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Uncertainty Quantification of Sea Waves - An Improved Approach

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Abstract

Sea waves are important dynamic loadings for the design of offshore structures. Casual observations will indicate that the sea waves are very unpredictable and may not be modeled deterministically. To capture the unpredictable nature of the sea waves, they are generally expressed in terms of a joint probability density function of mean zero crossing period T_z and the significant wave height H_s. Estimation of parameters of the joint distribution can be very challenging, particularly considering the scarcity of data. The joint probability density function (PDF) of T_z and H_s is generally represented as the multiplication of a conditional distribution for T_z given H_s and the marginal distribution of H_s. The estimation of parameters of the joint PDF is addressed in this paper. The available information on North Atlantic, as reported by Det Norske Veritas (DNV), is considered to document its applicability. DNV reported values for all the required parameters. They are considered as the reference values. Using the Maximum Likelihood Method (MLM), the three parameters of the Weibull distribution for the marginal distribution of H_s are estimated. Assuming H_s can be represented by a two-parameter Weibull distribution, they are also estimated. To compare different alternatives, their Root-Mean-Square-Error values are also estimated. It can be observed that the proposed MLM to estimate the parameters of H_s is superior to that of proposed by DNV.

Keywords: Wave height; Weibull distribution; Wave modeling; Lognormal distribution; Scatter diagram

Introduction

Sea waves are one of the most important dynamic loadings for the design of offshore structures. The dynamic characteristics of sea waves, in terms of the wave height and frequency contents, are extremely important for the design. Casual observations will indicate that the sea waves are very unpredictable and may not be modeled deterministically. For the successful design of offshore structures, the sea wave loading needs to be mathematically represented as accurately as possible. The authors believe that the most efficient way to quantify the uncertainty in the sea wave loading is statistically. It will require the collection of sea state data at a particular site. And then try to fit a mathematical model to the data. Obviously, the precision of the prediction will increase with the size and types of the data collected and their accuracy. Since the mathematical model will represent the actual sea states, it is expected to be acceptable to the engineering community. At present, the reliability analyses of both onshore and offshore structures are expected from the engineering profession so that communities are not exposed to unacceptable risk [1-4].

The sea wave data important to this study are the significant wave height H_s which is the average of the upper third of the wave heights and the mean of the zero up-crossing period T_z which is the average value of the time between successive up-crossing of the still water. The information on them is usually recorded in three-hour duration and both parameters are estimated for this duration. After collecting the data over a period, the information is summarized in a tabular form, generally known as the scatter diagram. Table 1 shows the scatter diagram for the North Atlantic [5]. It represents the total number of observations for each pair of H_s and T_z. The values of H_s and T_z shown in Table 1 are the center of each interval, i.e., when H_s is 1.5 m, it represents the interval of 1 and 2 m. T_z is defined similarly (Table 1).

Uncertainty Quantification - Methodology

With the availability of the data shown in Table 1, the major challenge is to find an appropriate mathematical model. A literature review will indicate the joint probability density function (PDF) of T_z and H_s is generally represented as the multiplication of a conditional distribution for T_z given H_s and the marginal distribution of H_s [6-10]. It can be mathematically represented as:

$$f_{T_Z,H_S}(t_Z,h_S) = f_{T_Z|H_S}(t_Z \mid h_S) f_{H_S}(h_S) \quad (1)$$

The joint distribution model represented by Eq. 1 is very helpful since it includes both frequency content and amplitude of the sea waves [11]. The most common distribution used for the conditional PDF (the first term on the right hand side of Eq. 1) is lognormal [12]. Weibull and Rayleigh distributions are generally used for the marginal distribution of H_s (the second term on the right hand side of Eq. 1) [13,14]. Subsequently, it was reported that lognormal distribution for conditional distribution T_z given H_s and Weibull for the marginal distribution of H_s are the best choices [15,16]. Even though this model was initially developed for the Norwegian Sea, it was shown that this model could be adjusted for any locations [17]. Finally, Det Norske Veritas (DNV) [18] proposed lognormal distribution for the conditional distribution and Weibull distribution for the marginal distribution of H_s and showed that these were the reliable mathematical models, and they are considered in this paper.

