**Robust Control Strategy of Human Assistant Robot**

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**Abstract**

Human assistant robot is a humanoid type robot which consists of a body with waist joint, arms designed for safety and a wheeled inverted pendulum mobile platform. The equation of motion is derived considering the non-holonomic constraint of the two-wheeled mobile robot with parameter uncertainties. A robust state feedback control method is applied for basic mobile controls wherein the control gain is calculated by linear quadratic control with proportional plus integral gain (LQRI). Through several experiments of balancing, linear running and steering, it was confirmed that the robot could realize stable mobile motion in a real environment by the developed controller.

**Keywords:** Human assistant robot; Robust LQRI design; LMIs

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**Introduction**

Recently, robots have been used in many applications including industry and human ordinary life. Robots can make the human life easier by doing the dangerous works such as working in a toxic environment or by doing the hard works such as carrying heavy objects. In addition, robots can add more precision and repeatability in many tasks such as welding and assembly tasks. Human assistant robot (HAR) is a humanoid type robot which consists of a body with waist joint, arms designed for safety, and a wheeled inverted pendulum mobile platform. Although the arms are designed at low-power and lightweight for safety, it is capable to perform tasks that require high power by utilizing its self-weight, which is the feature of a wheeled inverted pendulum mobile platform.

HAR is modeled as a three dimensional robot; with controls of inclination angle, horizontal position, and steering angle to achieve high mobile capability. The motion equation is derived considering the non-holonomic constraint of the two-wheeled mobile robot. Wheeled inverted pendulum is considered highly unstable robot, thus achieving the stability is the key that controller approaches are designed for. Two wheeled mobile robot balancing controller has been tackled as a linearized model [1] or as a nonlinear model [2]. Another work tackled the nonlinear disturbance [3] by developing a nonlinear observer. In [4] a two wheeled mobile robot is designed and a comparison between the linearized and nonlinear model is done experimentally. An MPC controller resented by [5] was designed to maintain the robot at balance.

One of the key challenges in using robots is the uncertainty. This uncertainty can be in the robot parameters such as its mass, inertia and the friction factors or in the sensors readings such as the robot speed and inclination angle. And also uncertainty can be in the working area such as the carried object mass, inertia and size. The challenge is how to increase the performance of such robots with the existence of this uncertainty. By developing more robust and fast controller algorithms can tackle uncertainty [6-8].

In this paper, we consider the modeling and robust control design of the human assistant robot and generate an appropriate linearized model with parametric uncertainties. Then we develop a robust control design method based on linear quadratic control with integral term (LQRI) design and establish an LMI formalism yielding a proportional plus integral state-feedback gain. The design procedures are cast into the format of feasibility problem over linear matrix inequalities (LMIs). By this way, effective computational method is established yielding guaranteed quality solution. Simulation studies are performed on experimental setup and the ensuing results yield good performance [9,10].

**Human Assistant Robot**

Figure 1 shows the case study HAR and it is consisting of two arms with one elbow for each arm which can be used to maintain stability of the robot in further research and two gyro sensors specified to define the acceleration and the inclination.
angle relative to the flat surface of the earth. Two wheels are
for movements in two degree of freedom.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>( M_g )</td>
<td>25.26 kg</td>
</tr>
<tr>
<td>( m_g )</td>
<td>125 kg</td>
</tr>
<tr>
<td>( \xi_s )</td>
<td>0.4005 m</td>
</tr>
<tr>
<td>( b )</td>
<td>0.16 m</td>
</tr>
<tr>
<td>( r_w )</td>
<td>0.1 m</td>
</tr>
<tr>
<td>( l_s )</td>
<td>1.408 kg m²</td>
</tr>
<tr>
<td>( l_w )</td>
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</tr>
<tr>
<td>( l_{ax} )</td>
<td>0</td>
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<tr>
<td>( l_{vd} )</td>
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</tr>
<tr>
<td>( g )</td>
<td>1</td>
</tr>
<tr>
<td>( c_r )</td>
<td>0.1 N m/s/ rad</td>
</tr>
<tr>
<td>( c_x )</td>
<td>0.1 N m/s/ rad</td>
</tr>
<tr>
<td>( G )</td>
<td>9.81</td>
</tr>
</tbody>
</table>

Table 1: Numerical Data.

