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Bifurcation Analysis and Multi-objective Nonlinear Model Predictive Control of an Epidemiological Model

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Abstract

Bifurcation analysis and multi-objective nonlinear model predictive control calculations are performed on an epidemiological model that involves susceptible, infected, and recovered subjects that take into account into account the level of awareness. The bifurcation analysis confirms the existence of the oscillation causing Hopf bifurcations. An activation factor involving the tanh function is shown to eliminate the Hopf bifurcations. The multi-objective nonlinear model predictive control (MNLMPC) calculations demonstrate that it is possible to minimize the infected and maximize the recovered population simultaneously. Bifurcation analysis was performed using the MATLAB software MATCONT while the multi-objective nonlinear model predictive control was performed by using the optimization language PYOMO.

Keywords: Bifurcation; Optimal Control; Multi-Objective; Epidemiological; Periodic Oscillatory Behavior

Abbreviations: MNLMPC: Multi-objective Nonlinear Model Predictive Control; NLP: Nonlinear Program; LP: Limit Points; BP: Branch Point; HB: Hopf Bifurcation Points

Introduction

The Covid problem has caused the development of epidemiological models and it is important to analyze and optimize these models to minimize the damage and ensure that the devastating effect of the disease is controlled effectively avoiding unnecessary wastage of scarce resources. The Covid pandemic will have caused around approximately 6.5 million deaths in 2022, it is necessary to develop effective techniques to prevent and control infectious diseases [1,2]. Al Basir et al. [3] provided techniques for Optimal Control of Malaria Using Awareness-Based Interventions. Wakefield et al [4] discuss the use of mass media campaigns to change health behavior. Al Basir F [5] explore the effects of awareness and time delay in controlling malaria disease propagation. Karimi et al. [6], investigate the effect of individual protective behaviors on influenza transmission. Abraha et al. [7] investigate farming awareness-based optimum interventions for crop pest control. Jones et al [8] assess the anxiety and behavioral response to novel swine-origin influenza. Ahorsu [9] discuss the Effect of COVID-19 Vaccine Acceptance, Intention, and/or Hesitancy and Its Association with Our Health. Samanta et al. [10] developed a mathematical model describing the effect of awareness programs by media on the epidemic outbreaks. Misra et al. [11], Zhao et al. [12] and Maji et al [13], developed a mathematical model involving Covid propagation with time delay. Misra and co-workers [11,14], investigated the effect of awareness programs in controlling the prevalence of an epidemic with time delay. Cui et al [15] studied the impact of media on the control of infectious diseases. Sharma and Misra [16] modelled the impact of awareness created by media campaigns on vaccination coverage in a variable population. Kiss et al [17] investigated the impact of information transmission on epidemic outbreaks. Roy and co- workers [18]. Roy et al. [18], studied the effect of awareness programs in controlling the disease HIV/AIDS using an optimal control approach.

Agaba et al. [19] studied the dynamics of vaccination in a time-delayed epidemic model with awareness while Basir and co-workers [20], studied the role of media coverage and delay in controlling infectious diseases. Sharma, et al. [21] modelled the media impact with stability analysis and found optimal solutions of the SEIRS epidemic model. Yuan and Li [22], developed Optimal control strategies with and cost-effectiveness analysis for a COVID-19 model with individual protection awareness Heffernan et al. [23] provided perspectives on the basic reproductive ratio. 2005, 2, 281-293. Al Basir and co-workers [24] modelled the effects of awareness-based interventions to control the mosaic disease of Jatropha curcas Greenhalgh et al. [25] developed a multiple delay induced mathematical model for infectious diseases while Hove-Musekwaa, and co-workers studied the dynamics of an HIV/AIDS model with screened disease carriers. Recently, Al Basir et al. [26], demonstrated the existence of Hopf Bifurcations and performed Optimal Control of studies an Infectious Disease with awareness model (SIRM model) This model involves susceptible, infected recovered subjects and takes into account the level of awareness. The main aim of this work is to use an activation factor to eliminate the oscillation causing Hopf bifurcations and perform multi-objective nonlinear model predictive control (MNLMPC) calculations using this model. This paper is organized as follows. The SIRM model is first described. This is followed by a description of the bifurcation analysis and multi-objective nonlinear model predictive control (MNLMPC) procedures. The results are then presented followed by the conclusions.

Model Description

In the SIRM model, the unaware susceptible population is represented by $S_U(t)$ and the aware susceptible population by $S_A(t)$, I(t) and R(t) are the infected and recovered population and M(t) is the level of awareness caused by the media campaign.

