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Effect of Heat Generation on Heat and Mass Transfer Over a Radially Stretching Surface Embedded in Porous Medium with Chemical Reaction and Activation Energy

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Abstract

The effect of heat generation on heat and mass transfer over a radially stretching surface embedded in a porous medium with chemical reaction and activation energy are numerically discussed. The governing boundary layer equations are formulated and transformed into ordinary differential equations using a suitable similarity transformation. The resulting ordinary differential equations are solved numerically by applying the fourth order Runge-Kutta method with the shooting technique. The influence of the different parameters on the velocity, temperature, and concentration are discussed and analyzed. The skin friction coefficient, the Nusselt number, and Sherwood number are also computed and investigated for different embedded parameters in the problem statements.

Keywords: radially stretching surface, heat and mass transfer, porous medium, chemical reaction and activation energy, heat generation

Introduction

The investigation of flow and heat transfer occurred by stretching surface exist in a lot of industrial applications such as plastic, glass, and rubber sheets. The quality and the specification of the final product are controlled by the criteria of stretching and the mechanism of heat transfer. The radial stretching is one of the stretching methods takes place in different investigations.

The exact similarity solution for the governing equations deals the natural convection on a vertical radially stretching surface [1]. The study of mixed convection for Jeffery fluid show that the momentum boundary layer thickness enhanced while the thermal and concentration get worse [2]. The existence of magnetic field in the axisymmetric flow of Carneau nanofluid enhances the thermal and concentration boundary layer thickness [3] and over a radially stretching/shrinking disk [4]. The influence of thermal radiation and magnetic field is investigated numerically using Homotropy analysis for the Jeffrey nanofluid [5] and numerically [6]. The existence of Joule heating for steady radially permeable stretching/shrinking sheet takes place for MHD axisymmetric flow [7] and unsteady radially stretching surface [8]. The effect of nanoparticle shapes on the nanofluid over a radially stretching rotating disk takes place [4].

The present paper introduces the influence of heat generation, activation energy in the existence of porous media and chemical reaction on the flow over stretching surface radially.

Mathematical

The surface is stretched radially in two-dimensional inducing steady, and laminar flow. The surface is embedded in porous medium with influences of heat generation. The choice of coordinate system is made as shown in Figure1 such that r-axis is taken along the surface radially in the direction of motion with velocity $U_w = U_o$ r, where $U_o > 0$ is rate of stretching surface and z-axis is perpendicular to it by keeping the origin fixed. The plane z = 0 represents the surface. The flow in the region z >0.

Assuming a species chemical reaction with finite Arrhenius activation energy. The equations of flow analysis can be expressed as: (Figure 1)

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial z^2}\right)-\frac{\mu}{k}u \qquad (2)$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial z^2}\right)-Q_o(T-T_\infty) \qquad (3)$$

$$u\frac{\partial c}{\partial r}+w\frac{\partial c}{\partial z}=D\left(\frac{\partial^2 c}{\partial z^2}\right)-K^2 r\left(\frac{T}{T_\infty}\right)^n \exp\left(-\frac{E_a}{kT}\right)(C-C_\infty) \qquad (4)$$

where u and w are the velocity components in the r and z-direction respectively, μ is dynamic viscosity and α is thermal diffusivity of , ρ is the fluid density, K is the permeability, Q_0 is the heat generation or absorption coefficient such that $Q_0>0$ corresponds to heat generation while $Q_0<0$ corresponds to heat absorption, T∞ is the ambient temperature, T is the temperature, D is the Solutal diffusivity, $\left(\frac{T}{T_\infty}\right)^n \exp(E_a/kT)$ is the modified Arrhenius function, κ is the Boltzmann constant, E_a is activation energy, k_r^2 is the chemical reaction rate constant, where -1<n<1. With boundary Conditions

At z=0: $u=U_w=U_0 r$, w=0, $T=T_w$, $C=C_w$

$$Z\to\infty:\ u=0,\ \ w=0,\ \ T=T_\infty,\ \ C=C_\infty \qquad (5)$$

Where T_w is surface temperature and C_w is solute concentration, T_∞ and C_∞ are the temperature and concentration of the ambient fluid.

