



# Robust Adaptive Sliding Mode Control for Mobile Manipulators



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## Abstract

This paper proposes a robust adaptive sliding mode control for the trajectory tracking of a nonholonomic wheeled mobile manipulator in task space coordinate. The proposed algorithm is robust adaptive control strategy where parametric uncertainties and disturbances are compensated by adaptive update technique. The adaptive law is designed based on the Lyapunov method. Simulations results are given to illustrate the effectiveness of the proposed robust adaptive based controller in comparison with a robust sliding mode based controller.

**Keywords:** Hybrid sliding-mode fuzzy neural networks(HSMFNN); Nonholonomic; Lyapunov

## Introduction

A Mobile manipulator consists of a mobile platform carrying a standard robotic arm. Such robotic system merges the dexterity of the manipulator with the increased workspace capabilities of the mobile platform and thus is particularly suited for field and service robotic applications which require both locomotion and manipulation abilities. A number of papers have been presented to address the issue of the trajectory tracking of mobile manipulators, for example the nonlinear feedback control [1], computed torque control [2], decoupled task and null space dynamic control [3], input output decoupling control [4].

Most previous approaches require a precise knowledge of the dynamics of the system and ignore external disturbances. These issues make the proposed schemes inappropriate for realistic applications. For such case, adaptive and robust control can be used to handle these difficulties. The sliding mode control technique is a robust approach which has many attractive features such as the insensitivity property toward the parametric uncertainties and external disturbances. Because of these properties, sliding mode control has applications in robotics. In [5], a sliding mode adaptive neural-network controller for trajectory following of non holonomic mobile modular manipulators is presented. In this work, sliding mode control and direct adaptive technique are combined together to suppress bounded disturbances and modeling errors caused by parameter uncertainties. A sliding mode control method for

a two-wheel welding mobile manipulator is proposed in [6], in order to track a smooth 3D curved welding path. In [7], an adaptive algorithm based on the sliding mode control for the tracking control of robot manipulators is presented. It is shown that the robustness of the developed algorithms is guaranteed by the sliding mode control law in the presence of disturbances and modeling uncertainties.

In [8], an adaptive fuzzy sliding mode controller for robotic manipulators is proposed. An adaptive single-input single-output fuzzy system is applied to calculate each element of the gain vector in a sliding mode controller. The chattering and the steady state errors in the sliding surface are eliminated and satisfactory trajectory tracking is achieved. In [9], a robust variable structure control and sliding mode planes has been considered for the robot manipulators in presence of parametric variations, transported load and external signals disruption. It is shown that this technique is suitable for nonlinear system with imprecise models. In [10], an adaptive fuzzy system combined with sliding mode control to solve the chattering problem for robotic manipulators is presented. In the design of the controller, no prior knowledge is required about the system dynamics.

In [11], a hybrid sliding-mode fuzzy neural networks (HSMFNN) controller for mobile manipulators in generalized coordinate was presented. The proposed control law consists of a kinematic velocity part for the control of the mobile

platform and the onboard arm separately and a robust tracking control system based on hybrid sliding-mode fuzzy neural networks (HSMFNN) to ensure the velocity tracking ability under dynamic uncertainties. Most previous approaches were used for control of manipulators, few works are found on non holonomic mobile manipulators.

Furthermore, most of them were designed in joint space, but few in task space. However, in practical applications, the trajectories are described in task space coordinate. In this work an integrated adaptive robust control methodology based on the sliding mode is developed for the trajectory tracking of a wheeled mobile manipulator in task space coordinate. The proposed approach possesses the advantages of both adaptive and robust control and does not rely on precise prior knowledge of dynamics parameters. However, it can suppress disturbances and modeling errors caused by parameters uncertainties.

This paper is organized as follows. Section 4 is devoted to kinematic and dynamic modeling of the mobile manipulator with non holonomic constraints. Section 5 presents the design of the robust adaptive controller. Section 6 presents computer simulation results to illustrate the effectiveness of the proposed theory. Conclusions are formulated in Section 7.

**Modeling of a Mobile Manipulator**

**Kinematic modeling**

Consider the mobile manipulator system depicted in Figure 1 [12].

