Challenging a Methodology to Analyse the Cycling of Materials and Induced Energy use Over Time

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Submission: October 13, 2020; **Published:** November 11, 2020

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Abstract

This article aims to answer the following question: 'How to account for anthropogenic material cycling, energy use and greenhouse gas emissions? More specifically, the challenge is to understand how to model and conceptualise dynamically an input-output model structure that can represent an evolution from the current industrial structure towards a renewable energy driven industrial structure, using an optimal or least-energy (or emission) intensive pathway.

Keywords: Feedback loops; Input-output analysis; Physical accounting; System dynamics; Time delays

Introduction

There are multiple environmental and societal issues in our society, each with different magnitudes, specificities and characteristics. One of them consists to explore the variety of accounting and policy frameworks that relate to material use and climate change in order to design viable decarbonisation and dematerialisation scenarios. Tackling climate change is a collective responsibility and effort to be made, to avoid a collapse of our environmental base in the long term [1]. The decarbonisation challenge requires multidisciplinary solutions, as it is interlinked with almost all sectors of society.

One of the main driving forces of climate change is the use of fossil energy. We use energy to extract and transform materials for different goods, to heat or cool our homes, to extract fertilizer and produce food, to run electric appliances and to transport a variety of goods and services. The possibility to extract, transport, transform and recycle materials is thus strongly dependent on the energy available to do so. Therefore, the utilization of materials should — in light of the decarbonisation challenge — be contextualized within a broader view on how we produce and use energy.

Beyond the fact that it's necessary to provide a systemic perspective, aiming at understanding the decarbonisation challenge on a global level while pointing out interdependencies and long-term evolutions, the main question of this article concerns the methodological box (methods and data) that we are supposed to use to improve understanding of how to assess material, energy and emission flows and how to organize data collection. Because we use energy in all our activities, almost every product or activity embodies a certain amount of 'embodied energy'. When fossil energy reserves are used to generate this energy (without carbon capture and storage or reutilization), this entails a certain amount of 'embodied greenhouse gases (GHG)'. In the industrial sphere one of the main solutions to tackle climate change is thus to reduce the amount of embodied greenhouse gases of economic activities, and shift to either fossil energy with carbon capture or renewable energies such as solar, wind, wave, tidal, geothermal, forward osmosis, nuclear and biomass. The challenge is to co-design wisely a transition scenario and decarbonize our economy in an intelligent manner, carefully considering trade-offs between different energy and material utilization choices and associated impacts.

In this article, we suggest that input-output framework is the best suited starting point and 'methodological backbone' to theoretically advance the accounting of environmental impacts and physical understanding of our economy. Four reasons could explain this choice.

i. Existing institutional linkages and prospects for advancing official statistics (system of national accounts and input-output analysis). In a world with strongly interlinked supply chains and worldwide economic interactions, having an internationally standardized methodology to account for economic activities is imperative to increase our understanding of material exchanges and calculate reliable footprints. The methodological framework that adheres most to international standards and provides a platform to progress towards unified datasets and reliable accounting of supply chains, is the system of national accounts. The institutional linkage make it also the best suited framework that is able to link to statistical institutes and inform policy making, because of it's strong ground in empirical data collection and institutionalisation of data collection. Although the framework is frequently used to carry out monetary and economic analysis in contemporary institutions, the input-output modelling framework has its roots in physical accounting.

ii. Actor-Attribution with Consumption-Based Accounting (widening the system Boundaries). Because of the large extent of international material and energy exchange and interlinkage of supply-chains, harmonized datasets attributing physical impacts of production (CO_2-emissions, raw material extraction, environmental impacts) to final consumption is a necessary first step in understanding the full supply-chain impact of consumption. Despite the fact that legislative and political processes mainly take place at national or supranational level and that policies are inevitably contained within territorial borders, analysis of downstream and upstream impacts and increased international collaboration are a prerequisite for a successful mitigation of environmental problems, specifically climate change. The reason for this is a rather ethical one, as it is argued that beneficiaries of products and services should be held accountable for the material and environmental impact of consumption. A national or regional assessment of material consumption of an open economy in a globalized world should strive towards indicators with worldwide system boundaries which attribute material impact to the country or region where the goods are consumed. This can only be achieved with harmonized data exchange, which therefore points to a large extent to the IO framework (Poor & Nemecek 2018). There should be a political and institutional willingness to collect and provide those data, so the insights can be used in developing environmental policies embedded in trade legislation. This notion of consumption- versus production-based accounting has been previously reviewed by Peters [2] and further advanced by Malik et al. [3].

iii. Price- Data versus physical: For a complete and reliable understanding of the material and energy exchanges in the economy, physical data is unsurprisingly best directly sourced from the different actors in the industrial system and collected in internationally harmonized datasets. Unfortunately, this type of data does not (yet) exist to the level required to attribute environmental impacts reliably to final consumption, although

different communities are pursuing this collaboratively and advances are made in this direction. Some reasons explain this situation:

a) the difficulty of collecting such data;

b) privacy and intellectual property concerns;

c) it has not been the primary interest of governments and statistical institutes to provide enough resources to do so;

d) it is not straightforward to create a harmonized data collection and accounting system for a wide array of factors that are adapted to different types of measurement and analysis, from monetary valuation on macro-level, to tracing the amount of a specific rare earth metal within the economy.

Starting from monetary data to derive physical quantities using a price-weight relationship is problematic for many reasons. Firstly, the assumption of homogeneity of aggregated sectoral commodity prices across all uses is not guaranteed [4]. If there is a transaction occurring with a different price than average, this decreases the quality of the estimation [5]. Secondly, physical IO tables derived from monetary IO tables could deliver biased results due to the violation of mass balance or absence of mass balance principles in monetary tables [6]. Thirdly, imbalances result from aggregating in homogenous products consumed in different proportions by the users [7]. Fourthly, a more fundamental problem arises from the fact that with monetary input-output data there is only accounting for flows that have a price, not for flows that are difficult to account for in monetary values such as grazed biomass and fuel wood from forests [8].

A commodity flow can be analysed in monetary value or in weight of traded goods. For the purpose of physical flow analysis and calculation of recycling efficiencies in the whole economy, while avoiding the shortcomings of monetary analysis, the most accurate assessment would be making direct use of physical exchange data of goods between the different sectors in weight units (tonnes, kg, …) for each material. To some extent, this entails going back to one of the first papers of Leontief - the inventor of the input-output method, in which he described the economy as a "Circular Flow" [9,10].

Another option is to estimate weights based on a price-weight relationship, based on the assumption that each sector produced homogeneous products and that there is a linear relationship between the price and the quantity of goods produced. However, as explained before, these assumptions rarely hold when using aggregated product categories.

The main obstacles for a valuable and useful analysis of both the monetary and physical structure of the economy are data availability and accounting frameworks, the compatibility of accounting frameworks in monetary and physical units and lack of transparency and inter-institutional collaboration. Efforts are pursued out to enable consistent analysis of relationships both

in monetary and physical flows, for example in the System of Environmental Economic Accounting (SEEA) [11]. To cope with different conventions and definitions for Physical Input-Output Tables (PIOTs) and Monetary Input-Output Tables (MIOTs), Többen [12] set the principle of maximum entropy to statistical inference as a least biased estimator for a system under study to estimate simultaneously physical and monetary commodity flows from partial and incomplete data, different levels of aggregation and mismatching commodity classifications.

Level of aggregation: To achieve a full understanding of the material and energy exchanges in the economy, to design policies that take into account sectoral distinctions and to attribute impacts to final consumers, in principle there should exist a database which describes material exchange from and to the industrial system, but also between sectors in the economy. At the moment, a balance is to be sought between higher levels of aggregation that enable international comparison and harmonisation (for example, the IO-output framework), and tailored product- and sector-specific analysis that provides reliable information in a certain context (for example, Life Cycle Analysis). In an ideal situation, these two methods converge to one framework where all material and energy flows are being recorded between different actors in the economy.

The level of aggregation determines the level of detail in which transactions, monetary or physical, are registered or used. Currently, this has consequences for the type of analysis which is to be carried out. For example, if speciality metals are the focus of study, input-output tables are generally too aggregated to look at these specific flows. The high level of aggregation in sectors of the input-output framework does not allow to look at specific materials [5] but efforts are being made to break down the sectors to a finest level within a harmonized system.

All these issues will be explored in the following article. Firstly, we will present the input-output analysis (IOA) in the system of national accounts. Secondly, we will explain the challenge to change the unit from monetary input-output analysis to physical input-output analysis. Thirdly, we will introduce a methodology to account for total system energy and emission impacts of material recycling. Finally, we will discuss the question of static and dynamic approach of input-output analysis

Input – Output Modelling and the System of National Accounts

On a global level, data compilation on economic activity is regulated in the international System of National Accounts (SNA), an internationally agreed standard set of recommendations on how to compile measures of economic activity. The SNA has been developed and revised by the Inter Secretariat Working Group on National Accounts (ISWGNA) and is issued by the UN Statistics Division of the UN Department of Economic and Social Affairs (UN DESA) (Eurostat, International Monetary Fund, OECD, & United Nations 1993; United Nations, European Communities, International Monetary Fund, Organisation for Economic Co-operation and Development, & World Bank 2009) [13].