The equation of continuity is satisfied if we choose a stream function $\psi(r,z)$ such that $u=-\frac{1}{r}\frac{\partial\psi}{\partial z}$, $w=-\frac{1}{r}\frac{\partial\psi}{\partial r}$, and $R_e=\frac{rU}{\nu}$ is the local Reynolds number. The velocity components are obtained as $u=Uf'(n)$ and $w=-2UR_e^{-\frac{1}{2}}f(n)$ Introducing the following similarity variables: $\psi(r,z)=-r^2UR_e^{-\frac{1}{2}}f(n)$,

$$\eta=\frac{z}{r}R_e^{\frac{1}{2}},\ \ U=U_o r, \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \ \varphi=\frac{C-C_\infty}{C_w-C_\infty}, \qquad (6)$$

Equations (2), (3), and (4) transformed into

$$f'''-f'^2+2ff''-Mf'=0 \qquad (7)$$

$$\theta''+2\Pr f\theta'+\Pr\gamma\theta=0 \qquad (8)$$

$$\varphi''+2Scf\varphi'-Sc\sigma^2(1+n\delta\theta)\exp\left(-\frac{E}{(1+\delta\theta)}\right)\varphi=0 \qquad (9)$$

with boundary conditions

$$\eta=0: f=0, f'=1, \theta=1, \phi=1$$

$$\eta\to\infty: f'=0,\quad \theta=0, \phi=0 \qquad (10)$$

where the primes denote differentiation with respect to η, $M=\mu a/k$ is the permeability parameter, $Pr=\nu/\alpha$ is the Prandtl number, $\gamma=Q_0/U_0$ is the heat-source/sink parameter, $Sc=\nu/D$ is the Schmidt number, $\delta=(T_w-T_\infty)/T_\infty$ is the temperature relative parameter, and $\sigma=k_r/\sqrt{a}$ is the dimensionless chemical reaction rate constant and $E=E_a/kT_\infty$ is the non-dimensional activation energy.

The non-dimensional quantities are the local skin friction coefficient C_{fx}, the local Nusselt number Nu_x, and local Sherwood number Sh_x which are expressed as $C_{fx}=\frac{\mu\left(\frac{\partial u}{\partial z}\right)z=0}{\rho U^2}$,

$Nu_x=-\frac{r\left(\frac{\partial T}{\partial z}\right)z=0}{(T_w-T_\infty)}$, $Sh_x=-\frac{r\left(\frac{\partial C}{\partial z}\right)z=0}{(C_w-C_\infty)}$, such that

$$C_f\sqrt{\text{Re}}=-f''(0), \frac{Nu}{\sqrt{\text{Re}}}=-\theta'(0), \theta\frac{Sh}{\sqrt{\text{Re}}}=-\varphi'(0) \qquad (11)$$

Results and Discussions

The Mathematica program solves the equations from (7) to (10) are solved. The numerical method is fourth order Rung-Kutta. The shotting method predicts the missing boundary terms. In these results, Graphical representation shows the change in

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the flow profiles, and tabulates the non-dimensional parameters at different settings. Profile of the velocity is affected with M parameter shown in Figure 2, for any selected η increasing *M* parameter decreases the velocity (Figure 2) (Figure 3).

The profile of temperature is affected by M Pr, and γ parameters as shown in Figures 3, 4, and 5 respectively. Figure 3 shows that increasing M increases the temperature, while it decreases with increasing Pr number in Figure 4 & Figure 5 ensures that the heat source increases the temperature (Figure 4) (Figure 5).

Figure (6) to Figure (11) shows the variation of the profile of concentration against the presented parameters E, M, n, δ, Sc and σ respectively. Figure 6 increasing E increases the concentration while Figure 7 increasing M makes an inconsiderable increase. Figure (8) and Figure (9) show an inconsiderable decrease in the concentration with the n and δ parameters, respectively. While Figure (10) and Figure (11) Sc and σ parameters make a decrease in the concentration with, respectively (Figure 6) (Figure 7) (Figure 8) (Figure 9) (Figure10) (Figure 11) (Table 1).