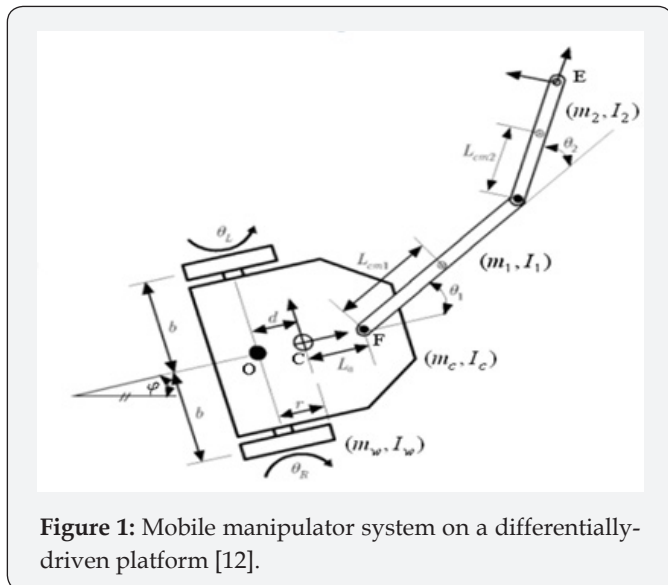


Figure 1: Mobile manipulator system on a differentially-driven platform [12].

For the mobile platform, the kinematic equation relating linear velocity of to the wheel velocities is:

$$\begin{bmatrix} \dot{x}_F \\ \dot{y}_F \end{bmatrix} = \begin{bmatrix} \frac{r}{2b}(bC_0 + (d + L_a)S_0) & \frac{r}{2b}(bC_0 - (d + L_a)S_0) \\ \frac{r}{2b}(bS_0 - (d + L_a)C_0) & \frac{r}{2b}(bS_0 + (d + L_a)C_0) \end{bmatrix} \begin{bmatrix} \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix} \dots\dots\dots (1)$$

Where  $\dot{\theta}_R$  and  $\dot{\theta}_L$ , are the angular velocities of the right and the left wheels respectively. With the notation  $C_0 = \cos(\phi)$  and  $S_0 = \sin(\phi)$ .

The linear velocity of the end-effector is found using the fact that its base velocity is known and given by Equation (1). Therefore, the end-effector velocity is written as:

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \end{bmatrix} = \begin{bmatrix} \dot{x}_F \\ \dot{y}_F \end{bmatrix} + \begin{bmatrix} C_0 & -S_0 \\ S_0 & C_0 \end{bmatrix} \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 + \dot{\phi} \\ \dot{\theta}_2 \end{bmatrix} \dots\dots\dots (2)$$

Where  $J_{ij}$  ( $i, j=1,2$ ) terms are the elements of the fixed-base Jacobian of the manipulator employed, given by:  $J_{11} = -L_1S_1 - L_2S_{12}$ ,  $J_{12} = -L_2S_{12}$ ,  $J_{21} = L_1C_1 + L_2C_{12}$ ,

$J_{22} = L_2C_{12}$  and  $\theta_1, \theta_2$ , are the joint variables of the manipulator. With the notation:  $C_i = \cos(\theta_i)$ ,  $S_i = \sin(\theta_i)$ :  $C_{ij} = \cos(\theta_i + \theta_j)$ , and  $S_{ij} = \sin(\theta_i + \theta_j)$ ,  $\forall i, j=1, 2$ .

Combining Equations (1) and (2), the forward differential kinematics of the mobile manipulator is obtained as:

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{x}_F \\ \dot{y}_F \end{bmatrix} = \begin{bmatrix} C_0 & -S_0 & 0 & 0 \\ S_0 & C_0 & 0 & 0 \\ 0 & 0 & C_0 & -S_0 \\ 0 & 0 & S_0 & C_0 \end{bmatrix} \begin{bmatrix} \frac{r}{2} - \frac{r}{2b}J_{11} & \frac{r}{2} + \frac{r}{2b}J_{11} & J_{11} & J_{12} \\ -\frac{r}{2b}(d + L_a + J_{21}) & \frac{r}{2b}(d + L_a + J_{21}) & J_{21} & J_{22} \\ \frac{r}{2} & \frac{r}{2} & 0 & 0 \\ -\frac{(d + L_a)r}{2b} & \frac{(d + L_a)r}{2b} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_R \\ \dot{\theta}_L \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \dots\dots\dots (3)$$

which can be expressed in the following form:

$$\dot{x} = (RJ_a)v = Jv \dots\dots\dots (4)$$

And it's time derivative as:

$$\ddot{x} = \dot{J}v + J\dot{v} \dots\dots\dots (5)$$

**Dynamic modeling**

The dynamics of a mobile manipulator subject to nonholonomic constraints can be obtained using the Lagrangian approach in the following form:

$$H(q)\ddot{q} + V(q, \dot{q}) + A^T(q)\lambda = E(q)(\tau + \tau_d) \dots\dots\dots (6)$$