The infected individuals recover through treatment at a rate r, and after recovery, a fraction p of the recovered individuals join the group S_U while the remaining fraction (1-p) join the group S_A. Several parameters are used to describe the model. These include b, which is the constant recruitment rate in the susceptible population; λ, the disease transmission rate; d, the natural mortality rate of the population; δ, the disease-induced mortality rate of the infected population; and r, the recovery rate. γ is the rate at which the recovered class becomes susceptible after immunity loss and the transfer rate from S_A to S_U is denoted as β (Table 1).

$$\frac{\partial S_U}{\partial t} = b - \alpha M S_U - \lambda I S_U + \left(\frac{\beta S_A}{1+M}\right) - dS_U + p\gamma R \quad (1)$$

$$\frac{\partial S_A}{\partial t} = \alpha M S_U - \left(\frac{\beta S_A}{1+M}\right) - dS_A + (1-p)\gamma R \quad (2)$$

$$\frac{\partial I}{\partial t} = \lambda S_U I - \left(\frac{\beta S_A}{1+M}\right) - (d+\delta)I + rMI \quad (3)$$

$$\frac{\partial R}{\partial t} = RMI - dR + \gamma R \quad (4)$$

$$\frac{\partial M}{\partial t} = \omega + \eta I - \theta M \quad (5)$$

Table 1: Base Values of Parameters.

Parameter	Value(day^{-1})
b(Constant recruitment rate)	12
λ (Disease transmission rate)	0.0005
A (Contact rate between unaware susceptible with media)	0.002
ω (Rate of media campaigns by global sources)	0.03
d (Susceptible class natural death rate)	0.01
δ (Additional death rate due to infection)	0.007
β (Rate at which aware human becomes unaware)	0.0025
R(Rate of recovery of infected human)	0.01
γ (the rate at which the recovered class becomes susceptible after immunity loss)	0.0015
P(Portion of recovered class becoming susceptible unaware class)	0.3
η (Rate of awareness programs by local sources)	0.25
θ (Depletion rate of awareness program)	0.015

Bifurcation Analysis

There has been a lot of work in chemical engineering involving bifurcation analysis throughout the years. The existence of multiple steady-states and oscillatory behavior in chemical processes has led to a lot of computational and analytical work to explain the causes of these nonlinear phenomena. Multiple steady states are caused by the existence of branch and limit points while oscillatory behavior is caused by the existence of Hopf bifurcations points.

One of the most commonly used software to locate limit points, branch points, and Hopf bifurcation points is MATCONT [27,28]. This software detects Limit points(LP), branch points(BP) and Hopf bifurcation points(HB). Consider an ODE system

$$\dot{x} = f(x, \beta) \quad (6)$$

$x \in R^n$ Let the tangent plane at any point x be [v1, v2, v3, v4 ,....v_{n+1}]. Define matrix A given by

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$$A=\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \ldots\ldots & \frac{\partial f_1}{\partial x_n} & \frac{\partial f_1}{\partial \beta} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \ldots\ldots & \frac{\partial f_2}{\partial x_n} & \frac{\partial f_2}{\partial \beta} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \frac{\partial f_n}{\partial x_4} & \ldots\ldots & \frac{\partial f_n}{\partial x_n} & \frac{\partial f_n}{\partial \beta} \end{bmatrix} \quad (7)$$

β is the bifurcation parameter. The matrix A can be written in a compact form as

$$A=\left[B \mid \frac{\partial f}{\partial \beta}\right] \quad (8)$$

The tangent surface must satisfy

$$Av=0 \quad (9)$$

For both limit and branch points the matrix B must be singular. For a limit point (LP) the n+1th component of the tangent vector $v_{n+1}=0$ and for a branch point (BP) the matrix $\begin{bmatrix} A \\ v^T \end{bmatrix}$ must be singular. For a Hopf bifurcation, the function $\det\left(2f_x\left(x,\beta \odot I_n\right)\right)$ should be zero. $\odot$ indicates the bi-alternate product while I_n is the n-square identity matrix. More details can be found in Kuznetsov [29,30] and Govaerts [31]. Sridhar [32] used Matcont to perform bifurcation analysis on chemical engineering problems.

Activation Factor

The tanh activation factor is used in neural networks [33-35] and in optimal control problems to eliminate spikes in the optimal control profile [36-39]. Sridhar [40] found a correlation between singular points (limit and branch points) and multi-objective Optimal Control. However, so far the integration of activation functions with bifurcation analysis has never been done. Hopf bifurcations cause periodic oscillatory behavior. In chemical processes, oscillatory behavior is detrimental to product quality and also causes equipment damage. This work uses the tanh activation function to eliminate oscillatory-causing Hopf bifurcations. The bifurcation parameter (control variable) ξ is replaced by with $\frac{\xi \tanh(\xi)}{\varepsilon}$ where ε is an arbitrary constant.