A similar accounting structure, the System of Environmental-Economic Accounting (SEEA) is used to derive indicators and statistics to describe the interactions between the economy and the environment. The main pillars of the SEEA are the Central Framework (SEEA-CF) [14] - an international statistical standard for environmental-economic accounting incorporating relevant environmental information concerning natural inputs, residual flows and environmental assets - and the Experimental Ecosystem Accounting framework (SEEA-EEA) [15]. The SEEA-EEA is a measurement framework starting from the perspective of ecosystems, integrating biophysical data, tracking changes in ecosystems and linking those changes to economic and other human activity. Examples of applications of the SEEA framework are provided in United Nations, European Commission, FAO, OECD, & World Bank [16].

The development of this environmental accounting system is coordinated at the international level by the UN Committee of Experts on Environmental-Economic Accounting (UNCEEA), established by the UN Statistical Commission at its 36th session in March 2005. The UNCEEA is group of high-level experts from national governments and international organizations with a broad range of experience in statistics and in the uses of environmental-economic accounts. The mandate of the UNCEEA is to mainstream environmental-economic accounting and related statistics, advocate for the SEEA to become an international standard and advance its implementation in countries.

The UNCEEA is assisted by several technical groups of which the most important one is the London Group on Environmental Accounting, established at the 27th Statistical Commission in 1993 as a City Group. When the UNCEEA was formed in 2005, their role as an expert body in charge of methodological issues was reconfirmed [17]. In March 2014, the Bureau of the UNCEEA inaugurated two other technical groups, the Technical Committee of the SEEA Central Framework and the Technical Committee on Experimental Ecosystem Accounting, to advance the work and development of core tables, accounts and associated technical notes for the SEEA-CF and SEEA-EEA. A more informal group consisting of practitioners, the Expert Forum on Experimental Ecosystem Accounting, also debates the development and advancement of the SEEA.

To provide methodological support and provide guidance to the implementation of the SEEA in countries of Eastern and South-Eastern Europe, the Caucasus and Central Asia, the ECE Joint Taskforce on Environmental Statistics and Indicators was established in 2009. They serve under the oversight of the Committee on Environmental Policy (CEP) and the Conference of European Statisticians (CES).

A working group specifically focusing on climate change indicators is the Task Force on a set of key climate change-related statistics and indicators using SEEA, established in 2014 [15] under the umbrella of the United Nations Economic Commission for Europe (UNECE). The objective of the Task Force is to define an internationally comparable set of key climate change-related statistics and indicators that can be derived from the System of Environmental-Economic Accounting.

These global rules and conventions are adapted at the European level [18]. The main body that is responsible for harmonizing statistical data on a European level is the European Statistical System Committee (ESSC), established by Regulation (EC) No 223/2009 of the European Parliament and Council of 11 March 2009 on European statistics. The ESSC is chaired by the Commission (Eurostat) and composed of the representatives of Member States' National Statistical Institutes and is tasked with setting priorities and harmonisation of statistical data collection and dissemination.

In the System of National Accounts, economic activities are classified depending on the type of analysis which is undertaken. The highest level of aggregation is usually on the level of industries, composed of different elementary units that undertake the same activity (agriculture, mining, …). These elementary units are commonly named establishments or local kind-of-activities (KOA) and they are commonly situated in a single location and carry out a single production activity. These establishments or local KOA are chosen to be homogeneous with regard to their activity. The UN has a set of guidelines in place to guide the classification of economic activities, the UN International Standard Industrial Classification (ISIC). In the EU, the classification used for grouping these elementary units is adopted from ISIC and further refined in the European Classification of Economic Activities (NACE) framework [19], after which it is adopted by the different Member States (Figure 1). ISIC and NACE have exactly the same items at the highest levels, where NACE is more detailed at lower levels [20]. On a lower level of aggregation, the classification of products (both goods and services) follows a similar logic with a UN Central Product Classification (CPC) which is implemented on EU level in the form of the European Classification of Products (CPA), subsequently implemented on national level. The classification of goods and services on international, European and national level is embedded in the structure of economic activities.
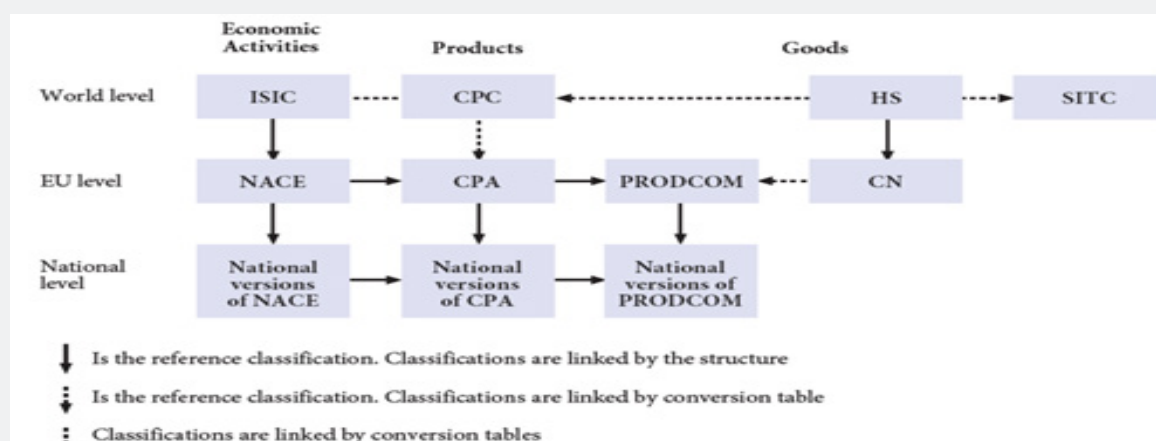

Figure 1: International system of classifications. Adapted from Eurostat [16].

For the purpose of organising international trade, a separate classification system - the Harmonized Commodity Description and Coding System (HS) - is maintained by the World Customs Organization to specifically classify goods that are traded. This classification is implemented at EU level as a Combined Nomenclature (CN) and feeds, together with the CPA, into a classification and database of manufactured goods (PRODCOM). A separate coding system based on the HS is used by the UN - the Standard International Trade Classification (SITC) - to allow for international comparison of commodities and manufactured goods.

From Monetary to Physical input – Output Analysis

A supply table (top half of Figure 2) contains the flows related to production, generation and supply of natural inputs, products and residuals. The use table (bottom half of Figure 2) contains the flows relating to the consumption and use of natural inputs, products and residuals. Supply and use tables give a detailed overview of the production process, interdependencies in production, use of goods and services and generation of income. Based on certain assumptions, these tables can be converted to symmetric input-output tables which can be used for input-output analysis.

Activities (*) which are supplying products (•) can be aggregated in a product-by-activity supply table $(V_{\cdot *})$. Product requirements of these activities are recorded in a use table $(U_{\cdot \cdot})$ and factors of production (★) requirements are recorded in an extension table $(G_{\cdot *})$ [21]. This extension table describes all requirement flows that cannot be fulfilled by the 'techno-sphere' within a given time period. A column h represents the

final consumption of products by households, governments and capital stock formation [22]. Inputs and outputs of industries can be recorded as observed, without specifying allocation (ex: supply of E and heat to electricity plant would be recorded as separate flows in the supply table and total use of fuel would be noted as one entry in the table –> no assumptions needed on attribution of inputs to different co-products) [23].

Physical SUTs

Although currently most data are collected in monetary units, physical supply and use tables are suggested to be used in the future to compile environmental accounts [14]. The current framework established by United Nations et al. [14] is based on the classification of monetary supply and use tables (MSUT) and adds additional columns and rows with physical flows - a physical supply and use table (PSUT) - that can record flows (a) from the environment or natural inputs, (b) within the economy or products and (c) back to the environment or residuals. Three different subsystems are used for material flow accounting (products, air emissions, solid waste and other residual flows), water flows and energy flows to allow for specific aggregation needs and different unit conventions. This allows for material, water and energy flows to be respectively expressed in mass, volume or energy content. Within each of the subsystems greater refinement can be obtained, which is specifically relevant for the distinction between different material flows. A full articulation of all flows is generally most relevant for energy and water, where all flows can be meaningfully expressed in a single unit (for example joules or cubic metres). The basic structure of a PSUT is given in Figure 2.

Figure 2: Structure of a Physical Supply and Use Table. Source: UN [15].