The  $m$  nonholonomic constraints can be expressed as:

$$A(q)\dot{q} = 0 \dots\dots\dots (7)$$

where  $q = [q_1 \dots q_n]^T \in \mathfrak{R}^{n \times 1}$  is the generalized coordinates  $H(q) \in \mathfrak{R}^{n \times n}$  is a symmetric, positive definite inertia matrix,  $V(q, \dot{q}) \in \mathfrak{R}^{n \times 1}$  represents the vector of centripetal and Coriolis forces terms,  $A(q) \in \mathfrak{R}^{m \times n}$  is the constraint

matrix,  $\lambda \in \mathfrak{R}^{m \times 1}$  is the Lagrange multiplier which denotes the vector of constraint forces,  $E(q) \in \mathfrak{R}^{n \times (n-m)}$  is the input transformation matrix,  $\tau \in \mathfrak{R}^{(n-m) \times 1}$  is the vector of input torques and  $\tau_d \in \mathfrak{R}^{(n-m) \times 1}$  is the vector of unknown external disturbance.

In order to eliminate the constraint force  $\lambda$ ,  $S(q)$  let be a full rank matrix ( $p = n - m$ ) formed by a set of smooth and linearly independent vector fields spanning the null space of  $A(q)$ , i.e.,  $S^T(q)A^T(q) = 0$ . From Equation (7) we can find a joint velocity input vector  $v = [\dot{\theta}_R, \dot{\theta}_L, \dot{\theta}_1, \dot{\theta}_2]^T$ , such that for all  $t$ :

$$\dot{q} = S(q)v \dots\dots\dots (8)$$

Where  $q = [x_c, y_c, \phi, \theta_R, \theta_L, \theta_1, \theta_2]^T$  is the generalized coordinate for the considered mobile manipulator system and the matrix  $S(q)$  is defined as:

$$S(q) = \begin{bmatrix} \frac{r}{2b}(bC_0 + dS_0) & \frac{r}{2b}(bC_0 - dS_0) & 0 & 0 \\ \frac{r}{2b}(bS_0 - dC_0) & \frac{r}{2b}(bS_0 + dC_0) & 0 & 0 \\ \frac{r}{2b} & -\frac{r}{2b} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots\dots\dots (9)$$

Substituting (8) and its derivative into (6) and left multiplying, the dynamic model can be reduced as follows:

$$\bar{H}(q)\dot{v} + \bar{V}(q, \dot{q})v + \bar{G}(q) = \bar{\tau} + \bar{\tau}_d \dots\dots\dots (10)$$

Where  $\bar{H} = S^T H S$ ,  $\bar{V} = S^T H \dot{S}$ ,  $\bar{G} = S^T V$ ,  $\bar{\tau} = S^T E \tau$

And  $\bar{\tau}_d = S^T E \tau_d$

We note that Equation (10) reflects only the feasible motions allowable by satisfying the constraints.

Using Equation (4) and (5), the reduced dynamic system (10) can be further written in task space as:

$$H_x(q)\ddot{x} + V_x(q, \dot{q})\dot{x} + G_x(q) = \tau_x + \tau_{dx} \dots\dots\dots (11)$$

Where  $H_x = J^{-T} \bar{H} J^{-1}$ ,  $V_x = J^{-T} (\bar{V} - \bar{H} J^{-1} \dot{J}) J^{-1}$ , and  $\tau_{dx} = J^{-T} \bar{\tau}_d$

The dynamic system (11), possess a number of important properties that facilitate analysis and control system design. Among these are:

**Property 1:** The inertia matrix  $H_x(q)$  is symmetric and positive definite, i.e.,  $H_x^T = H_x > 0$ .

**Property 2:**  $\dot{H}_x(q) - 2V_x(q, \dot{q})$  is skew symmetric, i.e.,  $x^T (\dot{H}_x - 2V_x)x = 0, \forall x \in \mathfrak{R}^p$

