Introduction

For many years, the trajectory tracking problem in mobile robots has been addressed by the research community [1-5]. One of the reasons is that the tracking control approach results more appropriate, since the nonholonomic constraints and other control tasks (obstacle avoidance, minimum travel time, minimum fuel consumption) are implicitly included in the path-planning procedure.

This paper presents a novel methodology to deal with the problem of trajectory tracking in WMR. This new approach uses the dynamic model of WMR and allows perfect velocities tracking. Generally, the classic control approaches applied to the dynamic model of the robot solve the problem of trajectory tracking in two stages. In the first one is designed a controller that ensures the perfect tracking of the velocities considering only the kinetic structure of the model. The second stage solves the problem of tracking velocities using the dynamics of the system [1,2]. In our work, the procedure arises naturally when looking for the conditions that a system must meet in order to have an exact solution. The control action is obtained by solving a linear system, even though the original system model is nonlinear. Compared with other literature methods (back stepping, look-ahead methods, finite-time techniques, etc.), only algebra knowledge is needed to understand and apply this methodology. The controller performance is evaluated and compared with other controllers of the literature through laboratory experiments in PIONEER 3DX mobile robot.

The paper is ordered as follows. In Section 2 are described the mathematical model of the mobile robot and the control design of the proposed controller. Section 3 presents the experimental results using a mobile robot Pioneer 3DX. Finally, Conclusions are detailed in Section 4.

Dynamic Model-Control Design

In this section is described the dynamic model used (1). It model has been used in several works of the literature [1,2]. This model allows that the linear and angular reference velocities are now considered as the input signals. These signals are usual inputs in commercial robots. The dynamic model of the mobile robot is given by:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi} \\
\dot{u} \\
\dot{\omega}
\end{bmatrix} = \begin{bmatrix}
ucos\psi \\
u\sin\psi \\
\omega \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
u_c \\
\omega_c
\end{bmatrix}
\]

Where \(u_c\) and \(\omega_c\) are the control action of the system, its represent the linear and angular velocity commands given to the robot. The variables \(\dot{x}, \dot{y}, \dot{\psi}, \dot{u}, \dot{\omega}\) represent the variations of the horizontal and vertical position, orientation and linear and angular velocities respectively. The identified parameters
\[ \theta = \left[ \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6 \right] \] of the dynamic model are functions of the physical parameters of the robot.

The aim is to find the values of the \( u \) and \( \omega \), such that the mobile robot follow a predefined reference trajectory \((x_r, y_r)\) with minimal error, for that is proposed a control law based on the parameterized dynamic equation of the model (1). Then a Proportional replacement \( \dot{x} = X_r + K e \) is proposed, where, \( X_r = [x_{r1}, y_{r1}, \psi_{r1}, \omega_{r1}, \dot{\psi}_{r1}, \dot{\omega}_{r1}]^T \). The variables in \( X_r \) are the derivatives functions of the desired trajectory to be followed, the elements of \( \psi, \omega \) and \( dX = \dot{x}, \dot{y}, \dot{\psi}, \dot{\omega} \) is the final time of the desired trajectory.

The experimentation is a test recommended in Roth & Batavia [7] and is formed by three different circle-shaped. The reference trajectory and the results of each controller are shown in Figure 1. As can be seen, all controllers reach and follow the desired trajectory. Figure 2 show the tracking error in the x-coordinate and y-coordinate for each controller. The errors for all controllers remains bounded and close to zero when the robot reaches the reference trajectory. The tracking cost calculated for each controller according to (6) is: \( C_1 = 0.92; C_2 = 1.41; C_3 = 4.19 \). As can be seen, the proposed algorithm \( C_1 \) present the lowest value.

### Experimental Results

The experiment was performed using a PIONEER 3DX mobile robot. The controller developed \( C_1 \) has been compared with two developments, \( C_2 \): presented in Scaglia et al. [4] and \( C_3 \): presented Cassius et al. [3]. \( C_1 \) was adjusted with \( k_1 = 1.1; k_2 = 1.07; k_3 = 0.92; k_4 = 1.98; k_5 = 2.06 \). \( C_2 \) and \( C_3 \) were implemented in the same PIONEER 3DX. For three controller the performance was evaluated using the cost functions defined in (6) for each experiment realized. Let \( \Phi \) be a desired trajectory, where \( t_f \) is the final time of the desired trajectory. Thus, the cost function can be represented by

\[ C^e = \frac{1}{2} \left( \int (x(t) - x_f(t))^2 dt + \int (y(t) - y_f(t))^2 dt \right) \]

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### Conclusion

A methodology based on numerical methods and linear algebra to controller design for mobile robot taking into account the nonlinear multivariable dynamic model has been presented. The methodology finds the control expressions that
minimize the tracking errors using algebraic techniques without requiring the linearization of the dynamic model. Thus, the controller is independent of the operating point. In addition, as the controller structure comes from the mathematical model of the system it can be implemented in much other system. The experimental test shows the good performance of the proposed controller.

References