A model of pendulum robot is built on two dimensional models with steering angle created from rotation and inclination angle caused by linear movements, see Figure 1. In addition, linear position of coordinates. The origin of vehicle coordinates \( \Xi_v \) is located at the midpoint on the line that connects the axes of the two wheels, and the origin of the body coordinates \( \Xi_b \) coincides with the \( \Xi_s \). The angle between \( z_h \) and \( 2\psi \) represents the inclination angle of the body \( \psi \), and the angle between \( x_v \) and \( \chi_0 \) axis of the global coordinates represents the steering angle \( \phi \). In all the coordinates, CCW direction is positive. The parameters in the model are described in Table 1 in the appendix.

Following the work of [11-16], the nonlinear robot model \( x = f(x,u,w) \) can be conveniently cast into the uncertain linearized model:

\[
x(t) = (A + \delta A)x(t) + (B + \delta B)u(t) + \Gamma w(t) \quad \text{......... (1)}
\]

\[
z(t) = Cx(t) + Du(t) + \Phi w(t) \quad \text{......... (2)}
\]

\[
y(t) = Cx(t) + \Psi w(t) \quad \text{......... (3)}
\]

Where \( x(t) \in \Re^n, u(t) \in \Re^m, y(t) \in \Re^p \) are the state, the control input and the measured output vectors and \( \delta A, \delta B \) accounts for the parametric uncertainties and for convenience they are expressed as:

\[
[\delta A \delta B] = H_i F [E_j E_k], F F I \leq 1 \quad \text{......... (2)}
\]

The state variables are \( x_v, \Phi, \Psi, x_v, \Phi, \Psi \) where \( x_v \) is the linear position of the vehicle (m) \( \phi \) is the inclination angle of center of gravity (CoG) (rad) \( \psi \) and \( \chi_0 \) is the steering angle (rad). The matrices \( H_i, E_j, E_k \) are selected to reflect the amount of deviation in model parameters from nominal levels. Observe that modeling errors and/or parametric uncertainties are incorporated in (1) to reflect some practicality in replacing the nonlinear model \( x = f(x,u,w) \).

By quasi-linearization at appropriate operating model \( x = f(x,u,w) \), the matrices \( A, B, C \) are given by:

\[
\begin{align*}
A &= \begin{bmatrix} 0 & I \\ A & B \end{bmatrix} C C \Gamma \quad \text{......... (3)}
\end{align*}
\]

Where the parameter definitions and values are given at the end.

Control Design

In control system terminology, the problem under consideration is that of determining feedback controller that makes system (1) stable over a wide range of operation while achieving a prescribed performance measure. Next, we provide design techniques to achieve this goal.

LQR: Proportional-integral gain

A formulation of the performance criterion is considered which contains the standard linear-quadratic regulator (LQR) plus a term representing the integral of the deviation of the output from its initial state \( z(t) = \int_0^\infty y(t)dt \). This formulation will be referred to as the LQRI.

\[
J = \int_0^\infty (y(t) y(t) + \rho u(t) u(t) + \sigma z(t) z(t))dt 
\]

Treating \( z = C \chi \) as additional state variable, we define

\[
\eta = \begin{bmatrix} x \chi \end{bmatrix}
\]

The augmented system becomes

\[
\begin{align*}
\dot{\eta} &= (A \quad 0) \eta + (B \quad 0) u \\
\chi &= (C \quad 0) \eta = \hat{C} \eta \\
\end{align*}
\]

And hence we can re-write (4) as:

\[
J = \int_0^\infty \left( \begin{bmatrix} y(t) & y(t) \end{bmatrix} \begin{bmatrix} y(t) & y(t) \end{bmatrix} \right) \begin{bmatrix} \rho & 0 \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} y(t) & y(t) \end{bmatrix} dt 
\]

\[
\hat{Q} = \begin{bmatrix} CC & 0 \\ 0 & \sigma I \end{bmatrix}, \hat{R} = \rho I, \rho, \sigma > 0
\]
Next, we present an LMI-based formulation to the LQI control of system (5) while minimizing the quadratic cost (6). We proceed to determine a linear optimal control $u = L x$ that achieves this goal.