Balance principles

There are two important balancing principles which relate to the conservation of mass and energy in PIOTs.

a.   The supply-use-identity states that the total supply of a given flow type is equal to the total use of the same flow type. For PIOTs, this can be applied to production (amount produced = amount consumed), natural input use and supply and use of residuals.

b.   The input-output identity states that the physical flow into the economy (natural inputs, imports, residuals) is identical to the physical flow out of the economy (residuals, exports) plus net additions to stock (inventory changes, accumulation, residuals generated by industries).

Development of a Methodology to Account for Total-System Energy and Emission Impacts of Material Recycling

Input-output tables depict the exchange of goods between different sectors in the economy, expressed in monetary of physical units. These exchanges can be expressed using a set of n linear equations with n unknowns, which can be easily represented in matrix notation. Traditionally, the focus is on a country (such as the System of National Accounts), but these matrices can be constructed for any particular economic region. An example of a typical input-output transactions table for a national economy is given in Figure 3. Historically, the idea of systemic interconnections in the economy was first developed by Petty [24] and was later formalized by Quesnay [25] and further developed

as a paired accounting and modelling framework to account for indirect effects of inter-sectoral relationships by Leontief [26]. An interesting note here is that the Leontief framework described

below was originally developed to analyse sectoral exchange in physical units.

Figure 3: Input-Output Transactions Table. Source: Miller & Blair [10].

The flows of products from one sector to each of the other sectors are inter-sectoral (or interindustry) flows, measured for a certain time period in a certain unit (monetary or physical). For example, the monetary transaction between sector i to sector j can be represented as z_{ij} (one of the grey cells in Figure 3). Inter-sectoral transactions typically equal out, as the demand for inputs for a certain sector j from other sectors will normally be related to the number of goods produced by that sector j. On the other hand, there are *exogenous* sales to purchasers or consumers who are external to the industrial sectors that produce (termed *final demand*, depicted on the right in Figure 3). Examples of these are the government, households and foreign trade. In this case, demand is generally unrelated to the amount produced and goods are being used or consumed and are not used as input to another industrial or sector.

For the development of the fundamental relationships, the simplified table below will be used:

Sec*tors*	1	...	*j*	...	*n*	*Final Demand*	*Total Output*
1	z_{11}	...	z_{1j}	...	z_{1n}	f_1	x_1
2	z_{21}	...	z_{2j}	...	z_{2n}	f_2	x_2
⋮	⋮		⋮		⋮	⋮	⋮
n	z_{n1}	...	z_{nj}	...	z_{nn}	f_n	x_n
Labor	v_1	...	v_j	...	v_n	f_{n+1}	x_{n+1}

Hence, in an economy of n sectors and with f_i the final demand of the products of sector i, the total output of sector i x_i can be written as:

This final demand of a sector corresponds to one of the horizontal *producer's* rows in Fig 10. For each of the n sectors, such an equation can be formulated:

$$x_1 = z_{11} + ... + z_{1j} + ... + z_{1n} + f_1$$
$$\vdots$$
$$x_i = z_{i1} + ... + z_{ij} + ... + z_{in} + f_i$$
$$\vdots$$
$$x_n = z_{n1} + ... + z_{nj} + ... + z_{nn} + f_n$$

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$$\vdots$$
$$x_i = z_{i1} + ... + z_{ij} + ... + z_{in} + f_i$$
$$\vdots$$
$$x_n = z_{n1} + ... + z_{nj} + ... + z_{nn} + f_n$$

{#eq:io-final-demand-sector-all}

In matrix notation, this can be written as

$$x = Zi + f \qquad (1)$$

With

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad Z = \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix} \quad and \quad f = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix} \qquad (2)$$

Linking this with the general input-output transactions table in Fig 10, the column vector **x** corresponds to the list of producers, matrix **Z** corresponds to the grey matrix and column vector **f** corresponds to the final demand. In the following, a lower-case bold letter (**x**) will be used for column vectors, with **x**^{T} the corresponding row vector, and matrices will be written as upper case bold letters (**Z**). In eq. 1, the "summation" column vector of 1's **i** is used to create a column vector whose elements are the sum of the rows of a matrix, as displayed for each sector individually in eq. ¿**eq:io-final-demand-sector?** Similarly, pre-multiplication with the row vector of 1's **i**^{T} creates a row vector with the column sums of a matrix.

In monetary terms, the j'th column of **Z** in eq. 2 represent the sales to sector j from all the different sectors. Apart from these *inter-* and *intra* industry (within the same sector) flows, a sector also pays for employees, business owners and taxes to the

government, termed the *primary inputs* or *added value* of sector *j*. These also include imports from outside the national economy. The combination of primary inputs (or added value) **v** and imports **m** are lumped together as the *payments* sector. In Figure 3, this is the area *value added* under the grey matrix.

In the system of national accounts, the final demand f_i is typically further divided into consumer or household purchases (C), purchases for private investment purpose (I), government purchases (G) and sales abroad (E). These are often grouped as *domestic* final demand (C+I+G) or *foreign* final demand (C+I+G+E).

The payments sector, consisting of primary inputs and added value, is typically divided in employee compensation (L) and

other value-added items such as government services, capital, land, profit, ... (N). If there are imports used by the sector, these are traditionally recorded also in the payments sector (M). The *total value-added payments* of a sector $i(v_i)$ is thus equal to $l_i + n_i$, and the *total expenditure* of the payment sector is thus equal to

In the simplified two-sector flow table for a national economy given in Figure 4, the *z*-values are the inter-sectoral exchanges **Z** (the grey area in Figure 3) and for each of these processing sectors, the sum of *z, c, i, g* and *e* equals the total output of the sector *x* (see eq. ¿**eq:io-final-demand-sector?** and eq. ¿**eq:io-final-demand-detail?**).

		Processing Sectors		Final Demand			Total Output (**x**)
		1	2				
Processing	1	z_{11}	z_{12}	c_1 $\quad$ i_1	g_1	e_1	x_1
Sectors	2	z_{21}	z_{22}	c_2 $\quad$ i_2	g_2	e_2	x_2
Payments	Value Added (**v'**)	l_1	l_2	l_C $\quad$ l_I	l_G	l_E	L
Sectors		n_1	n_2	n_C $\quad$ n_I	n_G	n_E	N
	Imports	m_1	m_2	m_C $\quad$ m_I	m_G	m_E	M
Total Outlays (**x'**)		x_1	x_2	C $\quad$ I	G	E	X

Figure 4: Flow table of two-sector economy. Source: Miller & Blair [10].

The three different types of payment (employee compensation, other value-added items and imports) are paid by both the suppliers (processing sectors *z*) and the consumers (households *c*, private investment *i*, government *g* and sales abroad *e*). For example, employee compensation *l* can be paid by the processing sector (l_1), domestic help for households (l_c) or government workers (l_G). Imported items can be used by the processing sectors (m_1), the government (m_G) or re-exported (m_E).

Reading the table horizontally, the totals on the right represent the total output (or production or revenue) of each of the processing sectors (**x**) to other processing sectors and payment sectors, and the total payment of employee compensation (L), other value-added items (N) and imports (M) by the different sectors.

When reading the table vertically, the totals on the bottom of the table represent the total outlays (**x'**) or the sum of all the expenditures by the processing sectors (*x*) and the total household purchases (C), private investment (I), government expenditure (G) and exports (E) from the processing and payment sectors.

The total gross output throughout the economy X can be either

calculated vertically or horizontally, as the sum of payments (L, N and M) should equal the equal purchases by the different actors (C, I, G and E):

$$x_1 + x_2 + L + N + M = X + x_1 + x_2 + C + I + G + E$$

This can be rewritten as an equality between the *gross national income* (L + N, the total factor payments) and the totals spent on consumption and investment, government purchases and the total value of **net** exports as:

$$L + N = C + I + G + (E - M)$$

Assuming that there are no imports, another relationship can be observed, based on - in monetary terms - the added value *v*. In a physical input-output model, these values could be interpreted as the total physical inputs required for production in each of the sectors. The value v_j is the total required labour (in monetary terms) or inputs (in physical terms) required for the outputs of sector *j*, so:

$$x_j = \sum_{i=1}^{n} z_{ij} + v_j$$

or written in matrix form $$

Production functions and the input-output model

Demand-driven IO formulation (Leontief)

When it comes to analyse interdependencies between sectors, a frequently used concept is the concept of *technical coefficients*. That is, for a given z_{ij} (input from sector i to sector j) and x_j (total output from sector j), the ratio

$$a_{ij} = \frac{z_{ij}}{x_j} = \frac{value \,/\, quantity \;of\; \sec tor\; i\; bought\,/\,used\; by\; \sec tor\; j}{value \,/\, quantity\; of\; production\; of\; \sec tor\; j} \qquad (3)$$

represents the ratio between inputs from a sector i required for sector j to produce it's total output and the total output of sector j in a certain year. If - as in traditional IO analysis - monetary values are used, this technical coefficient represents the value worth of inputs from sector i to sector j per value worth of output of sector j. This relationship can be written for all the sectors in matrix form as

$$A = Z\hat{x}^{-1}$$

{eq:io-coefficient-basic-relation}

where $\hat{x}$ is a diagonal matrix with the elements of the vector along the main diagonal, and $\hat{x}^{-1}$ **is** the inverted matrix with the main diagonal filled with the elements $\frac{1}{x_n}$. This matrix **A** is called the *technical coefficient matrix*. The columns represent the *production recipes* per unit output for each of the sectors, in terms of its dependency on the other sectors.