Table 1 introduces tabulated results for the non-dimensional quantities local skin friction $Cf\sqrt{\mathrm{Re}}$, local Nusselt number $Nu\sqrt{\mathrm{Re}}$, and Sherwood number $Sh/\sqrt{\mathrm{Re}}$ as $-f''(0)$, $-\theta'(0)$, and $-\varphi'(0)$, respectively. E parameter decreases $-\varphi'(0)$, but it has no effect on $-f''(0)$ and $-\theta'(0)$. Prenumber has no effect on $-f''(0)$ and $-\varphi'(0)$ while it causes an increase in $-\theta'(0)$. n parameter has no effect on $-f''(0)$ and $-\theta'(0)$, while it causes an increase in -φ' (0). Sc parameter increases -φ' (0) without no effect on $-f''(0)$ and $-\theta'(0)$. γ parameter increases the $-\theta'(0)$ and decreases -φ' (0) without any effect on -f^'' (0).δ and σ parameters increase -φ' (0) and have no effect on $-f''(0)$ and $-\theta'(0)$. M parameter increases $-f''(0)$, while it decreases $-\theta'(0)$ and -φ^' (0).

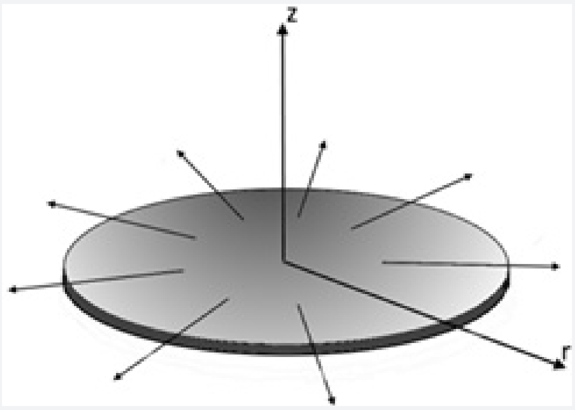

Figure 1: The problem schematic.

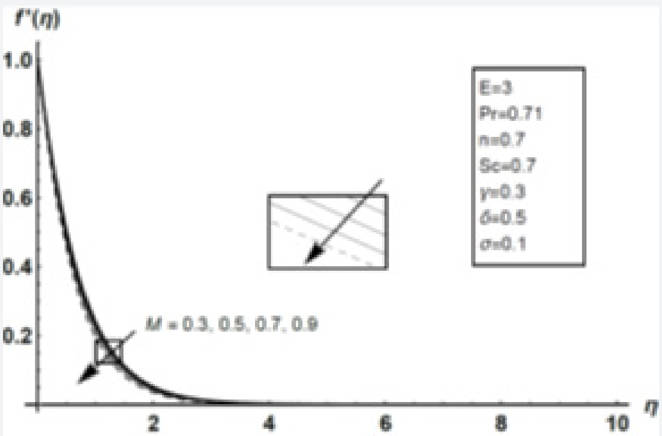

Figure 2: f^' (η) Profile at different M.

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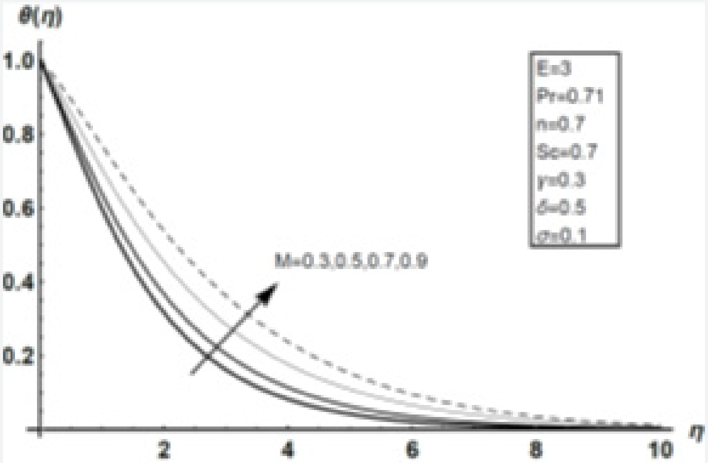

Figure 3: θ(η) Profile at different M.