The tanh factor is very effective in eliminating spikes that occur in control profiles. Hopf bifurcation points cause oscillatory behavior which are similar to spikes and the examples described demonstrate the effectiveness of the tanh factor in eliminating the Hopf bifurcation by preventing the occurrence of oscillations. Figures 6a and 6b demonstrate this fact. Figure 1a shows sinx vs x and the waves (oscillations) are clearly visible, However a plot of sin x vs tanhx (Figure 1b) demonstrates an absence of the oscillations. The tanh factor takes all the variables to (-1,1) causing the oscillations to disappear resulting in a non-oscillatory curve and thereby eliminating the Hopf bifurcations..

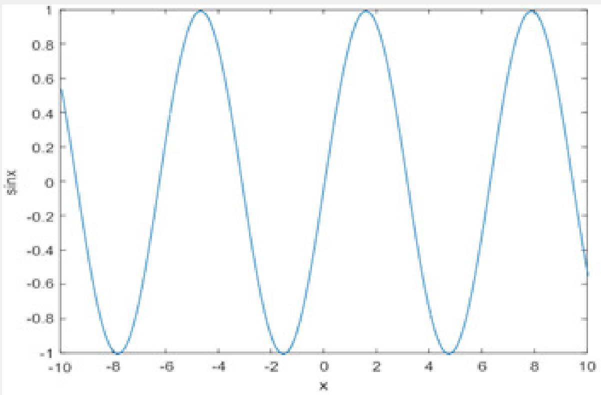

Figure 1a

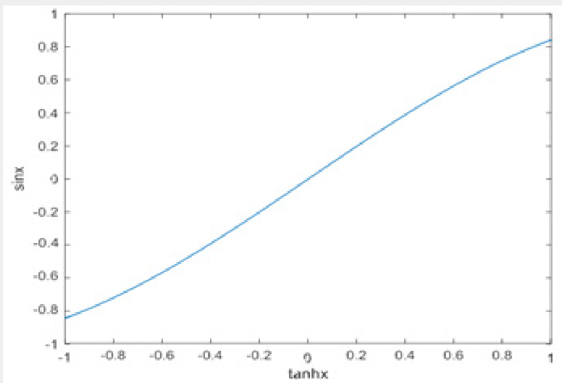

Figure 1b

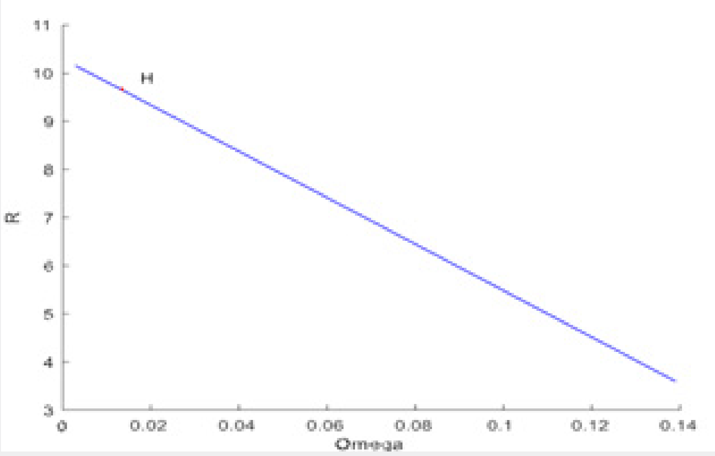

Figure 1c

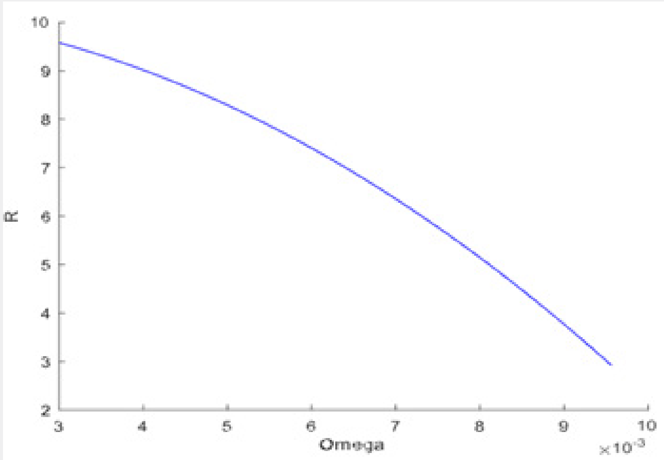

Figure 1d

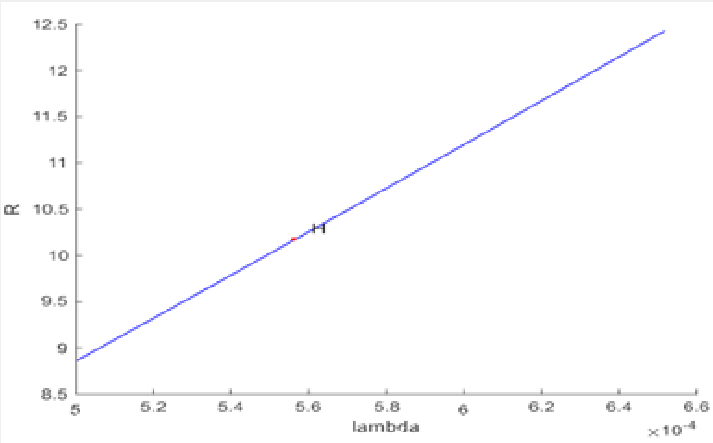

Figure 1e

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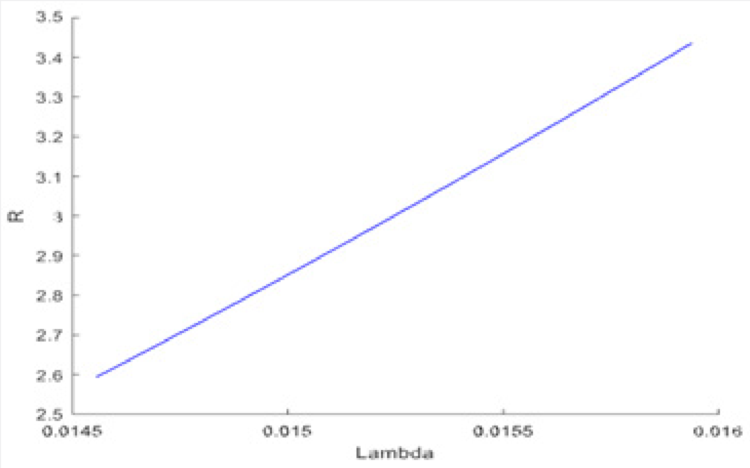

