

Fractal-Numerical Duality in Materials Science



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Opinion

The article sets forth the results obtained by the authors of applying a new mathematical model called the numerical asymmetry of fractals, which reflects the size effects that I.V. Tananaev proposed to consider as an additional degree of freedom in the transformations of matter.

Keywords: Fractals; Duality; Size effects; p -Adic numbers; Mathematical modeling; Materials science

Introduction

Today, materials science is studied through models of mathematical physics, which is not able to take into account the qualitative changes in substances produced by dimensional effects. In addition, according to the current state of research methods, it is obvious that matter has a systemic property. This means that it requires, for its description, dissimilar physical and chemical theories that cannot be reduced to one another - electromagnetic, thermal, elastic-plastic, theories of atomic-molecular structure, etc. The corresponding parameters of the samples and products are interrelated. Therefore, a proportion of the full-scale experiment in materials science is large, but theoretical models are fragmented, constructed *ad hoc*. There is not any consistent theory of matter, that connects its different states and transformation, for to be fruitful base for variety of modern technologies, problems, and interrelations with other sciences.

The Received Results

The authors conducted research both in the field of materials science and in applied mathematics, which made it clear to see the possibility of constructing a formal systemic model of matter based on the empirical facts and theory of fractals developed in recent decades. The duality of the fractal model of matter consists, on the one hand, by its range of size changes-from fine ones to sample sizes, and, on the other hand, by the presence in mathematics two basic numerical systems : real R and p -adic Z_p numbers . These systems are combined into a self-dual system $U = RZ_p$. The choice of $p=2$ is associated with the interpretation of S. Ulam

of p -adic numbers as invariants of the infinite divisibility of matter-everything is always divided by 2. Two numerical systems are formal analogues of a universal natural pair processes: continuity and divisibility, contraction and expansion, inflation and deflation, convergence and divergence, energy and entropy, aggregation and dissipation, and other synonymous to them, known in all sciences. Thus, Euclidean space is expanded by a dynamic pair of processes generated by universal forces, named attraction and repulsion, which act on all scales. An example is modern computers, the state space of which is 2-adic, and therefore, a computer is not just an auxiliary tool, but has theoretical significance-all the images obtained, for example, in computer tomography, are numerical fields and can be studied analytically via mathematical methods. On the other hand, a powerful technique of image analysis and processing is involved in the model, which, in turn, is based on an iterative-functions system (IFS), which is an isomorphic to Z_2 . The computer reproduces a universal pair of processes in the form of numerical calculations and forming different images [1].

This universal pair of forces exists in materials design as "bottom-up" and "top down" processes. The mentioned pairs of oppositions are connected by involutorial anti-isomorphism and form duality as asymmetry. It corresponds to the asymmetry of real and p -adic numbers and all mathematical constructions based on them. In particular, the often-mentioned duality of energy and entropy in thermodynamics completely fits into the proposed scheme. It has been shown that the extensive and intensive variables of thermodynamics connected via Legendre transforma-

tion correspond to two types of values of numbers, which are related by a power-law dependence that often appears in materials science

$$|x|_{\infty} = c \cdot |x|_2^{-d}$$

(the left side is the real number, the right side is 2- adic one, d is fractal dimension).

One of the important topics in materials science is the modeling of the internal space of non-crystalline materials. *p*-Adic numbers as invariants of infinite divisibility play role of coordinatization of the internal space, complementary to Euclidean. From the position of numerical asymmetry, it is possible to construct a quasilattice in the Lobachevsky space, which, replaces the Fourier transform, and implements a topologically equivalent pair of contravariant Galois correspondences. Using *p*-adic numbers it is possible to prove the existence of 5-fold symmetry and regular polyhedra with any number of faces [2].

Being a scale-invariant lattice, the inner space does not allow the use of classical probability, but it is consistent with the theory of possibility, which permit lattice operations. It models the gradual materialization of matter from atoms to the whole sample. This theory is free of tenets of classical probability [3]. Network presentation simulates the structure of the internal space and allows us to apply for a study of its movement via the theory of Kronecker matrices, in addition to the classical theory. In general, internal space is representable as a system - a holographic model of matter:

$$C \cong C_{matter} \cong \exp(C) \cong 2^C \cong Z_2 \cong [IFS \equiv \{0,1\}^N] \cong \\ \cong [Z_2 \rightarrow Z_2] \cong C(Z_2, Z_2) \cong H \cong C_{Bool} \cong C_{Stone} \cong C.$$

Here, from left to right - from top to bottom: matter with a fractal structure; the structure is invariant under its transformations; the truth set of Boolean algebra; 2-adic numbers; iterated function system as formal language (and its propositions); space of continuous functions (and their gradients/particle velocities); Hilbert space; Boolean algebra, Stone space, matter as a symbolic object (via Stone duality). Each member of this series of isomorphisms generates its own equation, which is equivalent to the amount of movement of a fractal sample C. Thus, obtained jointly

incompatible system of equations binds all parameters of a substance into an organically dependent composition [4].

Being mutually complementary, two numerical systems are mutually indefinite. They generate a representation of the measured number as

$$\|u\| = |x|_{\infty} \cdot |\xi|_2 \quad u = x \cdot \xi, \quad x \in R, \quad \xi \in Z_2$$

which is a synthesis determinacy and indeterminacy (stochasticity). Using this representation, the existence of the number of the golden ratio as defined by nature itself is proved, instead of the indefinable unit of standard man-made real numbers. Therefore, it makes sense to use a numerical system with the irrational basis of Bergman as the number of the golden ratio as its base, in replacement to of 2-adic numbers. Then it was possible to unite disparate results regarding the structure of the periodic table as the pseudo-sphere in the Lobachevsky space and the role of Fibonacci numbers in the distribution of elements, which as often discussed in scientific literature. The properties of this numerical system allow the existence of isomerism and isotopy of chemical elements. The duality of the numerical system of the model is consistent with the duality in chemistry - the combined action of matter (atoms) and fields (invisible, zero-dimensional formations, corresponds to Z_2).

Conclusion

The studies performed by the authors showed: firstly, the consistency of the model results with existing facts, and secondly, the possibility of expanding the arsenal of materials science with new results from the arsenal of chemical science.

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