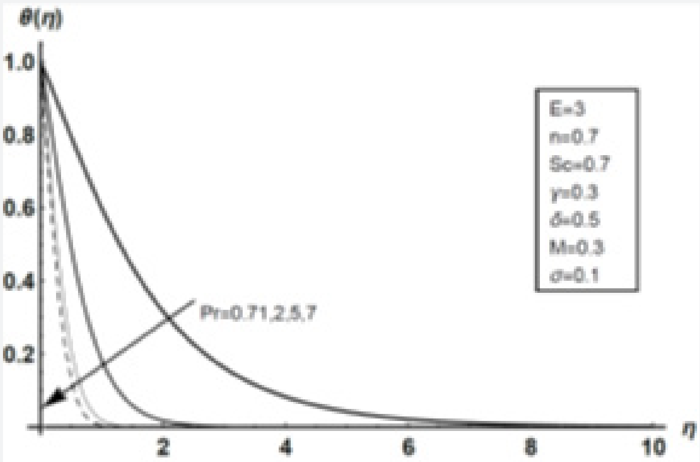

Figure 4: θ(η) Profile at different Pr.

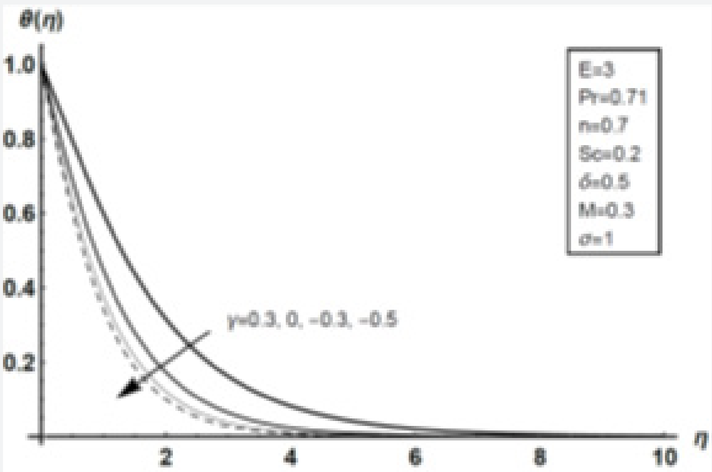

Figure 5: θ(η) Profile at different γ.

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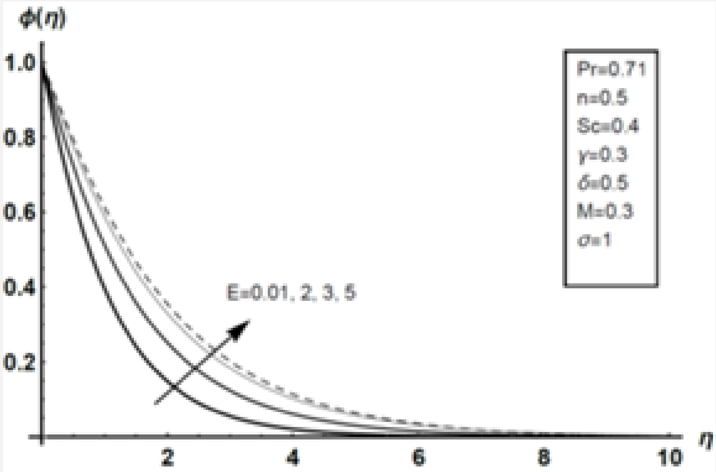

Figure 6: φ(η) Profile at different E.

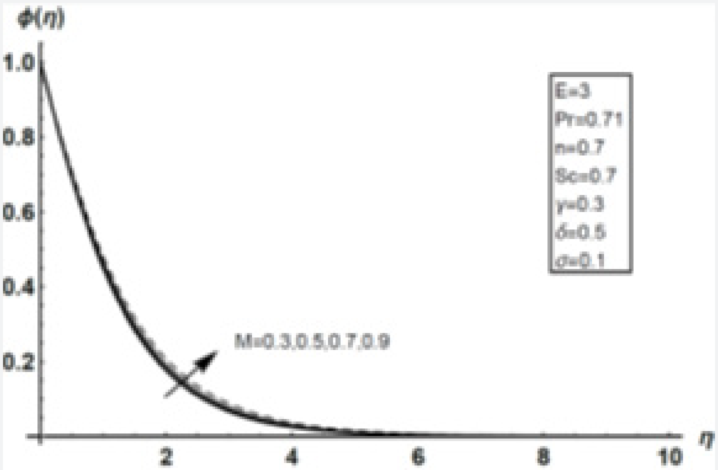

Figure 7: φ(η) Profile at different λ.