To be able to extrapolate this relationship in time or use the technical coefficient to derive the inter-sectoral exchange (z_{ij}) for a different total output (x_j) using the relationship $a_{ij}x_j = z_{ij}$, a fundamental assumption must be made that is not always guaranteed. When assuming that a_{ij} is a fixed relationship between a sector's output and inputs, economies of scale in production are ignored and the system operates under the assumption of 'constant returns to scale'. This is certainly problematic when assessing inter-industry relationships in monetary terms but could be assumed to be less problematic when using physical exchange units.

Technical coefficients can also be used to assess the supply of two different sectors i and k to sector j using the proportion $p_{ik} = \frac{z_{ij}}{z_{kj}} = \frac{a_{ij}x_j}{a_{kj}x_j} = \frac{a_{ij}}{a_{kj}}$.

The assumption of constant technical coefficients results thus in another assumption of *fixed proportions*, being that a fixed input proportion of goods from different sectors is required for a certain final output of the receiving sector. Here again, this assumption is harder to make for monetary values than for physical exchange, as it can be assumed that for most products the physical requirement will be linearly related to the total production or that additional input from either one of the two inputs will not result in an increase in total output because of the fixed input proportion. This is an assumption that is different from the traditional production function structure in economics (isoquants), where the assumption of diminishing marginal productivity results in a decreasing or increasing proportion of inputs depending on the quantity used. This Leontief production function thus requires inputs in fixed proportions where a fixed amount of each input is required to produce one unit of output.

When accepting these assumptions, the production functions or output of the different sectors (eq. ¿**eq:io-final-demand-sector-all?**) can be rewritten using $z_{ij} = a_{ij}x_j$ as:

$$\begin{aligned} x_1 &= a_{11}x_1 + \ldots + z_{1j}x_i + \ldots + z_{1n}x_n + f_1 \\ &\vdots \\ x_i &= a_{i1}x_1 + \ldots + z_{ij}x_i + \ldots + z_{in}x_i + f_i \qquad (4) \\ &\vdots \\ x_n &= a_{n1}x_1 + \ldots + z_{nj}x_i + \ldots + z_{nn}x_i + f_n \end{aligned}$$

Knowing these inter-sectoral relations, and assuming that the final demand is known, a relationship can be developed which focuses on the effect of a change in final demand on the production rates of the different sectors. The set of equations in eq. 4 allow to analyse the effect of changes in total demand (or output) on the output of each of the other sectors. If the f_i's and a_{ij}'s are known, after bringing the x terms to the left and grouping equal x terms in each of the functions, eq. 4 can be rewritten as:

$$\begin{aligned} (1-a_{11})x_1 - \cdots - a_{1i}x_i - \cdots - a_{1n}x_n &= f_1 \\ &\vdots \\ -a_{i1}x_1 - \cdots + (1-a_{ii})x_i - \cdots - a_{in}x_i &= f_i \qquad (5) \\ &\vdots \\ -a_{n1}x_1 - \cdots - a_{ni}x_i - \cdots - (1-a_{nn})x_n &= f_n \end{aligned}$$

These relationships can be rewritten in matrix form for the n x n system as a set of n linear equations with n unknowns (x_1, x_2, ..., x_n):

$$(I - A)x = f \qquad (6)$$

with **A** the *technical coefficient matrix* and **I** the n x n identity matrix with the value *1* on the diagonal. Whether there is a solution for this set of equations, depends on the fact whether or not (**I-A**) is singular. Otherwise stated, (**I-A**)$^{-1}$ should exist. From the definition of an inverse of a square matrix, it follows that $(I - A)^{-1} = \frac{adj(I - A)}{|I - A|}$ with $dj(I - A)$ the adjoin of the matrix (**I-A**). If $|I - A| \neq 0$, $(I - A)^{-1}$ an be found, the unique solution to eq. 6 is given by

$$x = (I - A)^{-1} f = Lf \qquad (7)$$

Or

$$\begin{aligned} x_1 &= l_{11}f_1 + \ldots + l_{1j}f_i + \ldots + l_{1n}f_n \\ &\vdots \\ x_i &= l_{i1}f_1 + \ldots + l_{ij}f_i + \ldots + l_{in}f_i \qquad (8) \\ &\vdots \\ x_n &= l_{n1}f_1 + \ldots + l_{nj}f_i + \ldots + l_{nn}f_i \end{aligned}$$

with $(I - A)^{-1} = L$ the *Leontief inverse* or the *total requirements matrix*. This relationship endogenously calculates the intermediate production and primary inputs (imports, value added, wages) required for a given exogenous final demand, and can thus be used to derive the total requirements of each of the sectors for a certain output f_j. The individual values l_{ij} of the Leontief inverse can also be formulated as the partial derivative of x_i to f_j ($\frac{\partial x_i}{\partial f_j} = l_{ij}$).

To summarize, the inter-sectoral relations in the economy can thus be represented by a set of linear relations. The basic relation is that intermediate (**Zi**) and final (**f**) production are equal to total production for each of the sectors (**x**), as in eq. 1. The second relationship relates the intermediate production (**Z**) to the total production $(\hat{x})$ by means of a technical coefficient matrix **A**, as in eq. ¿**eq:io-coefficient-basic-relation?**. When substitution the second relation in the first, the relation of total production (**x**) to final production (**f**) can be derived, as in eq. 7. In matrix form, columns should be read as outputs or a sum of total outputs and rows are the inputs to the different sectoral activities or a sum of total inputs. Import and export are typically accounted respectively as primary inputs and final demand. The model described above can thus be used with different types of units. In the system of national accounts, these are typically monetary units. When using monetary units, the model above can be used to capture the direct and indirect (what's the difference here??! see Altimiras-Martin [27] effects of a change in final demand on the required sectoral inputs using the *Leontief inverse matrix*.

Supply-driven IO formulation

Instead of using *technical input-coefficients*, Ghosh [28] suggested another way to look at the input-output structure to relate effects of a change in total inputs on the total output of each of the sectors, depending on the inter-sectoral matrix **Z**. He suggests using *direct output coefficients* or *allocation coefficients* b_{ij} represented in a matrix **B**, instead of the technical coefficient matrix **A**. In this matrix, the element b_{ij} represents the outputs of sector j that is used as an input to sector i. In matrix terms, this means that **B** is constructed by dividing each row (inputs) of the inter-sectoral matrix **Z** by the total output of each of each of the sectors, instead of dividing the columns (outputs) by the total output of each of the sectors.

In contrast to the technical coefficient matrix (eq. ¿**eq:technical-coefficient?**), this matrix **B** can be derived as:

$$B = \hat{x}^{-1}Z$$

with the elements b_{ij} describing the number of products used by sector j from sector i per output of sector i:

$$b_{ij} = \frac{z_{ij}}{x_i} = \frac{value\,/\,quantity\ of\ \ sec\,tor\ i\ bought\,/\,used\ by\ \ sec\,tor\ j}{value\,/\,quantity\ of\ production\ of\ \ sec\,tor\ i}$$

Combining this with the description of supply to each of the sectors (eq. 3), this can be rewritten as:

$$x^T = i^T \hat{x}B + v^T = x^T B + v^T \quad (and\ i^T \hat{x}\ =\ x^T)$$
$$x^T = v^T (I - B)^{-1}$$

In this relationship, the **output inverse** or **Ghosh matrix G** with elements g_{ij} can be defined in a similar manner as how the **input inverse** or **Leontief matrix** is defined for demand-driven models:

$$G = (I - B)^{-1}$$

Here, the element g_{ij} describes the total output of sector j per unit input of sector i. This can be interpreted in a monetary or physical way, as is the case for the Leontief inverse. This formulation allows to calculate the effect of a change in inputs **v** on the outputs of the different sectors **x**, which can be formulated in column or vector matrices:

$$\Delta x^T = (\Delta v^T)G$$
$$\Delta x = G^T(\Delta v)$$

Accounting for material use and recycling

To understand the amount of materials that passes through the economy and attribute it to final consumers, a coherent framework is needed. One of the most used frameworks at international level that serves at calculating and comparing indicators on material use is the framework developed by European Communities [29] & OECD [30]. A generic definition of material use is provided by the European Statistical Office Eurostat, defining societal material use as all raw materials - except water and air - that serve production and reproduction of humans, livestock, built infrastructure, durable and non-durable goods and services as input to the human system [31,32] The main raw material inputs are thus "*plant harvest for food, feed, other energy uses and material input to industrial production; sand, gravel and crushed stone mainly for construction; metals and non-metallic minerals for industrial production and fossil energy carriers for both energetic and material applications*" [33].