Figure 1f

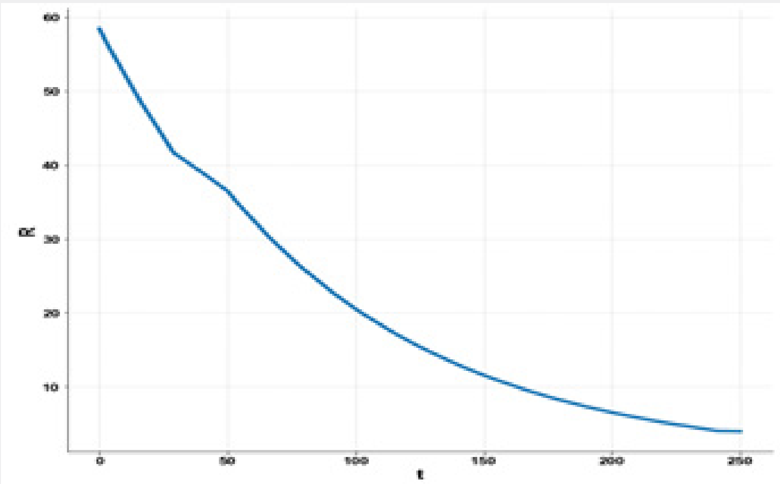

Figure 2a

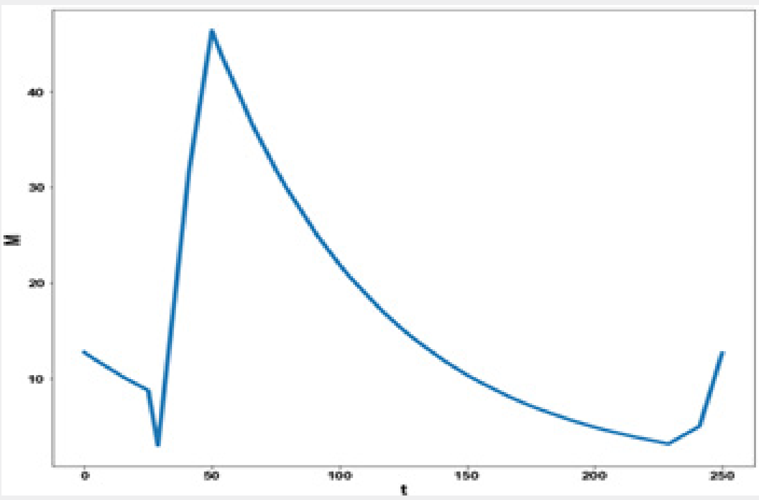

Figure 2b

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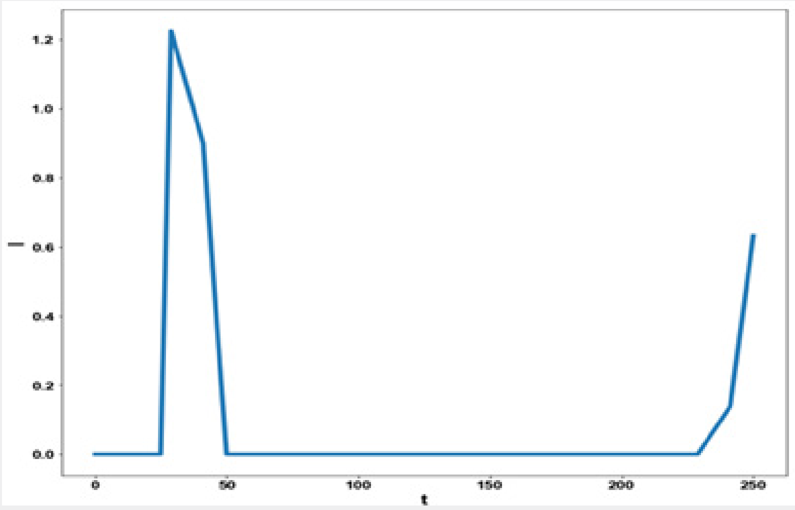