Conditional PDF - Lognormal Distribution

The conditional PDF of $f_{T_Z|H_S}(t_Z \mid h_S)$ considering it is lognormally distributed can be mathematically expressed as:

$$f_{T_Z|H_S}(t_Z \mid h_S) = \frac{1}{\sqrt{2\Pi}\zeta_{T_Z|H_S}(t_Z \mid h_S)} \exp\left[\frac{(\ln(t_Z \mid h_S) - \lambda_{T_Z|H_S})^2}{2\zeta^2_{T_Z|H_S}}\right] \quad (2)$$

where $\lambda_{T_Z|H_S}$ and $\zeta_{T_Z|H_S}$ are the two parameters of the lognormal distribution [19]. They are essentially mean and standard deviation of the natural logarithm of the original data.

For the conditional lognormal distribution, the two parameters can be shown to be $\lambda_{T_Z|H_S} = a_0 + a_1 h_s^{a_2}$ and $\zeta_{T_Z|H_S} = b_0 + b_1 \exp(b_2 h_s)$ [20]. Based on the data in Table 1 for each interval of h_s (each column in Table 1; there are 17 columns; in general it can be any other number), $\lambda_{T_Z|H_S}$ and $\zeta_{T_Z|H_S}$ are estimated. Using 17 values of $\lambda_{T_Z|H_S}$, the parameters a_0, a_1 and a_2 are estimated by curve fitting the data. Similarly, b_0, b_1 and b_2 are estimated using the 17 values for $\zeta_{T_Z|H_S}$.

Marginal Distribution of H_s - Weibull Distribution

The Weibull distribution is a type III extreme value distribution. As mentioned earlier, Weibull distribution is considered for H_s in this study. The PDF and cumulative distribution function (CDF) of H_s, $f_{h_s}(h_s)$ can be expressed as:

$$f_{H_S}(h_s) = \frac{\beta(h_s-\gamma)^{\beta-1}}{\alpha^\beta}.\exp\left\{-\left(\frac{h_s-\gamma}{\alpha}\right)^\beta\right\} \quad (3)$$

$$F_{H_S}(h_s) = 1-\exp\left\{-\left(\frac{h_s-\gamma}{\alpha}\right)^\beta\right\} \quad (4)$$

where α, β, γ are the shape (slope), scale and location (shift or threshold) parameters. α, β are always positive. When $\beta = 1$, Eq. 3 will represent the Exponential distribution, and when $\beta = 2$, it will represent the Rayleigh distribution. However, γ can be positive or negative. If it is zero, the three-parameter Weibull distribution will reduce to two-parameter Weibull distribution. γ is the minimum value for which the CDF is non-zero. According to the definition of H_s which is the average of the upper third of the wave heights, it is always positive and cannot be zero. Furthermore, there some observations for the first interval of H_s, for example, in Table 1, 13 observations exist for the range between zero and one. Consequently, the maximum acceptable value for γ is the upper range of the first interval, i.e., 1 for Table 1.

Table 1: Scatter diagram for the North Atlantic [5].

T_z (sec)	H_s (m)																
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5
3.5	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.5	1337	293	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5.5	8656	9860	1975	349	60	10	2	0	0	0	0	0	0	0	0	0	0
6.5	11860	49760	21588	6955	1961	510	126	30	7	2	0	0	0	0	0	0	0
7.5	6342	77380	62300	32265	13543	4984	1670	521	154	43	12	3	1	0	0	0	0
8.5	1863	55697	74495	56750	32885	16029	6903	2701	979	332	107	33	10	3	1	0	0
9.5	369	23757	48604	50991	38575	23727	12579	5944	2559	1019	379	133	44	14	4	1	0
10.5	56	7035	20660	28380	26855	20083	12686	7032	3506	1599	675	266	99	35	12	4	1
11.5	7	1607	6445	11141	12752	11260	8259	5249	2969	1522	717	314	128	50	18	6	2
12.5	1	305	1602	3377	4551	4636	3868	2767	1746	992	515	247	110	46	18	7	2
13.5	0	51	337	843	1309	1509	1408	1117	776	483	273	142	68	31	13	5	2
14.5	0	8	63	182	319	410	422	367	277	187	114	64	33	16	7	3	1
15.5	0	1	11	35	69	97	109	102	84	61	40	24	13	7	3	1	1

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16.5	0	0	2	6	13	21	25	25	22	17	12	7	4	2	1	1	0
17.5	0	0	0	1	2	4	5	6	5	4	3	2	1	1	0	0	0
18.5	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0