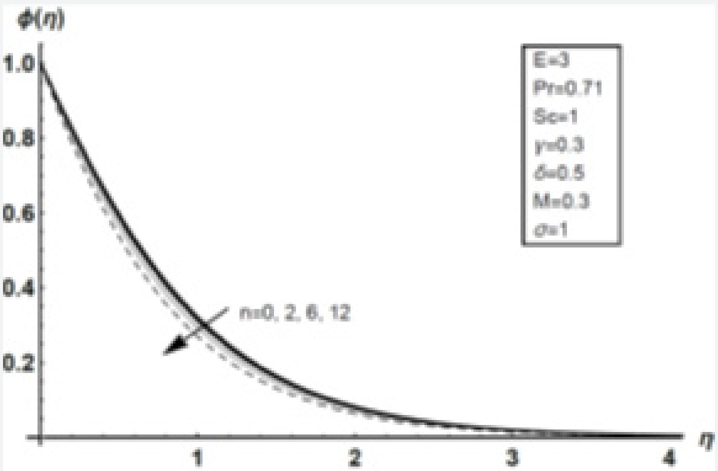

Figure8: φ(η) Profile at different n.

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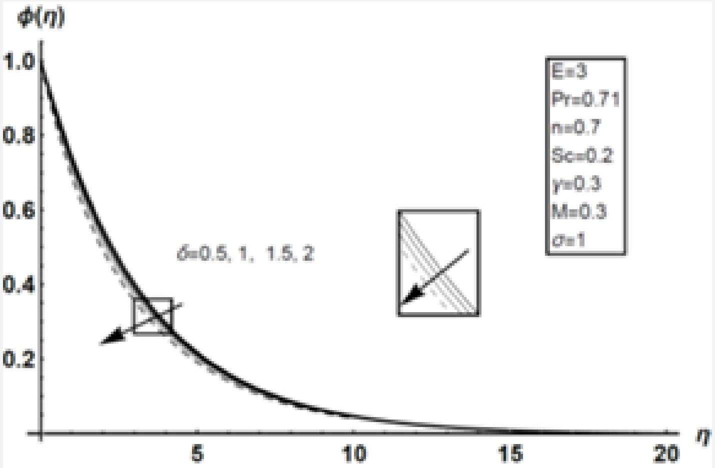

Figure 9: φ(η) Profile at different δ

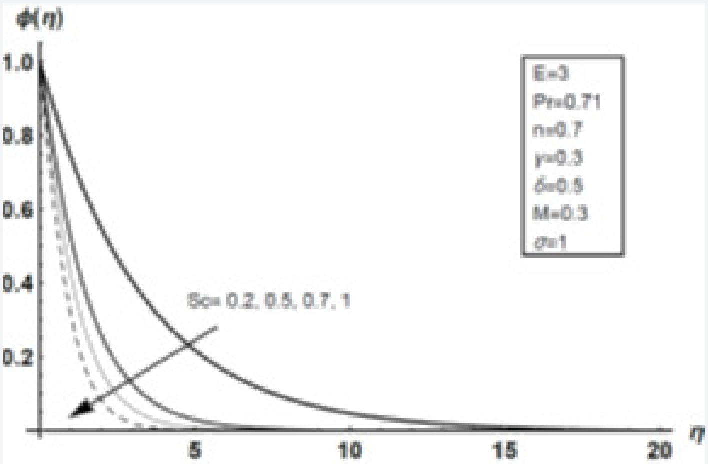

Figure 10: φ(η) Profile at different Sc.

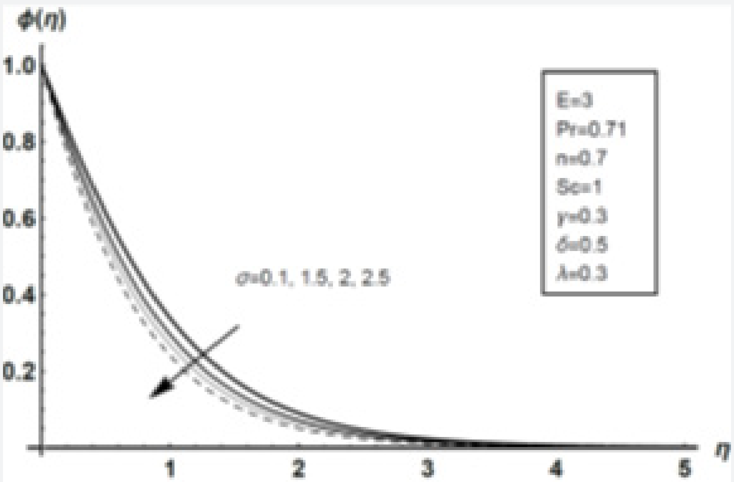

Figure 11: φ(η) Profile at different σ.