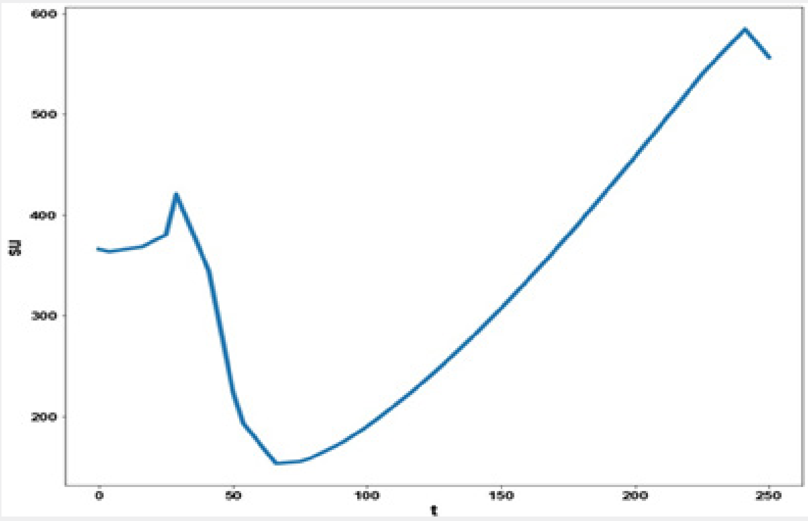


Figure 2c



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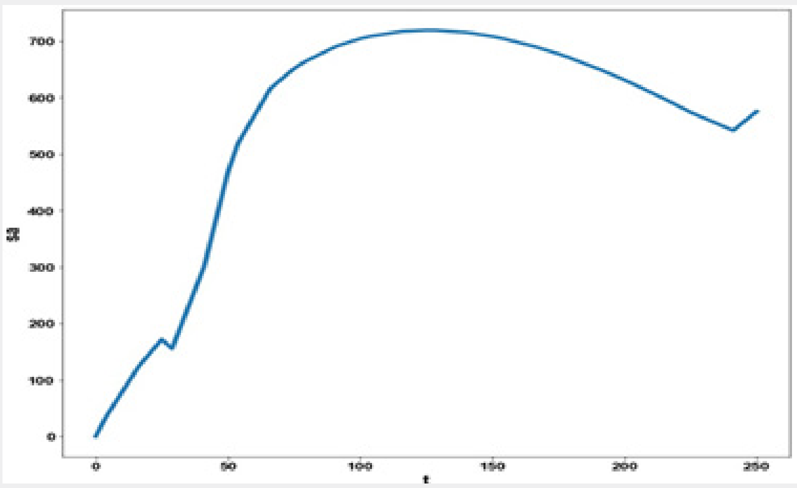

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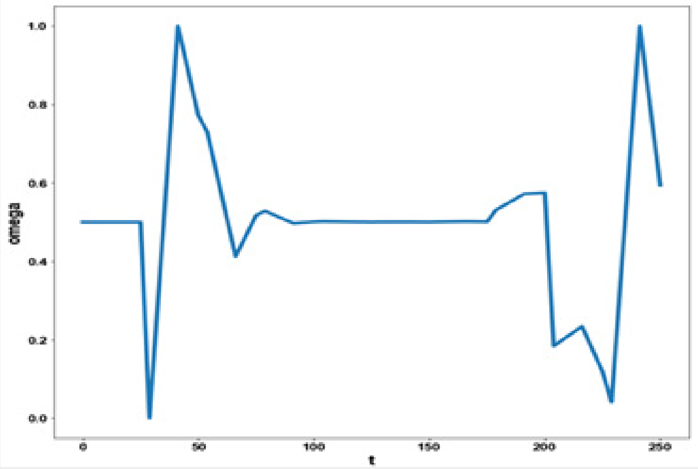