Maximum Likelihood Estimation

In this study, the estimation of parameters of the Weibull distribution is studied considering the type of information given in Table 1. There are several methods available to estimate the parameters of a distribution. However, estimation of the three parameters of Weibull distribution can be challenging. The level of difficulty increases significantly when the available data are grouped as shown in Table 1. The Maximum Likelihood Method (MLM) is considered to estimate the three parameters of Weibull distribution [20]. The basic concept of MLM is that if there are n observations of a random variable, then the estimated value of the parameter is the most likely to produce these values. The underlying mathematical concept can be developed in the following way. Suppose, X is a random variable with parameter which needs to be estimated with n data. Denoting the PDF of X as $f_X(x,\theta)$ and assuming the likelihood of observing x_i's is proportional to their corresponding PDF, the likelihood function can be defined as:

$$L(x_1,x_{2,}.....x_n;\theta)=f(x_1,\theta)f(x_2,\theta)......f(x_n,\theta)=\Pi_{i=1}^{n}f(x_i,\theta) \quad (5)$$

The data in Table 1 are grouped in k intervals, i.e. $[t_0,t_1^-],[t_1,t_2^-],\ldots,[t_{k-1},t_k^-]$. The probability of an observation falling in the j[th] interval is:

$$L_j=F(t_j,\theta)-F(t_{j-1},\theta) \quad (6)$$

where $F(t,\theta)$ is CDF of t for a given value of θ. Hence, the likelihood function of the Weibull parameters for a grouped sample can be expressed as:

$$L(X;\alpha,\beta,\gamma)=\Pi_{j=1}^{k}\left[F(t_j;\alpha,\beta,\gamma)-F(t_{j-1};\alpha,\beta,\gamma)\right]^{n_j} \quad (7)$$

where k is the total number of the groups, n_j is the frequency or the number of observations for the j^{th} group, and $\sum_{j=1}^{k}n_j=n$. The three parameters of the Weibull distribution are α, β and γ and they need to be estimated by the available data using MLM. Equation 7 can be re-written as:

$$L(X;\alpha,\beta,\gamma)=\Pi_{j=1}^{n}\left[e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}\right]^{n_j} \quad (8)$$

Since working with summation is easier than multiplication and taking logarithm from both sides does not change the location of the maximum values, which is of interest, the natural logarithm of Eq. 8 can be shown to be:

$$ln[L(X;\alpha,\beta,\gamma)]=\sum_{j=1}^{k}n_j ln\left[e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}\right] \quad (9)$$

By taking the derivative of Eq. 9 with respect to α,β and γ, respectively, one by one, the maximum likelihood estimation of three parameters can be obtained. This operation will result in the following three equations:

$$\frac{\ln[L(X;\alpha,\beta,\gamma)]}{\partial\alpha}=\sum_{j=1}^{k}n_j\frac{\beta}{\alpha}\frac{\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}}{e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}} \quad (10.a)$$

$$\frac{\ln[L(X;\alpha,\beta,\gamma)]}{\partial\beta}=\sum_{j=1}^{k}-n_j\frac{\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}\ln\left(\frac{t_{j-1}-\gamma}{\alpha}\right)e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}\ln\left(\frac{t_j-\gamma}{\alpha}\right)e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}}{e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}}$$

(10.b)

$$\frac{\ln[L(X;\alpha,\beta,\gamma)]}{\partial\gamma}=\sum_{j=1}^{k}n_j\frac{\beta}{\alpha}\frac{\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta-1}e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta-1}e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}}{e^{-\left(\frac{t_{j-1}-\gamma}{\alpha}\right)^{\beta}}-e^{-\left(\frac{t_j-\gamma}{\alpha}\right)^{\beta}}} \quad (10.c)$$

The data shown Table 1 are used to calculate the mean (μ) and standard deviation (σ) of the whole sample. Weibull parameters are estimated using MLM using Eq. 10. Assuming, the distribution of H_s can be described by two-parameter Weibull distribution, they are also estimated. The information is summarized in Table 2. The Weibull parameters estimated by DNV for the data shown Table 1 are available [5]. The information is summarized in Table 2. To compare different alternatives, the Root-Mean-Squared-Error (RMSE) of each model is calculated with respect to the data in Table 1. They are summarized in the last column in Table 2.