To attribute material use on the **national level**, European Communities [29] & OECD [30] distinguish different indicators. An overview of the below described indicators is given in Figure 5.

a)   Domestic Material Extraction (DME)

a.   Direct Material Input (DMI) = DME + imports

b.   Total Material Input (TMI) = DMI + unused domestic extraction

b)   Total Material Requirement (TMR) = TMI + indirect flows (used and unused) associated to imports

c)   Domestic Total Material Requirement = DME + unused domestic extraction (additive along countries)

d)   Domestic Processed Output (DPO) = material flows released to the environment by the economy

e)   Total Material Output (TMO) = DPO + unused extraction

f)   Domestic Material Consumption (DMC) = domestically consumed products = DMI - imports

g)   Total Material Consumption (TMC) = TMR - exports and associated indirect flows

h)   Physical Trade Balance (PTB) = trade balance in physical units (how much does economy rely on domestic extraction vs imports?)

i)   Net Addition to Stocks (NAS) = accumulation of materials within economy.
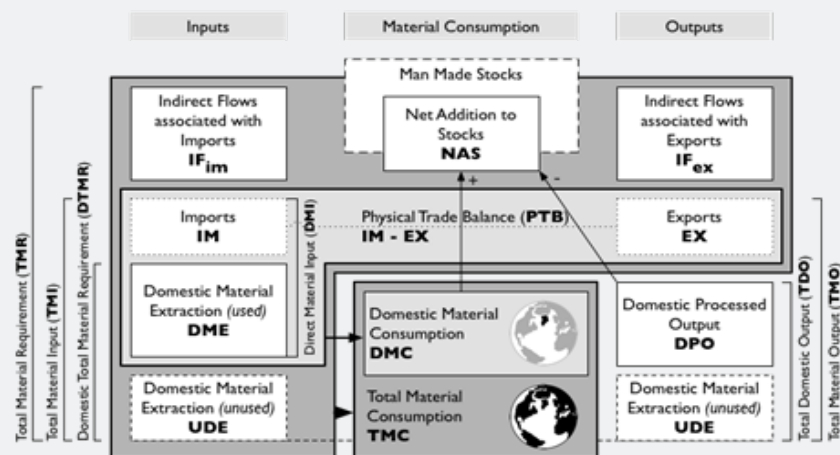
Figure 5: MFA Indicators.

On the national level, the accumulated amount of these materials is called the Domestic Material Consumption (DMC). To calculate reliable consumption-based indicators of material use - taking into account global supply chains and trade, the calculation framework and availability of reliable data becomes more critical.

Static vs Dynamics Systems

A model always starts with relationships between variables. An *independent variable x* is given, and the effect on a *dependent variable y* is measured under steady state conditions (all other variables influencing this relationship remaining equal). A line can be constructed that describe this relationship (a function), and the assumption could be made that this relationship can be used to predict or evaluate similar situations in the future or in another context. However, complications can arise in establishing

this relationship and rendering it useful for analysis in a different context [34]:

1. The quantity of *y* may depend on other variables

a. with a known quantity

b. unknown quantity or unknown existence;

2. There might be errors in the measurement of *y*, and possibly also *x;*

3. It might not be possible or practical to obtain enough data to establish a clear relationship between *x* and *y.*

Case (1.a) requires to approximate data points by a function of more than one variable, cases (1.b) and (2) require assumptions about the random nature and case (3) requires assumptions on the degree of confidence of the type of relationship.
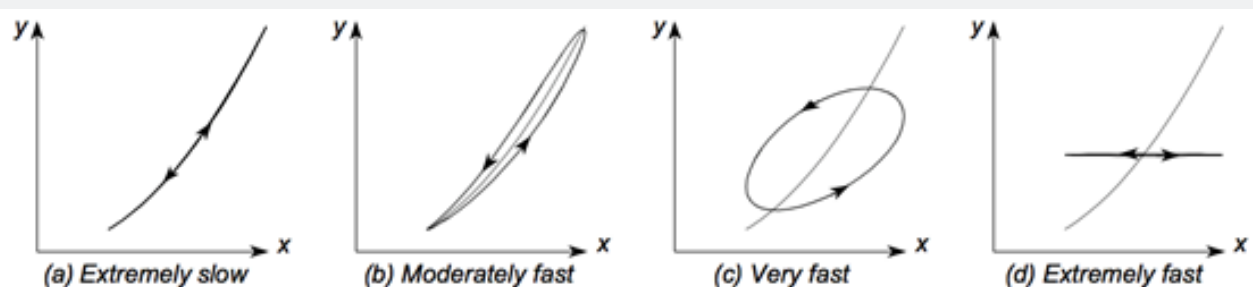

Figure 6: Dynamic relationship. Source: Benyon [34].

However, a fourth issue arises when the change of *y* depends on the **rate of change** of *x* over time. This is called a dynamic relationship of an independent variable on a dependent variable. An example of such a behaviour is given in Figure 6. Here, the effect of *x* on *y* is delayed, which becomes visible when changing the variable *x* fast enough. In cases where the rate of change of the

independent variable *x* can possibly change fast and where there is possibly a time-dependent effect of independent variables, it is relevant to consider a function in time. It can be assumed that all relationships - be it in engineering, ecology, economics or any other field - must show dynamic effects if only the independent variable is made to change fast enough. On the other hand, the

relevance of studying these dynamic effects depend on the cases considered. Another important aspect in assessing the behaviour of a system, is the difference between time-lags in reaction to the independent variable of the different components (dependent variables). If one such a relationship has a large time-lag, other relationships with a relative short time lag can be considered static relationships because the dynamic behaviour will never be apparent compared to other relationships in the considered system.

The practice of differentiating between static and dynamic relationships differs between different disciplines. However, a general common feature of dynamic relationships is the notion of storage [34]: "*We need to consider what it is about actual processes or components that gives rise to these dynamic effects. Since the effects consist, as we have seen, of a dependent variable being influenced not just by the value of an independent variable, but in addition by its speed and direction of change, there must be some means of sensing change. This implies some way of comparing current with immediate past values of the independent variable, and this in turn means retaining, somewhere in the system, something*

that is a measure or reflection of those recent past values. This amounts to saying that the system must incorporate some form of storage. We thus identify storage as being the key feature that we would expect to find in any mechanism, organism or process exhibiting dynamic effects".

All dynamic relationships can be represented by combining relationships that are either (1) instantaneous functions of more variables [static relation] and (2) a relation between a flow and an accumulated amount [dynamic relation]. Mathematically, this notion of storage and interdependency between these two type of relationships can be represented by taking the *integral* (accumulation S) of the net rate of change (N) of the dependent variable (inflow I - outflow O) and the effect over time of the total accumulation S on the outflow O:

$$N = I - O$$
$$S = \int N dt$$
$$O = F(S)$$

This notion of accumulation and rate of change is approached with different terminology in different fields (see table 1), but they all have the same conceptual foundation.

Table 1: Accumulation and rate of change - terminology differences in different academic fields.

	System Dynamics	Ecology	Economics	Process Control
Accumulation / variable of interest (dynamic)	Level	Compartment	Stock	State variable (not necessarily dynamic?)
Rate of change	Rate	Flow	Flow	

Conceptual link between system dynamics and systems's theory

The system dynamics methodology starts with working on a conceptual overview of interrelationships between variables, transferring it to a software environment (such as STELLA[1], VENSIM, …). These graphical environments are intuitive and accessible, but do not expose all the information necessary to accurately understand the structure of the system and its various behaviour when approaching it from a mathematical point of view. Any system dynamics model can be expressed as a system of differential equations. The main obstacle for comparison is the use of different terminology, symbols and definitions in system dynamics practice and classical mathematics / process control engineering. Therefore, below an overview will be given to explain the similarities between the different approaches.

The basic approach in system dynamics is to describe causal links in a causal loop diagram, using arrows to indicate the link between an independent to a dependent variable. These can form circular feedback loops, either increasing the rate of increase or decrease of a quantity of interest or balancing the rate of increase or decrease of this variable. These effects are traditionally termed *reinforcing* (positive feedback) or *balancing* (negative feedback). These causal loops are useful to derive the polarity and character of an interaction between variables Figure 7.