Figure 2f

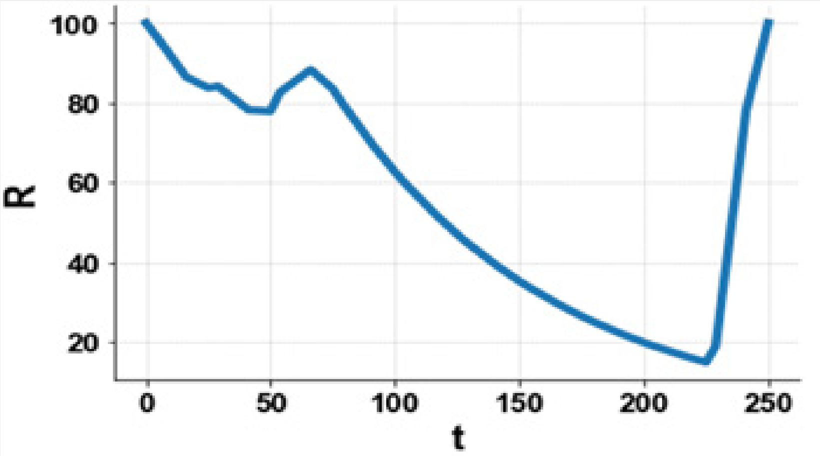

Figure 3a

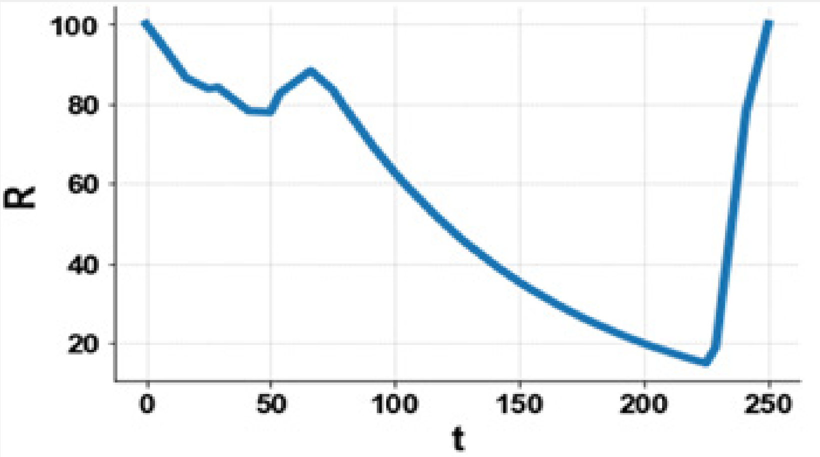

Figure 3a

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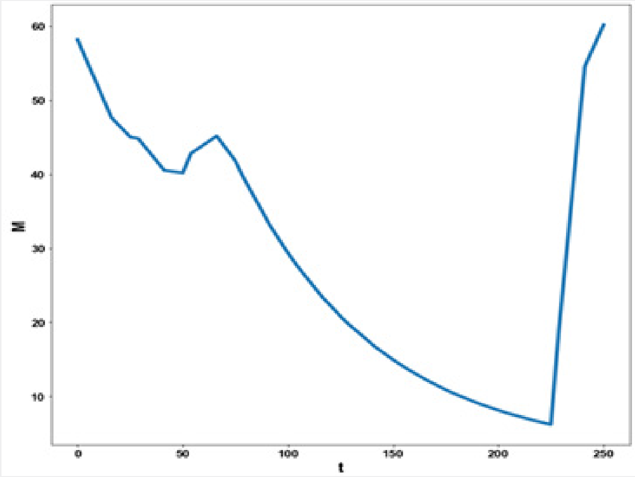