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Table 1: Nondimenstional quantities variation.

	E	Pr	n	S_c	Υ	δ	M	σ	$-f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
E	0.01	0.71	0.5	0.4	0.3	0.5	0.3	1	1.29207	0.37958	0.85633
	2										0.61329
	3										0.47434
	5										0.42627
Pr	3	0.71	0.7	0.7	0.3	0.5	0.3	0.1	1.29207	0.37958	0.64587
		2								1.03969	0.64569
		5								1.89231	0.64559
		7								2.31067	0.64557
n	3	0.71	0	1	0.3	0.5	0.3	1	1.29207	0.37958	0.88742
			2								0.93227
			6								1.0185
			12								1.14007
S_c	3	0.71	0.7	0.2	0.3	0.5	0.3	1	1.29207	0.37958	0.28136
				0.5							0.56121
				0.7							0.7209
				1							0.90323
Υ	3	0.71	0.7	0.2	0.3	0.5	0.3	1	1.29207	0.37958	0.28136
					0					0.65181	0.27645
					-0.3					0.81632	0.27452
					-0.5					0.90637	0.27365
δ	3	0.71	0.7	0.2	0.3	0.5	0.3	1	1.29207	0.37958	0.28136
						1					0.31063
						1.5					0.34582
						2					0.38473
M	3	0.71	0.7	0.7	0.3	0.5	0.3	0.1	1.29207	0.37958	0.64587
							0.5		1.36581	0.32925	0.63238
							0.7		1.43604	0.24738	0.61974
							0.9		1.50318	0.16945	0.60786
σ	3	0.71	0.7	1	0.3	0.5	0.3	0.1	1.29207	0.37958	0.82923
								1.5			0.99041
								2			1.10292
								2.5			1.23364

Table 2: Nomenclature.

C_f	Skin friction coefficient	U_w	Streching velocity
C_w	Solute concentration	μ	r-component of the fluid velocity
D	Solutal diffusivity	w	z-component of the fluid velocity
E	Activation energy parameter	r, z	Coordinates

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E_a	Activation energy	Greek Symbols	
f	Dimensionless velocity	α	Thermal diffusivity
k	Boltzmann constant	$\boldsymbol{\Upsilon}$	Heat generation/absorption parameter
k_r^2	Chemical reaction rate constant	$\boldsymbol{\delta}$	Temperature relative parameter
M	permeability parameter		
Nu_x	Local Nusselt number	$\boldsymbol{\eta}$	Dimensionless coordinator
n	Exponent rate Constant	θ	Dimensionless temperature
Pr	Prandtl number	μ	Dynamic viscosity
Q_o	Uniform volumetric heat generation	$\boldsymbol{\nu}$	Kinematic viscosity
Re	Reynolds number	ρ	Fluid density
Sh	Sherwood number	σ	Chemical reaction parameter
Sc	Schmidt number	φ	Dimensionless concentration
T	Fluid temperature	ψ	Stream function
T_∞	Ambient temperature	Subscribts	
T_w	Surface temperature	w	At the surface of the cylinder
U_0	Constant	∞	Far away from the cylinder

Conclusion

The Navier-stokes equations of the boundary layer flow over radial stretching surface is solved numerically using fourth order Runge-Kutta.

The velocity decreases with M parameter.

The temperature increases with M parameter, while it decreases with Pr and γ parameters.

The concentration increases with E and M parameter, while it deceases with n, Sc, δ, and σ parameters.

The skin friction coefficient increases with M parameter.

The local Nusselt number increases with Pr and γ parameters, while it decreases with M parameter.

The Sherwood number decreases with E, γ, and M parameters, while it increases with n, Sc, δ, and σ parameters (Table 2).

Declarations

Ethics Approval: This manuscript is original and has not been published elsewhere and is not submitted to another journal. All authors have contributed to the work.

Competing Interests: The authors declare no competing interests.

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