**Property 3:** The left hand side expression of (11) can be linearly parameterized in terms of the robot system parameters as follow:

$$H_x(q)\ddot{x} + V_x(q, \dot{q})\dot{x} + G_x(q) = Y(q, \dot{q}, \ddot{x})\theta \dots\dots\dots (12)$$

For any  $\ddot{x}$ ,  $\dot{x} \in \mathfrak{R}^p$ , where  $\theta \in \mathfrak{R}^l$  is the vector of uncertain (or unknown) parameters and  $Y \in \mathfrak{R}^{p \times l}$  is the so-called "regressor" matrix which contains known function. This property means that the dynamic equation can be linearized with respect to a specially selected set of system parameters.

### Design of the Robust Adaptive Sliding Mode Control

Suppose that the desired trajectories in task space are described by  $x_d$  and  $\dot{x}_d$ . In order to track the desired trajectories, we define the sliding surface:

$$s = \dot{e} + \sigma e \dots\dots\dots (13)$$

Where  $e = x - x_d$  is the tracking error and is a constant positive definite diagonal matrix.

Let us define the reference state as:

$$\begin{aligned} \dot{x}_r &= \dot{x} - s = \dot{x}_d - \sigma e \dots\dots\dots (14) \\ \ddot{x}_r &= \ddot{x} - \dot{s} = \ddot{x}_d - \sigma \dot{e} \end{aligned}$$

We now define a robust control input based on sliding mode theory in task space coordinate as:

$$\tau_x = H_x \ddot{x}_r + V_x \dot{x}_r + G_x - K_{s1}s - K_{s2} \text{sgn}(s) \dots\dots\dots (15)$$

Where  $K_{s1}$  and  $K_{s2}$  are a constant positive definite diagonal matrices. The proposed control input consists of a continuous nominal control part, and a discontinuous switching control part to make the system invariant to parametric uncertainty and external disturbances. The switching control part causes the chattering problem. To reduce chattering, the switching function can be replaced by a continuous approximation in the neighborhood of the sliding surface, such as sigmoid-like function. The proposed algorithm performs well if the dynamics of the system are known exactly [8]. On the other hand, the existence of disturbances and uncertainties influences the performance of the sliding mode based controller and make the closed loop system unstable. To handle these difficulties, the sliding mode based controller (15) is modified as:

$$\tau_x = \hat{H}_x \ddot{x}_r + \hat{V}_x \dot{x}_r + \hat{G}_x - K_{s1}s - K_{s2} \text{sign}(s) \dots\dots\dots (16)$$

Where  $\hat{H}_x$ ,  $\hat{V}_x$  and  $\hat{G}_x$  are estimate of unknown parameters  $H_x$ ,  $V_x$  and  $G_x$ .

According to property (3), we can rewrite Equation (16) as:

$$\tau_x = Y(q, \dot{q}, \ddot{x}_r, \dot{x}_r)\hat{\theta} - K_{s1}s - K_{s2} \text{sign}(s) \dots\dots\dots (17)$$

Where  $\hat{\theta}$  is estimates of unknown constant parametric vector. Substituting Equation (17) into Equation (11), we may have the closed-loop error dynamics:

$$H_x \dot{s} + V_x s + K_{s1}s + K_{s2} \text{sign}(s) = Y(q, \dot{q}, \ddot{x}_r, \dot{x}_r)\tilde{\theta} \dots\dots\dots (18)$$

Where  $\tilde{\theta} = \theta - \hat{\theta}$ . If an appropriate adaptive update law for  $\hat{\theta}$  can be selected, we may easily to prove the convergence

of the tracking errors to zero and the system stability.

Let us define a Lyapunov function candidate as:

$$L = \frac{1}{2} s^T H_x s + \frac{1}{2} \tilde{\theta}^T \Gamma \tilde{\theta} \dots\dots\dots (19)$$

Where  $\Gamma$  is a positive constant design matrix. The time derivative of  $L$  can be computed as:

$$\dot{L} = s^T H_x \dot{s} + \frac{1}{2} s^T \dot{H}_x s + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \dots\dots\dots (20)$$

Substituting the closed-loop error dynamics (18) in the above Equation, we get:

$$\dot{L} = -s^T K_{s1} s + s^T (\dot{H}_x - 2V_x) s - s^T \text{sign}(s) + \tilde{\theta}^T (s^T Y(q, \dot{q}, \ddot{q}, \ddot{q}_r) + \dot{\tilde{\theta}}^T \Gamma) \dots\dots\dots (21)$$