Several important observations can be made from the information summarized in Table 2. The estimated RMSE values clearly indicate that the proposed MLM-based parameter estimation for H_s using the three parameters Weibull distribution is superior to that of the method used by DNV, at least for the data for the North Atlantic region. The results also indicate that the very commonly used two-parameter Weibull distribution to model the marginal distribution of H_s is not as accurate as the three-parameter Weibull distribution. Six parameters of the conditional lognormal distribution are estimated next (Table 3). The information is summarized in three ways in Table 3. The values reported by DNV are summarized in the first row. They used ao = 0.7. Assuming ao = 0.7, the remaining 5 parameters are estimated, and the information is summarized in row 2 of Table 3. Without assuming a fixed of 0.7 for ao, all 6 parameters are estimated for the data given in Table 1. The information is summarized in the third row in Table 3 (Table 3).

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Table 2: Estimation of parameters for Weibull distribution.

S.NO		α	β	γ	μ	σ	RMSE
1	Data in Table 1				3.407	1.889	
2	Parameters of Weibull Dist. by DNV	3.041	1.484	0.661	3.41	1.885	0.0026
3	3-Parameter Weibull	2.959	1.453	0.724	3.406	1.876	0.0007
4	2-Parameter Weibull	3.85	1.926		3.415	1.847	0.0179

Table 3: Estimation of parameters for the conditional lognormal distribution.

S.NO		a_0	a_1	a_2	b_0	b_1	b_2
1	DNV Estimation	0.7	1.27	0.131	0.133	0.026	-0.191
2	a_0 similar to DNV	0.7	1.272	0.13	0.132	0.027	-0.174
3	No restriction on a_0	0.516	1.454	0.116	0.132	0.027	-0.174

To study the implications of different values of the parameters to define the joint PDF of T_z and H_s, they are plotted for different cases. The joint PDFs of the data given in Table 1 and parameters suggested by DNV are plotted in Figures 1a & 1b, respectively. They are essentially the same except at the region close to the origin (Figure 1). The same information is plotted in Figure 2 using the parameters proposed in the paper. Figures 2a & 2b are essentially the same. Figures 2c & 2d clearly indicate the deficiencies in using the two-parameter Weibull distribution for H_s as documented in Table 2. The authors recommend the MLM-based parameters estimation procedure demonstrated in this paper to quantify uncertainty in the sea waves (Figure 2).

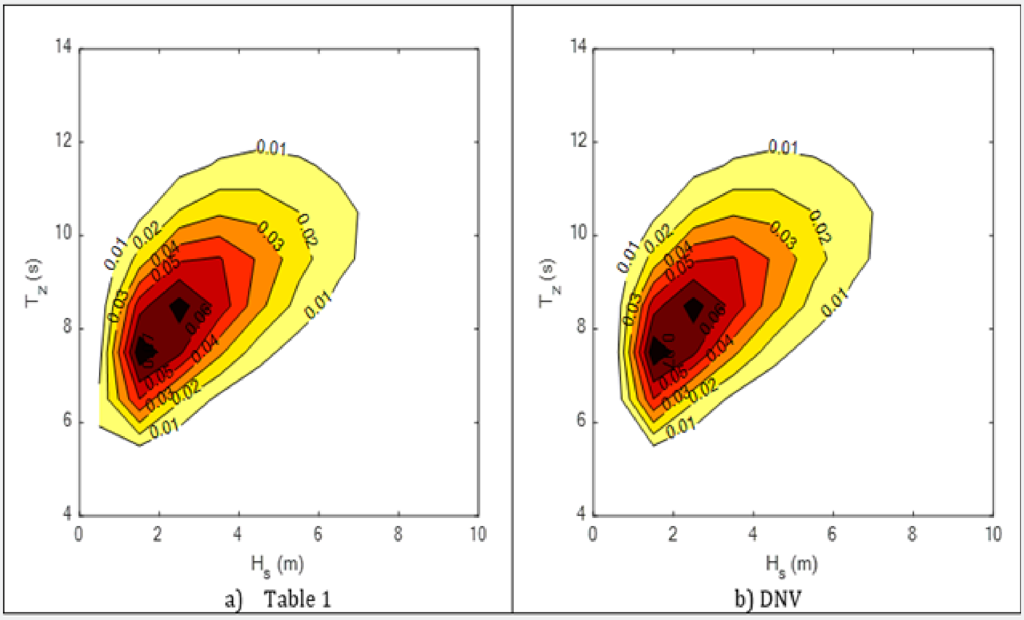

Figure 1: Comparisons of joint PDF of T_z and H_s using data in Table 1 and by DNV

