without using any learning algorithm, it has been observed that for a FCM with determined and constant weight matrix the use of Equation 1 and Equation 2 lead to the same output value no matter what the initial concept values are. Starting from this observation, and looking for a solution to this problem, it was estimated that, apart from the initial concepts' values, the initial disturbances are necessary in order to calculate the system output. A different disturbance would force the system to reach a different equilibrium point because it has a different impact on the system response.

Taking the above reasoning as a basic idea, the classical FCM equation, Equation 1, and the weight values' explanation in section 4.4 a new conception, was suggested in this paper, [39]. In section 4.4 it was mentioned that " If $w_{ji} > 0$ this means that an increase of the C_j value will cause an increase of C_i value." This means that in order to have a change in the C_i concept value, there should be a disturbance on the value of C_j , in other words there should be a change, DC_j . The new model suggested to be used in order to calculate the new concept values, A [39] is.

$$A_i(k+1) = A_i(k) + \sum_{j=1}^N (DA_j(k)W_{ji}) \quad (10)$$

Where

$$DA_j(k) = A_j(k) - A_j(k-1) \quad (11)$$

In the same paper [39], the sigmoid problem was also studied and a new sigmoid was proposed which is the same of the study [38] and is equation (9), given above. Please note that in study [39] the concepts remain all in the same vector. This method is applied on zero energy buildings. In this paper the concept values are described as A and not as C which is the case in study [39]. On the other hand on studies [37, 38,40] the concepts are separated to states, inputs and outputs. Thus in study [40] the problem of separating the concepts that are separated into states, inputs and outputs is addressed in

more details. A new approach of Dynamic Fuzzy Cognitive Knowledge Networks (DFCKN) is presented.

The two equations extracted from the classic FCM are the followings:

$$x(k+1) = f[Ax(k) + Bu(k)] \quad (12)$$

$$y(k) = f[Cx(k) + Du(k)] \quad (13)$$

Where $x(k) \in R^n$ is a state vector, $u(k) \in R^r$ is an exogenous known input vector, $y(k) \in R^m$ is the output vector and f is an activation function. The new model was implemented for first time in diagnosing meniscus injury in IFAC World Congress 2017 with very encouraging results [40]. This is an evolutionary type of Fuzzy Cognitive Maps (FCM) that arose from the need for updating classic methodology in order to overcome its drawbacks, concerning the single calculation rule equation (1), updating the weights, equation (3), stability and other real time problems. The new proposed (DFCKN) model is able to diagnose knee injuries and specifically meniscus injuries in a very simple way and to distinguish between acute and degenerative injury.

This new approach was tested for its accuracy in Decision Support Systems (DSS) in medicine by considering 17 real cases of patients. Subsequently we observe the evolution of the injury by administering a proposed treatment by the physician. Results of this new method, which are presented in detail, in [40] are very satisfactory for both two levels and treatment stage, and in total agreement with Magnetic Resonance Imaging (MRI) outcomes. The whole methodology is the outcome of a close collaboration between engineers and medical doctors and is significant because it is a promising tool which sets aside the main disadvantages of Fuzzy Cognitive Maps and allows us a wide use in many real time problems.

Another interesting problem and in some way a deficiency of today's FCM theories is the causality notion. The values of the weights W_{ij} for the interconnection between the concepts express the kind and degree of causality. Now is this related to the statistical correlation coefficient? And if yes how? Not an easy question to be answered. It would require a whole new paper. The research team of LAR is investigating this difficult problem. My own scientific feeling says that correlation does not necessary implies causality while the reverse is true. Thus causality always implies correlation.

Conclusion and Future Research

In this paper one of the most difficult and challenging problem in modelling, analyzing and controlling complex dynamic systems (CDS) has been seriously addressed. The analysis and efficient control of CDS are impossible without a formal model of the system. However today's technologies for building such models for CDS are not sufficient. Qualitative

description of most of the parameters of complex dynamic systems results inevitably in fuzziness, complexity, ambiguity and uncertainty. One of the challenges of accepting the "operation" of any complex dynamic system is the ability to make Decisions so the system runs efficiently and cost effectively.

It was shown that new conceptual and innovative approaches are needed. It is absolutely necessary to accept that Knowledge is the one and only one that can lead us in developing such models. And this knowledge must come from more than one expert who has extensive experience in observing and working on today's CDS. Decisions must be made by new Decision Making Support Systems (DMSS) which utilize new advanced and intelligent systems. Such a new approach is proposed to be Fuzzy Cognitive Maps (FCMs). FCMs offer the opportunity to produce better knowledge based on systems applications, addressing the need to handle uncertainties, fuzziness and inaccuracies associated with real CDS's problems. The illustrative examples have been provided in this paper and the obtained results are promising for future research efforts in this exciting field of research. However today's FCM theories, methods and algorithms have a number of drawbacks and deficiencies.

Some of them have been addressed in this paper and the related studies that the research team of the Laboratory for Automation and robotics has so far provided, are properly referenced. The reader is advised to look into these studies. Finally for the first time so many challenging questions have been raised for developing FCM models and theories when modelling, analyzing, controlling and optimizing complex dynamic systems. With so many questions been raised on this paper challenging future research directions include: formulate mathematically better the proposed separation of the concepts into states, inputs and outputs; based on this separation investigate the learning algorithms; generate new models of FCMs for CDS using learning methods; develop new DMSS using intelligent systems and advanced neural network theories; develop mathematical models using new advance FCMs for different applications and using a number of experts.

How causality is related to the statistical correlation coefficient; develop new software tools for various CDS and perform extensive simulations using real data from a large number of applications. Another interesting future research direction is to study and investigate control issues of Complex dynamic systems. There is very little being done on this control issues. Today's FCMs are more for modelling and making decisions for complex dynamic systems. FCM theories do not know the term and role of feedback control for complex dynamic systems. Finally there is a need to combine Intelligence and Cognition to a unified theory of Intelligent Cognitive Control (ICC).

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