Since  $(\dot{H}_x - 2V_x)$  is a skew symmetric, the above equation become:

$$\dot{L} = -s^T K_{s1} s - s^T \text{sign}(s) + \tilde{\theta}^T (s^T Y(q, \dot{q}, \ddot{q}, \ddot{q}_r) + \dot{\tilde{\theta}}^T \Gamma) \dots\dots\dots (22)$$

With selected update law as:

$$\dot{\tilde{\theta}} = -\Gamma^{-1} Y^T(q, \dot{q}, \ddot{q}, \ddot{q}_r) s \dots\dots\dots (23)$$

The time derivative of the Lyapunov function is negative  $\dot{L} \leq 0$  and the system is asymptotically stable.

The proposed robust adaptive sliding mode control system is shown in Figure 2.

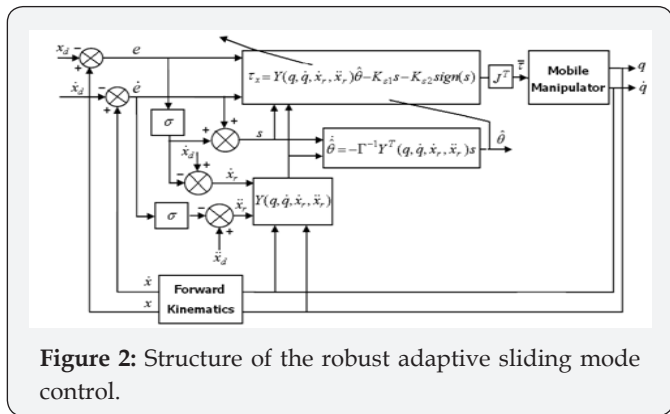


Figure 2: Structure of the robust adaptive sliding mode control.

**Simulation Results**

Let us consider the mobile manipulator system shown in Figure 1. In order to verify the effectiveness of the proposed robust adaptive sliding mode control, we compare the robust sliding mode controller given by Equations 13, 14 and 15 and the proposed robust adaptive sliding controller given by Equations 13, 14, 17 and 23.

The initial conditions are given as follow:

$$q(0) = [0 \ 0 \ \frac{\pi}{2} \ 0 \ 0 \ \frac{\pi}{4} \ -\frac{\pi}{2}]^T \text{ and } \dot{q}(0) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

Let the desired trajectory be:

$$x_d(t) = [x_{Ed}, \ y_{Ed}, \ x_{Fd}, \ y_{Fd}]^T = [0.2t + 0.3, \ 0.5 + 0.25 \sin(0.2\pi t), \ 0.2t, \ 0]^T$$

It consists of a sinusoidal path for the end-effector and a straight line for the mobile platform.

We assume the following external disturbances applied at each joint of the system as:

$$\bar{\tau}_d(t) = [1.5 \sin(3t) \ 1.5 \cos(3t) \ 3 \sin(1.5t) \ 3 \cos(1.5t)]^T$$

The control gains used in the simulation were selected as:

$$K_{s1} = 100I_{4 \times 4}, \ K_{s2} = 100I_{4 \times 4}, \ \sigma = 2I_{4 \times 4} \text{ and } \Gamma = I_{4 \times 4}$$

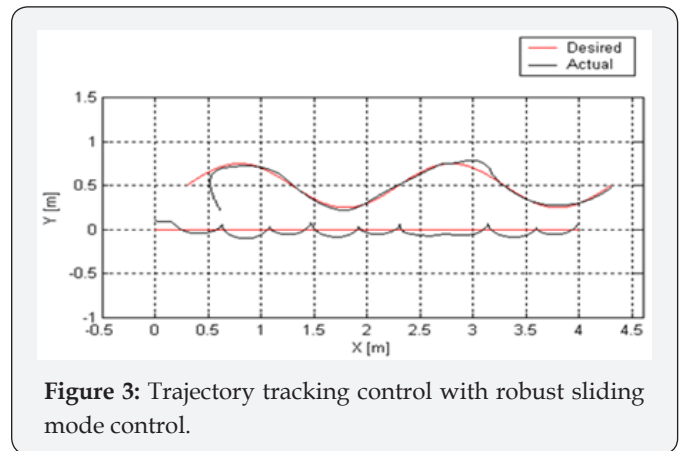


Figure 3: Trajectory tracking control with robust sliding mode control.