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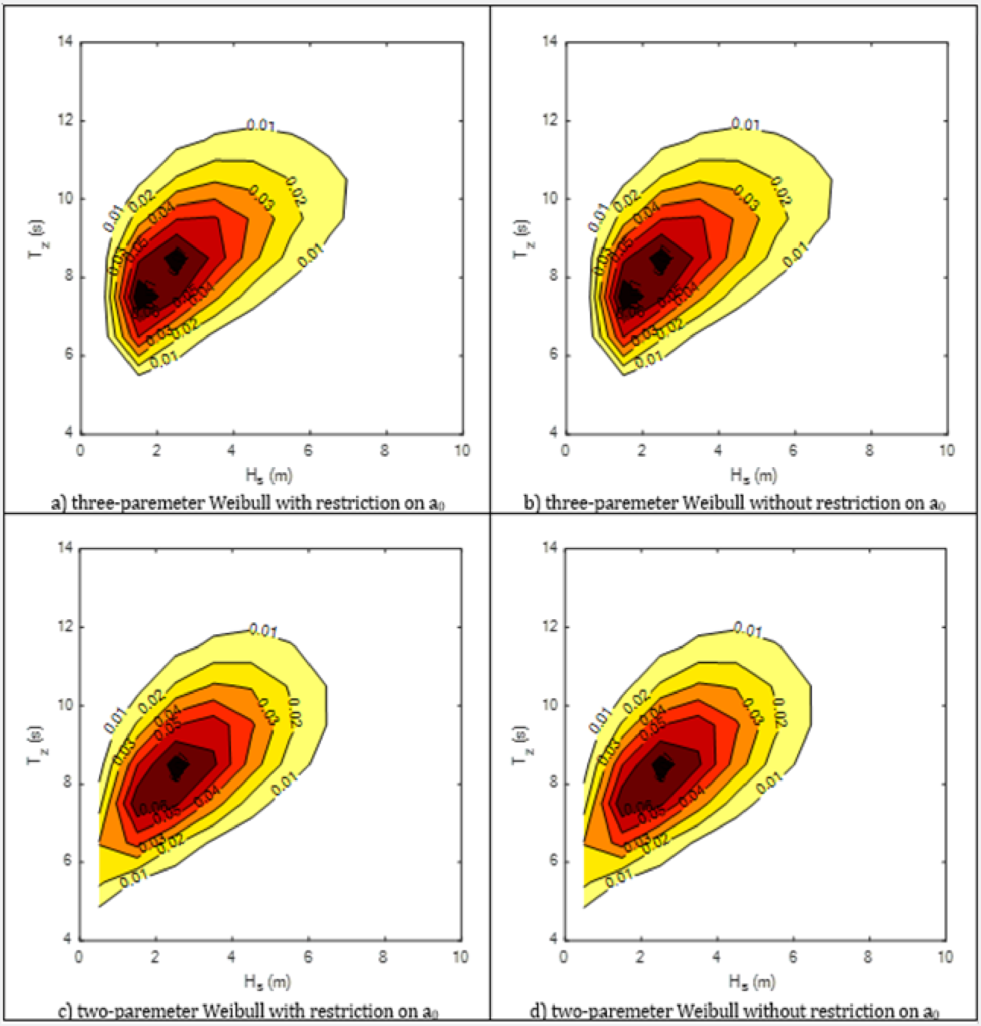

Figure 2: Comparisons of joint PDF of T_z and H_s using the proposed method

Conclusion

To capture the unpredictable nature of the sea waves, they are generally expressed in terms of a joint PDF of T_z and H_s. Estimation of parameters of the joint distribution can be very challenging, particularly considering the scarcity of data when T_z and H_s approach zero. The available information on North Atlantic, as reported by DNV, is considered in this study. DNV reported values for all the required parameters. They are considered as the reference values in this study. Using the MLM, the three parameters of the Weibull distribution for Hs are estimated. Assuming H_s can be represented by two-parameter Weibull distribution, they are also estimated using MLM. To compare different alternatives, their RMSE values are also estimated. It can be observed that the proposed MLM to estimate the parameters of H_s is superior to that of the method used by DNV. For the conditional lognormal distribution of T_z given H_s, assuming a specific value for a_0 or a data-based value may not be critical, particularly for the North Atlantic region. The authors recommend the MLM-based parameters estimation procedure demonstrated in this paper to quantify uncertainty in the sea waves.

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