Assumption 1: There exists a Lyapunov functional $\tilde{V}(x)$ which has the properties:

- $\dot{\tilde{V}}(x) = \eta^T \dot{P} \eta + \gamma > 0$
- There exist $\gamma^+ > 0$ such that $\eta^T \dot{P} \eta \leq \gamma$
- $\tilde{V}(x) \leq \eta^T \tilde{Q} \eta + \gamma$

The following theorem provides an LMI-based LQRI design for system (5) with linear control $u = L x$

Theorem 1: Given matrices $Q > 0, R > 0$ and scalar $n > 0$, system (5) $w(t) = 0$ and the LQRI control $u = L x$ is robustly asymptotically stable for all admissible uncertainties $F^T F \leq I$ and $J \leq \tilde{V}(x)$ if there exist matrices $S, Y > 0$ such that

$$\begin{align*}
\min_{\gamma^+, \gamma^-, S, Y} & \gamma^+ + \gamma^- \\
\text{subject to} & \\
& \gamma^+ \geq 0 \\
& \gamma^- \leq 0 \\
& \eta^T \dot{P} \eta \leq \gamma \\
& \tilde{V}(x) \leq \eta^T \tilde{Q} \eta + \gamma
\end{align*}$$

Has a feasible solution, then the proportional plus integral (PI) gain matrix is $L = SY^{-1}$.

Proof: By Assumption (1), and using control $u = L x$ in system (5), the inequality of the derivative of the Lyapunov functional is expressed as

$$\dot{\tilde{V}}(x) = \eta^T \dot{P} \eta + \gamma > 0$$

It is evident that (8) is satisfied if there exists $\gamma^+$ and such that

$$\begin{bmatrix} A + B L & P \end{bmatrix} \begin{bmatrix} A + B L & P \end{bmatrix}^T \tilde{Q} \leq \gamma \begin{bmatrix} \dot{L} + R L \end{bmatrix}^T \begin{bmatrix} \dot{L} + R L \end{bmatrix} \quad \text{subject to} \quad \gamma \geq 0$$

Simple computations on (6) in view of Assumption (1) yields $J \leq \tilde{V}(x)$. By minimizing the upper bound $\gamma^+$ on the cost $\eta^T \dot{P} \eta$, we obtain

$$\begin{align*}
\min_{\gamma^+, \gamma^-, S, Y} & \gamma^+ + \gamma^- \\
\text{subject to} & \\
& \gamma^+ \geq 0 \\
& \gamma^- \leq 0 \\
& \eta^T \dot{P} \eta \leq \gamma \\
& \tilde{V}(x) \leq \eta^T \tilde{Q} \eta + \gamma
\end{align*}$$

To convexify the above problem, we define

$$\begin{bmatrix} H & E' \\ 0 & E \end{bmatrix} = \begin{bmatrix} E \end{bmatrix}^T \begin{bmatrix} E \end{bmatrix} \quad \text{and express (9) as}
$$

$$\begin{bmatrix} \Phi \dot{X} + R \dot{P} & \tilde{Q} \end{bmatrix} \begin{bmatrix} \Phi \dot{X} + R \dot{P} \\ \tilde{Q} \end{bmatrix}^T \begin{bmatrix} \Phi \dot{X} + R \dot{P} \end{bmatrix} \begin{bmatrix} \tilde{Q} \end{bmatrix}^T \begin{bmatrix} \Phi \dot{X} + R \dot{P} \end{bmatrix} \begin{bmatrix} \tilde{Q} \end{bmatrix}^T < 0$$

This can be manipulated to yield the other part of (7).

Figure 2: Human assistant robot experiment.

The values of the basic elements are presented in Table I. Using the lab capabilities at KFUPM, Figure 2 illustrates two-wheeled inverted pendulum robot experiment. It consists of two motor drivers, two motor, a 32-bit microcontroller, an inertial measurement unit (IMU), and two encoders. 3-axis gyro sensor and 3-axis accelerometer are installed in IMU sensor. To operate the robot as shown in Figure 2, IMU sensor precedes the data of gyro sensor and accelerometer by Kalman filter in order to estimate the angular velocity and tilt angle of the body. The encoder is detected the rotation of both wheels then they are converted to position and velocity in Cartesian coordinate frame. Then, position, velocity, yaw angle, yaw rate, tilt angle, pitch rate are used to compute by control law. It will...
send the command to motor drivers in order that the motor drivers produce electronic current and voltage for motors.

Several simulation experiments of balancing, linear running, and steering have been carried out using the stabilization method LQRI. The uncertainties are added to the mass and inertia of the robot. ±10% of its nominal values. The ensuing results are depicted in (Figure 3), from which it was confirmed that the robot could realize stable mobile motion in a real environment by the developed controller.

**Conclusion**

This article introduced the linearized dynamic model of two wheeled inverted pendulum robot subject to uncertain parameters, robust control design and an experimental result. The control technique focused on linear quadratic methods with proportional plus integral using linear-matrix inequalities. Improved analytical results are developed and simulation results are presented.

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**References**

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