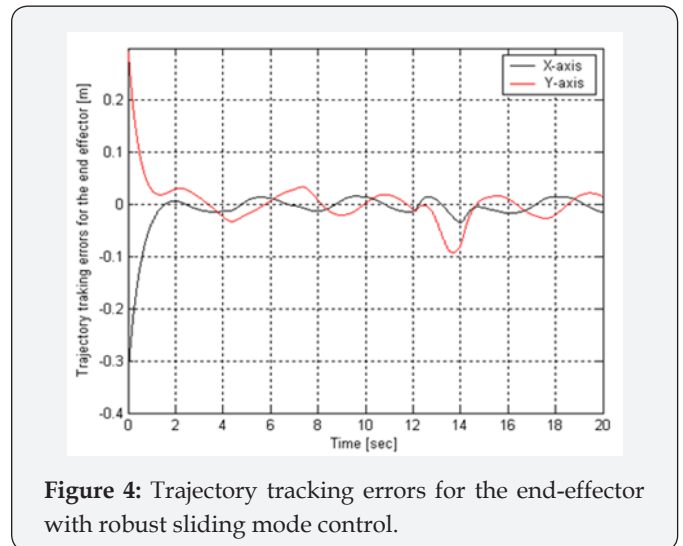
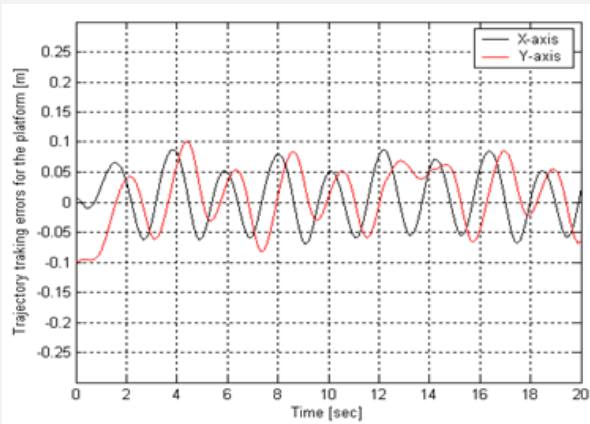


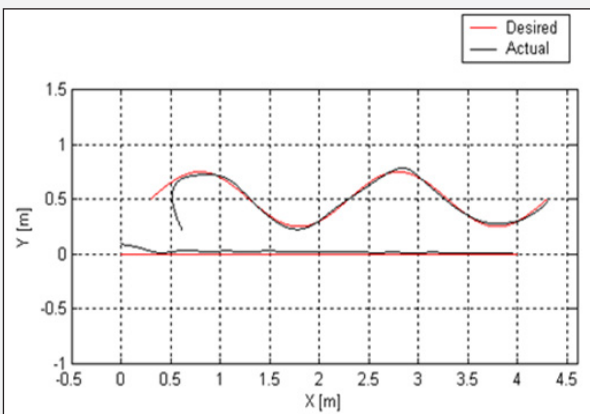
Figure 4: Trajectory tracking errors for the end-effector with robust sliding mode control.

The tracking performances of each control scheme are illustrated in Figures 3-8, respectively. From the comparison of both controls, it can be seen that the tracking results of the robust sliding mode control are not satisfactory and the tracking errors fluctuate greatly in comparison with the adaptive robust sliding mode control schemes which attain good control performance, and the tracking error is much small because of adaptive mechanism. The simulation result

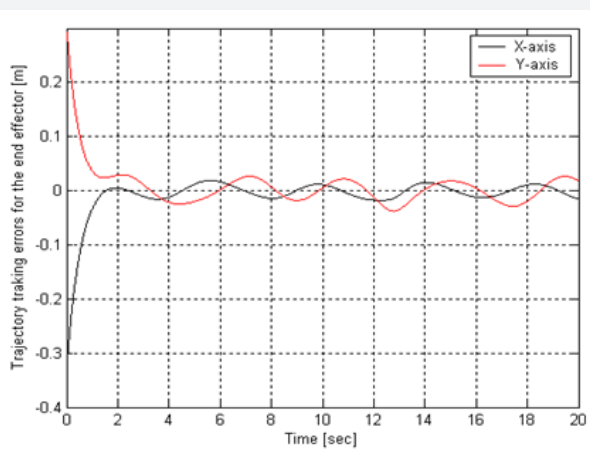
thus verifies the effectiveness of the proposed control in the presence of external disturbances and uncertainties.