To get more insights in the size and rate of change of a variable, stock and flow diagrams (SFD) are used. The difference with causal loops is that they focus explicitly on the rate of change on a centrally defined variable of interest. Conceptually they could be the same as causal loop diagrams, but the representation is different. They describe the relationship between two variables

---

[1]Different possibilities were explored by Victor & Jackson [35] to analyse an input-output framework in the STELLA environment. The first option is to use the link routine of STELLA. This function allows to export and import array values (input-output table) from and to an excel sheet between the different iterations in the STELLA environment. However, this approach appeared to be impractical and slow because the large amount of data imports and exports between each iteration. The function is rather designed to import excel values one time and not continuously during the iteration process. Another option explored by Victor & Jackson [35] is using the array function in STELLA to include an input-output table and a table with associated technical coefficients in the model. This also appears to be unpractical and impossible to use, because STELLA is unable to calculate the inverse of a matrix which is required to calculate the Leontief equation. Finally, the third option is to directly replicate the set of linear equations, which allows the calculation of the change in sector output for each sector based on the initial values of sectoral outputs, final demands for each product and the direct input requirements per unit produced. A major drawback here is that it is still impossible to calculate the inverse of the matrix, which means that it is not possible to analyse the indirect effects of changes in sectoral outputs on other sectors.

as derivative of a variable over time, as in system dynamics, time is always the independent variable. The main problem of defining *stocks* and *flows* is that this wording suggests that they apply to physical processes, but in system dynamics they are often used to describe a rate of change of any type of variable over time. If the in- and outflows of a stock-and-flow diagram are expressed as *x = in - out*, the **rate of change of a variable (stock)** over a specific time interval (the *time step*) can be expressed as:

$$\dot{x} = \frac{dx}{dt} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{\Delta t} \quad with \ x \ = \ \inf low - outflow$$

This gives an idea of how much the variable changes over time. The value of the stock (X) itself can be represented as the integral of the total in- and outflow (x) of a stock over a time *t*, taking into account the initial value of the stock itself (*x(0)*):

$$X(t) = \int_0^t x(t)dt + x(0)$$

When combining the stock-and-flow notation with extra variables influencing the values of the stocks, this results in a system of differential equations describing the rate of change of the stocks over time, depending on the influence or relation to other defined variables in the model. This influence can be a linear relationship to either the inflow or outflow, or they can be 'loops' resulting in a feedback when the value of the stock itself influences the in- or outflow. An example of such a system is given in Figure 8.
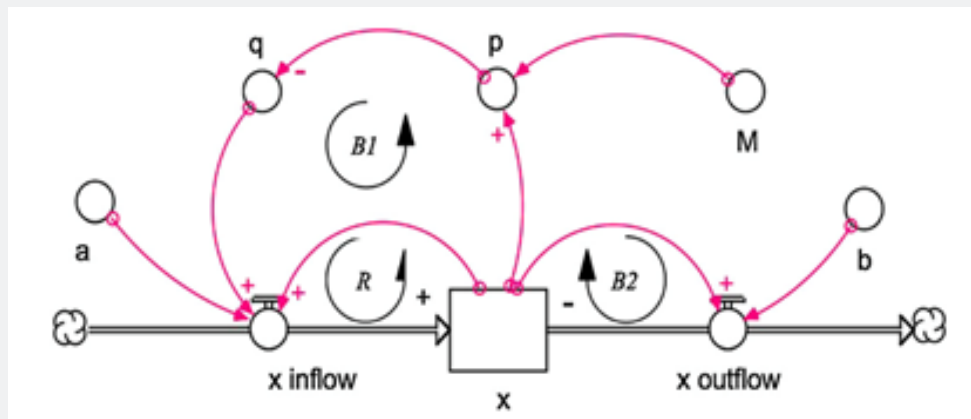

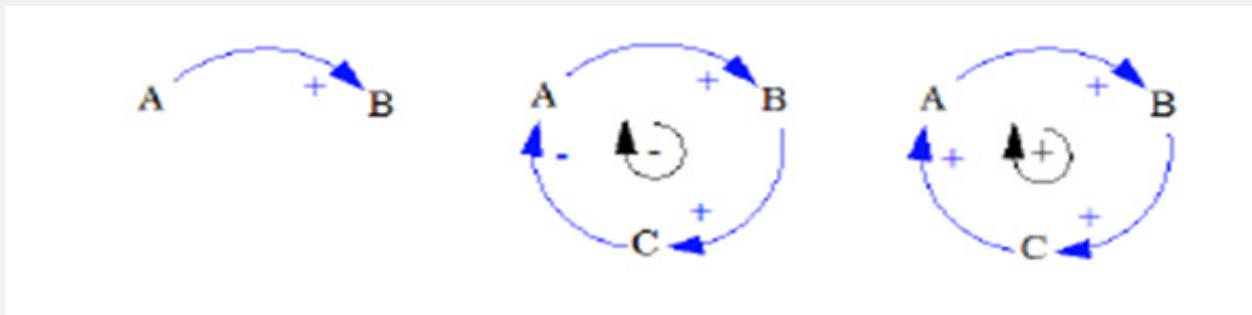**Figure 7:** System dynamics representation. Source: Hayward & Boswell [36].

Figure 8: Polarity Loops (+: Reinforcing Loop, -: Balancing Loop).

In this example, the change of x over time is dependent on the inflow ($axa = ax(1 - p) = ax(1 - \frac{x}{M})$) and the outflow bx. The change of the variable x in time can thus be written as:

$$\dot{x} = ax(1 - \frac{x}{M}) - bx$$

This is a first order differential equation (it only involves the first derivative of x), in which the input is determined by the effects of a reinforcing loop R (ax) which is countered by a balancing loop B1 (-ax²/M) and in which the output is determined by the balancing loop B2 (bx). When the derivative of x is zero (no input and no output), the non-zero equilibrium point is equal to $x_{eq} = M(1 - \frac{b}{a})$. This means that the content of the stock is stable if a>b, i.e. if there is enough growth compared to losses.

The second derivative gives information about the magnitude of change of the first derivative over time (i.e. the *loop impact*, see

Hayward & Boswell [36]). For example, if the second derivative is increasing, the slope of the tangent line to the function is increasing and the graph is concave up. For the formula above this gives:

$$\ddot{x} = \left[a(1 - \frac{x}{M}) - \frac{a}{M}x - b \right] \dot{x}$$

In this formula, the three terms $a(1-(x/M))$, (a/M) x and b are the magnitudes of change of respectively loop R, B1 and B2. This tells us that, as x increases, the magnitude of loop R decreases and the one of loop B1 increases. Therefore, loop R initially dominates (has an impact larger than the sum of the two balancing loops). To compute loop overall loop dominance, the magnitude of the loops of equal polarity (either R or B) should be summed up to have an idea of the overall effect. Depending on how system boundaries are defined, a there are exogenous and endogenous variables influencing the behaviour of the system. Exogenous variables are considered to be independent variables outside the system boundaries influencing the behaviour of the system by influencing the endogenous variables, and endogenous variables are generated inside the system as a result of interactions between the direct past of exogenous and endogenous variables (see Figure 9a). Even if there is an assumed delayed effect of endogenous variables on the system, these can always be represented as a delayed effect so that all exogenous variables always and only depend on the immediate past of the state of a system (Figure 9b).
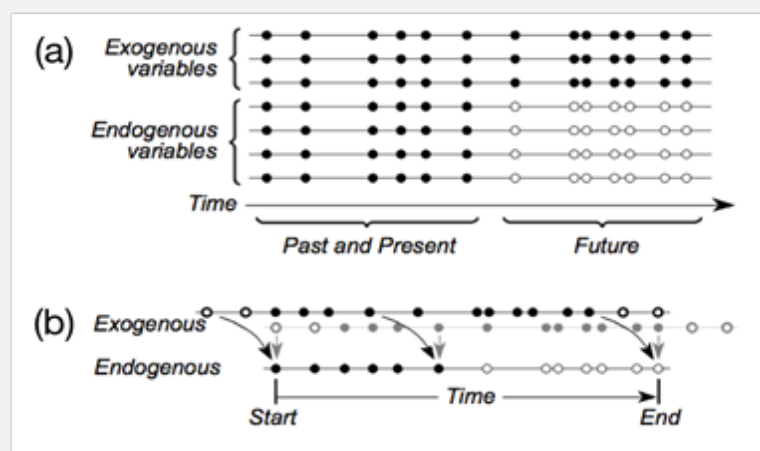

Figure 9: Adapted from Benyon [34].

If we consider the endogenous variables, these can be further broken down to the smallest possible subset of system variables that can represent the entire state of the system at a given time t [37], and of which the value from the past influences the behaviour of the system in the present. Otherwise stated, if there are variables that are dependent on an initial set of starting variables, they are not part of the set of state variables. The variables of interest are named *state variables*.