Figure 3b

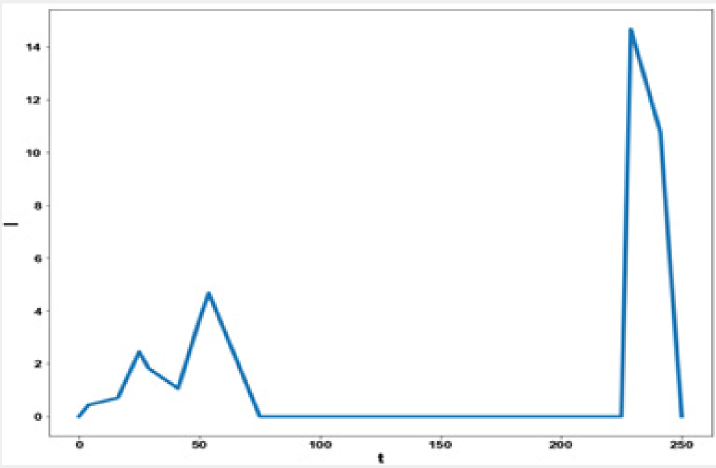

Figure 3c

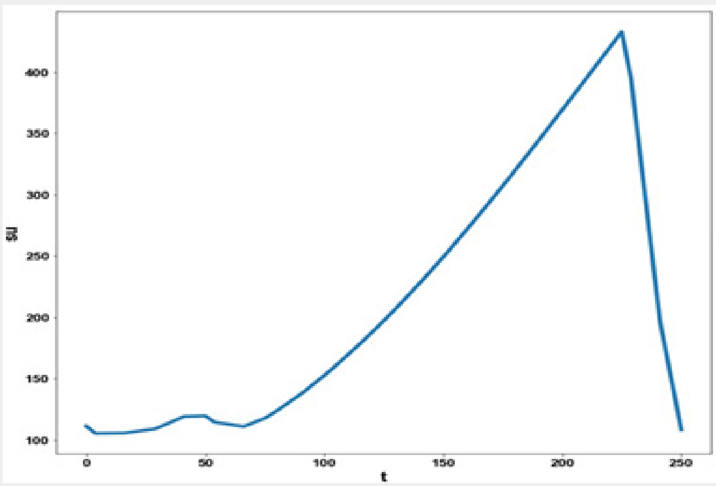

Figure 3d

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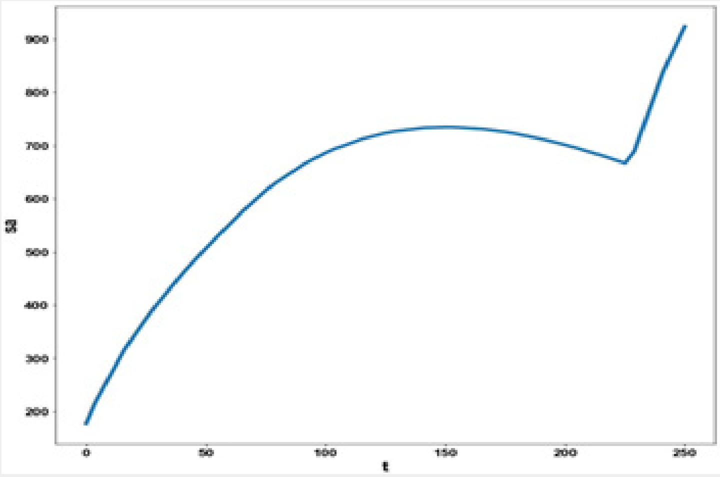

Figure 3e

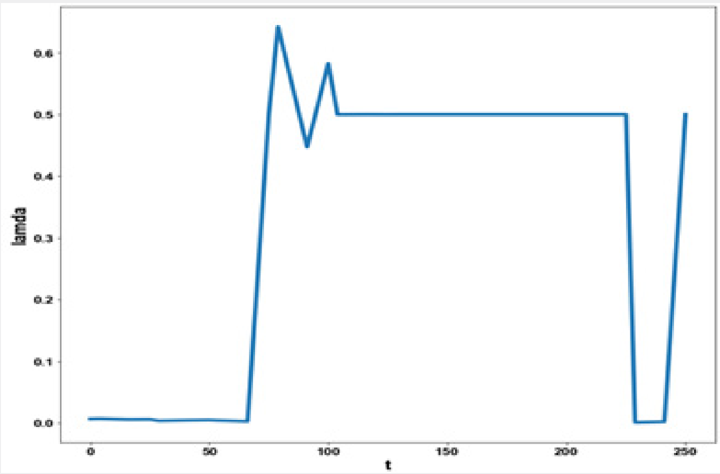

Figure 3f

MNLMPC (Multi-objective Nonlinear Model Predictive Control) method

The multi-objective nonlinear model predictive control strategy (MNLMPC) method was first proposed by Flores Tlacuahuaz [41] and used by Sridhar [42]. This method does not involve the use of weighting functions, nor does it impose additional constraints on the problem unlike the weighted function or the epsilon correction method [43]. For a problem that is posed as

$$\min J(x,u) = (x_1, x_2 \ldots . x_k)$$

$$\text{subject to} \frac{dx}{dt} = F(x,u)$$

$$h(x,u) \le 0 \quad (10)$$

$$x^L \le x \le x^U$$
$$u^L \le u \le u^U$$

The MNLMPC method first solves dynamic optimization problems independently minimizing/maximizing each x_i individually. The minimization/maximization of x_i will lead to the values x_i^*. Then the optimization problem that will be solved is

$$\min \sqrt{\left\{x_i - x_i^*\right\}^2}$$

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$$subject\,to \frac{dx}{dt} = F(x,u)$$
$$h(x,u) \le 0 \qquad (11)$$
$$x^L \le x \le x^U$$
$$u^L \le u \le u^U$$

This will provide the control values for various times. The first obtained control value is implemented and the remaining discarded. This procedure is repeated until the implemented and the first obtained control value are the same.