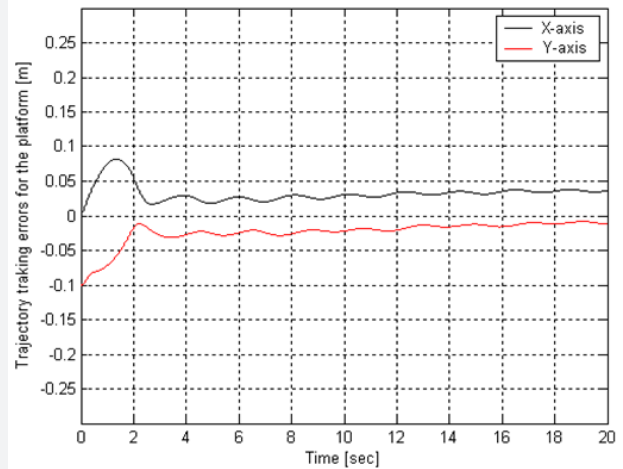
**Figure 5:** Trajectory tracking errors for the platform with robust sliding mode control.



**Figure 6:** Trajectory tracking control with robust adaptive sliding mode control.



**Figure 7:** Trajectory tracking errors for the end-effector with robust adaptive sliding mode control.



**Figure 8:** Trajectory tracking errors for the platform with robust adaptive sliding mode control.

## Conclusion

In this paper, a robust adaptive sliding mode control for mobile manipulator system in the presence of parametric uncertainties and external disturbances is proposed. The proposed control strategy was designed to drive simultaneously in task space coordinate desired end-effector and platform trajectories without violating the non holonomic constraints. The unknown parameters and the external disturbances are estimated by using update law in adaptive control scheme. The effectiveness of the proposed controller is verified by simulation.

## References

1. Yamamoto Y, Yun X (1996) Effect of the dynamic interaction on coordinated control of mobile manipulators, *IEEE Trans. On Robotics and Automation* 42(5): 816-824.
2. Papadopoulos E, Poulakakis J (2000) Planning and Model-Based Control for Mobile Manipulators. *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, Takamatsu, Japan*, pp. 1810-1815.
3. White G, Bhatt R, Tang C, Krov V (2009) Experimental Evaluation of Dynamic Redundancy Resolution in a Nonholonomic Wheeled Mobile Manipulator. *IEEE/ASME Transactions on Mechatronics* 14(3): 349-357.
4. Mazur A (2005) A New approach to designing input-output decoupling controllers for mobile manipulators. *Bulletion of the Polish academy of sciences, technical sciences* 53(1): 31-37.
5. Li Y, Liu Y (2004) Sliding Mode Adaptive Neural-network Control for Nonholonomic Mobile Modular Manipulators. 44(3): 203-224.
6. Tun T, Chung H, Kyeong S, Kim, Suk M (2004) Trajectory Tracking of Mobile Manipulator for Welding Task Using Sliding Mode Control. *Busan, Korea*.
7. Park J, Han G, Ahn H, Kim D (2001) Adaptive Approaches on the Sliding Mode Control of Robot Manipulators. *Transactions on Control, Automation and Systems Engineering* 3(1): 2001.

8. Guo Y, Woo P (2003) Adaptive Fuzzy Sliding Mode Control for Robotic Manipulators, Proceeding of the 42<sup>nd</sup> IEEE Conference on Decision and Control Maui 3: 2174-2179.
9. Khalal O, Mellit A, Rahim M, Salhi H, Guessoum M (2007) Robust Control of Manipulator Robot by using the Variable Structure Control with Sliding Mode. Athens, Greece.
10. Erbatur K, Kaynak O (2001) Use of Adaptive Fuzzy Systems in Parameter Tuning of Sliding-Mode Controllers. IEEE/ASME Transactions on Mechatronics 6(4): 2001.
11. Cheng M, Tsai C (2005) Hybrid Sliding-Mode Fuzzy Neural Network Tracking Control for a Wheeled Mobile Manipulator. Taipei, Taiwan.
12. White G (2006) Simultaneous Motion and Interaction Force Control of a Nonholonomic Mobile Manipulator. University of New York at Buffalo, USA.



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