The transition of the different state variables can be represented in a *state transition diagram*, indicating at which moment each variable changes over time. Although there are conceptual differences in considering time, the range of values a variable and the change itself as continuous or discrete, they can all be approximated by considering them all having discrete characteristics (see Figure 10). The distinction between discrete or continuous variables is a rather theoretical distinction, depending on whether it is appropriate to consider the system as continuously changing or only changing between certain time-intervals. Mathematically, complex systems are normally simulated in discrete time with approximation methods, but they can conceptually be considered continuous. If there are n different variables describing the state of the system, these can be represented as a vector in an n-dimensional Euclidian space or *state space* in which there is a different axis for each variable.

In a system, the **inputs** are the exogenous variables, or controlled or uncontrolled actions of the system's environment on the system. The state variables are all those endogenous values that are needed to determine the current values of the endogenous variables. The **outputs** are the remaining endogenous variables that are of interest for the purpose of monitoring the system, measurable or observable. The analogy with system dynamics here is that state variables can be interpreted as stocks, and the inflow of a stock (derivative of the state variable) will be in its most general case a function of all the state variables (stocks) and the inputs. The order of the system's set of defining deferential equations n is equal to the number of stocks (variables changing over time), as this formulation is conceptually identical to first derivative of the stock variable, or the change of the stock variable over time.
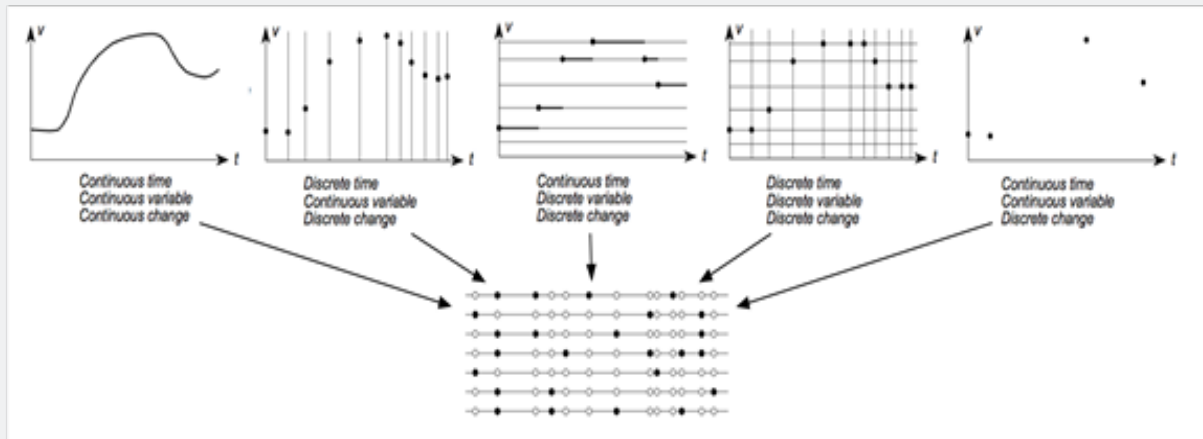
Figure 10: Continuous and discrete change systems can all be modelled as discrete time systems. Adapted from Benyon [34].

Solving strategies (process control)

In dynamic systems theory, a system $\sum$ is characterised by a set of state variables $x(t)$. These state variables are influenced by the input variables $u(t)$ that represent the actions of the environment on the system. The output variables $y(t)$ represent the observable or measurable aspects of the the system's response. A basic representation of such a system is given in Figure 11.
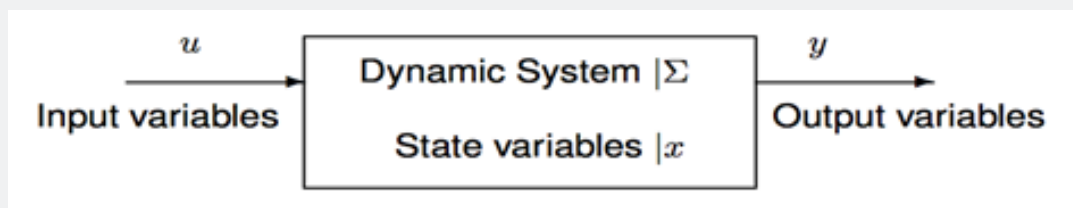

Figure 11: General dynamic system model.

This model can be used in five different ways [38]:

1.   Analysis or **simulation:** Given a future trajectory of the inputs $u(t)$ over time, what would be the future outut of the system $y(t)$?

2.   Model identification: Given a history of inputs $u(t)$ and outputs $y(t)$, how does the system and its state variables $x(t)$ look like? A 'good' model is one that is consisent with a large variety of inputs and outputs. This approach is often used in machine learning.

3.   State estimation: Given a system $\sum$ and a history of inputs $u(t)$ and outputs $y(t)$, what are the state variables $x(t)$ that best describe the behavior of the system? In this case, an algorithm is searched to estimate unmeasurable states if not every state is measurable.

4.   System design: Given an input $u(t)$ and a desired output $y(t)$, what would be the best possible system such that the input results in the desired output? This approach is a typical engineering approach, useful to test prototypes of assumptions on the best design of a system.

5.   Control synthesis: Given a system $\sum$ with a current state $x(t)$ and a desired output $y(t)$ what would be the inputs $u(t)$ such that the system produces the desired output?

This approach is frequently used when trying to control the energy or material flow of a system

The fifth method of control synthesis is thus very useful to look at the energy and material flows in an economy, and to determine what the possible downstream effects are when simulating a decrease or increase of final energy and material use. In this sense, a (physical) input-output model can be formulated as a dynamic relationship between inputs and outputs, related through the design of the system $\sum$ (with the structure of the input-output relationships as input-output analogy) and the size

of the variables $x(t)$ describing the system (with the *technical coefficient matrix* or *Leontief-matrix* as input-output analogy).

State-space representation (process control)

If the outputs $y(t) \in \mathbb{R}^q$ of a system $\sum$ can be described as a linear combination of the inputs $u(t) \in \mathbb{R}^p$ and state variables $u(t) \in \mathbb{R}^n$ and if the rate of change of the state variables (first derivative $\dot{x}$) is linearly dependent on the state variables and input variables, the most general representation of a state-space of a first-order linear system is:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

With *A* the *state or system matrix* (dim[A]=*n x n*), *B* the *input matrix* (dim[B]=*n x p*), *C* the *output matrix* (dim[C]=*q x n*) and *D* the *feedthrough matrix* (dim[D]=*q x p*). In cases, there is no direct feedthrough, the matrix D is equal to zero. This state-space model can be either in continuous or discrete time, and matrices are allowed to be time-variant. A block-diagram representation of such a system is represented in part (a) of Figure 12:
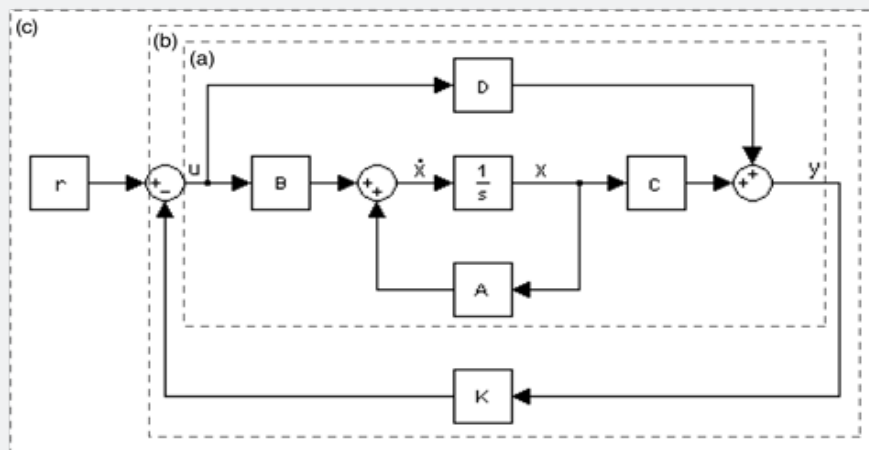

Figure 12: Block diagram representation of the state-space model. Adapted from Wikimedia Commons [39].

Dynamic input-output analysis

In traditional input-output analysis, the technical coefficient matrix **A** measures the flows between sectors, to serve current production in a particular time interval (or time-step, generally a year). Each of the inter-sectoral flows z_{ij} serves as the input for the final output x_j at a certain moment, and these are reflected in the technical coefficients $a_{ij} = z_{ij} / x_j$ of matrix **A**. However, if the inputs contribute to the production but are not immediately used during the production during the specified time interval, they can be represented as stocks. This has also relevance when a sector needs a permanent stock of a certain input to function. An input from sector *i* that is held by sector *j* can be represented as the *stock k_{ij}* and from this stock a coefficient $b_{ij} = k_{ij} / x_j$ (traditionally named *capital coefficient*, or *stock coefficient*) can be derived that represents the amount of produce from sector *i* that is held as a stock per unit output of sector *j*. These coefficients form together the *stock coefficient matrix* **B** [10] and they might be considered as monetary or physical material stocks. For energy analysis, the stock concept might apply to the storage of energy, which is only meaningful if there are energy storage capabilities which exceed the time-step of the model.