The optimization package in Python, Pyomo (Hart et al. 20170, where the differential equations are automatically converted to a Nonlinear Program (NLP) using the orthogonal collocation method [44]. The Lagrange-Radau quadrature with three collocation points is used and 10 finite elements are chosen to solve the optimal control problems. The resulting nonlinear optimization problem was solved using the solvers IPOPT [45] and confirmed with Baron [46] To summarize the steps of the algorithm are as follows

i. Minimize/maximize x_i subject to the differential and algebraic equations that govern the process using Pyomo with IPOPT and Baron. This will lead to the value x_i^* at various time intervals t_i. The subscript i is the index for each time step.

ii. Minimize $\sum\{x_i - x_i^*\}^2$ subject to the differential and algebraic equations that govern the process using Pyomo with IPOPT and Baron. This will provide the control values for various times.

iii. Implement the first obtained control values and discard the remaining.

Repeat steps 1 to 4 until there is an insignificant difference between the implemented and the first obtained value of the control variables.

Results and Discussion

Bifurcation analysis of SIRM model

For the Bifurcation analysis the two bifurcation parameters used were ω and λ. When the bifurcation analysis was performed using MATCONT on the SIRM model with ω as the bifurcation parameter and $\lambda = 0.0005$ a Hopf bifurcation point was obtained at the $(S_U, S_A, I, R, M, \omega)$ values of (315.334240 873.661132 0.789886 9.661823 14.0667120.013529) (Figure 1c). When ω was replaced by $\omega(\tanh(\omega))/0.0006$ the Hopf bifurcation point disappears (Figure 1d).

When the bifurcation analysis was performed using MATCONT on the SIRM model with λ as the bifurcation parameter and $\omega = 0.03$ a Hopf bifurcation point was obtained at the $(S_U, S_A, I, R, M, \lambda)$ values of (300.357643 888.145473 0.779884 10.17108114.998061 0.000556) (Figure 1e). When λ was replaced by $\lambda(\tanh(\lambda))$ the Hopf bifurcation point disappears (Figure 1f).

Multi-objective nonlinear model predictive control

In the SIRM model, $\sum I_i$ was minimized, leading to a value I_{min} $\sum R_i$ and was maximized leading to a value R_{max}. For the MNLMPC problem the objective function to be minimized was $(\sum I_i - I_{min})^2 + (\sum R_i - R_{max})^2$ and each time the first obtained control values was implemented and the remaining discarded. The procedure was repeated until there was an insignificant difference between the implemented and the first obtained value of the control variables. The two control variables used were the same as the bifurcation parameters ω and λ. This demonstrates that the oscillatory behavior can be easily controlled using an activation factor. This is the first conclusion of this paper.

When ω is the control parameter, the minimization of $\sum I_i$ led to a value of 0, while the maximization of $\sum R_i$ led to a value of 62.377. The MNLMPC calculation involved the minimization of $(\sum R_i - 62.377)^2 + (\sum I_i)^2$ and resulted in the Utopia point where the objective function was 0. The model predictive control value of ω obtained was 0.4999. In this case the profiles of the various variables are shown in (Figures 2a-2f).

When λ is the control parameter, the minimization of $\sum I_i$ led to a value of 0, while the maximization of $\sum R_i$ led to a value of 200. The MNLMPC calculation involved the minimization of $(\sum R_i - 200)^2 + (\sum I_i)^2$ and resulted in the Utopia point where the objective function was 0. The model predictive control value of λ obtained was 0.006. The profiles of the various variables are shown in (Figures 3a-3f).

In both cases the Utopia point was obtained, demonstrating that one can maximize the number of recovered subjects and minimize the infected subjects at the same time using the MNLMPC strategy and this is the second important conclusion of this paper [47].

Conclusions

The main conclusions of this work are

i. The use of an activation factor that involves the tanh function is successful in eliminating the oscillation causing Hopf bifurcations in the SIRM model.

ii. Multi-objective nonlinear model predictive control is effective in can maximizing e the number of recovered subjects and minimizing the number of infected subjects simultaneously.

These two issues can be very useful in controlling the spread of diseases.

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