A general way of working is to measure the products of sector *i* that are held as stocks in sector *j* in a certain year (k_{ij}) and derive the stock of sector *i* needed to produce one unit (or kg) of sector *j's* output (b_{ij}). One could also assume that the number of new products from sector *i* that will be used as a stock for sector *j* in the next year (t+1) will be linearly dependent on the difference between current and new production $(bij(x_j^{t+1} - x_j^t))$. Taking into account the final demand for products from sector *i* (f_i), the equation for the production of sector *i* in period *t* is thus:

$$x_i^t = \sum_{j=1}^n a_{ij}x_j^t + \sum_{j=1}^n b_{ij}(x_j^{t+1} - x_j^t) + f_i^t$$

or rewritten in function of final demand:

$$x_i^t = \sum_{j=1}^n a_{ij}x_j^t + \sum_{j=1}^n b_{ij}x_j^t - \sum_{j=1}^n b_{ij}x_j^{t+1} = f_i^t$$

This can be generalized in matrix form for all the different sectors:

$$(I - A)x^t - B(x^{t+1} - x^t) = f^t$$
$$(I - A + B)x^t - Bx^{t+1} = f^t$$
$$Bx^{t+1} = (I - A - B)x^t - f^t$$

This is a difference equation, representing the change over time in discrete time intervals. As with any difference equation, this relation can be converted to a continuous differential equation

when the time-step is made very small (and the difference $x_j^{t+1} - x_j^t$ approaches the derivative $\frac{dx_j}{dt} = \dot{x}_j$. The continuous analog of the previous relationship is thus

$$x_i = \sum_{j=1}^{n} a_{ij} x_j + \sum_{j=1}^{n} b_{ij} \dot{x}_j + f_i$$

which can be rewritten in matrix form as

$$B\dot{x} = (I - A)x - f$$
$$x = Ax + B\dot{x} + f \qquad (9)$$

Here, $\dot{x}$ denotes the time derivative of the production rates

for the different producing sectors, f denotes the final demand for each of the sectoral outputs, **A** is the technical coefficient matrix with the elements *ij* indicating the ratio of products of sector *i* to sector *j* per output of goods produced by sector *j*, **B** is the stock coefficient matrix where the elements *ij* indicate the rate of the stock of goods produced by sector *i* that is held by sector *j* to the total output of sector *j*. To make the link with system dynamics clear, this formulation can be visually represented in a system dynamics diagram as: (Figure 13)
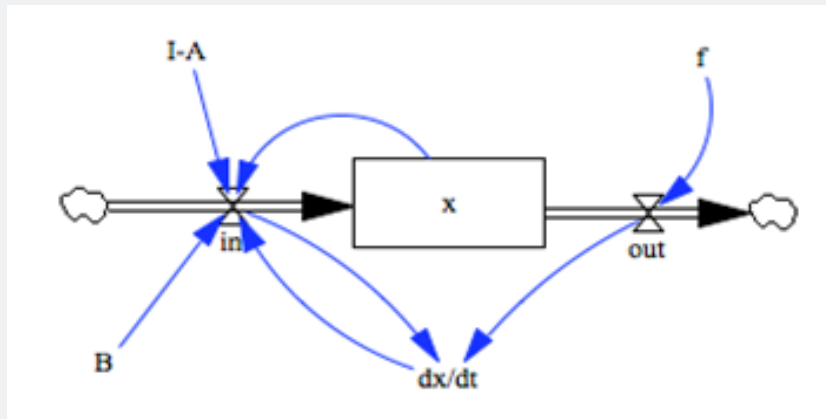

Figure 13: System dynamics representation of dynamic input-output model.

Extension of the dynamic Leontief model with renewable resources

An extension of the traditional Leontief demand-driven model to include the effect of renewable resources on dynamic inter-sectoral links was done by Dobos & Tallos [40]. Their main methodological aim is to assess whether the use of renewable resources can be controlled by altering the consumption, from a process control perspective, and how the rate of regeneration of the renewable resource influences the growth rate of consumption and production. To analyse this, eq. 9 should be complemented with a relationship describing the stock of renewable resources, rates of regeneration and input coefficients describing the use of natural resources in the economy. Therefore, the following relationship is used:

$$\dot{n} = \hat{g}n - Ex \qquad (10)$$

with $\dot{n}$ an m-dimensional positive vector of renewable resources, $\hat{g}$ an m-diagonal matrix with the rates of regeneration of the renewable resources on the diagonal, *E* the *m x n* matrix of input-coefficients of resources to the different sectors in the economy, with the element e_{ij} indicating the requirement of resource *i* to produce one unit output in sector *j* used to determine the extraction of renewable resources Ex.

The two relationships, eq. 9 describing the inter-sectoral

relations and rate of change of the sectoral stocks and eq. 10 describing the depletion of renewable resources, can be summarized in matrix form as:

$$\begin{pmatrix} B & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} 1-A & 0 \\ -E & \hat{g} \end{pmatrix} \begin{pmatrix} x \\ n \end{pmatrix} - \begin{pmatrix} I \\ 0 \end{pmatrix} f$$

This system is controllable. That is, the system can be steered from any initial state to any other state in a finite time period by means of suitable choice of control function (which is in this case, consumption **f**). A mathematical proof of this is given in Dobos & Tallos [40]. Because the controllability does not exclude negative control (consumption) values and negative state variables, the model is complemented with an additional assumption that there is a balanced growth path of both consumption and production. That means that $x = x_0 e^{\alpha t}$ and $f = f_0 e^{\alpha t}$ with $\alpha \geq 0$ the growth rate. Combining these with eq. 9, learns that the initial output x_0 of the balanced growth path depends on the growth rate and the initial consumption c_0:

$$(I - A - \alpha B)x_0 = c_0$$
$$with \; x_0(\alpha, c_0) = (I - A - \alpha B)^{-1} c_0$$

In a similar manner, the evolution of the stock of renewable resources can be derived by substituting the balanced growth formulation of *x* into eq. 10:

$$n(t) = e^{\hat{g}t} n_0 - (e^{\hat{g}t} - e^{\alpha It})(-\hat{g}\alpha I)^{-1} E x_0(\alpha)$$

which learns the intuitive thing that the renewable resources will not be fully exhausted if the consumption and production growth rate (α) is lower than the regeneration rate of the renewables $(\hat{g})$. This system can be represented in a system dynamics formulation as [41]: (Figure 14)
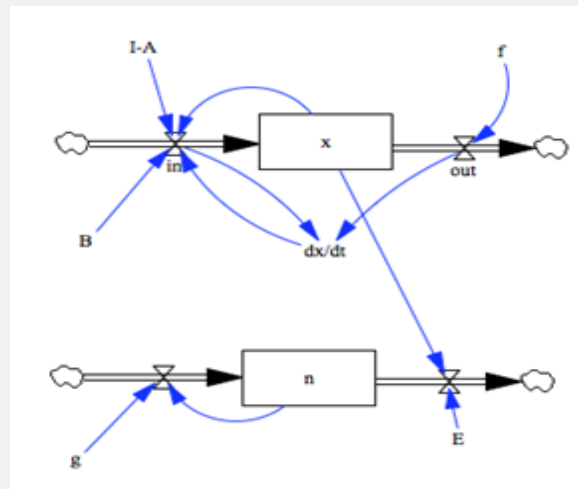

Figure 14: System dynamics representation of dynamic input-output model with stock of renewable resources.

Conclusion

To mitigate material and climate impacts along the supply chain, an integrated monitoring framework is necessary that leverages the knowledge from all actors in the economy - from producers to consumers. Such a system is imperative to design intelligent environmental and social policies. Such a system would need to be integrated and have institutional backup at different levels. Considering the interlinkages of our global economy, the UN seems to be the first go-to actor to lay the foundations for such an accounting framework.

At the same time, different aspects are important in determining the feasibility and progress towards global and regional decarbonisation. The speed at which we will be able to decarbonise, will depend on a variety of factors, but the baseline of such a transition is the availability of a labour force, the exchange of knowledge and expertise on renewable energy technologies, collaboration between nation states and regions, and last but not least, the availability of sufficient material resources to organise such a transition.

In this article, we insist on the fact that methodological box is quite important to understand how to assess material, energy and emission flows and how to organize data collection. Because we use energy in all our activities, the challenge is to design a transition scenario and decarbonize our economy, considering trade-offs between different energy and material utilization choices and associated impacts. We suggest that input-output framework is the methodological backbone to account environmental impacts and physical footprint understanding of our economy.

An interesting approach would be to bring the input-output analysis closer to System Dynamics. The economic sectors would continue to provide a relevant representation of economic reality. However, a flow - stock analysis, integrating feedback loops and time lags, would make it possible both to account for the different pathways of decarbonisation policies and to switch to physical accounting. In the European System of Accounts, physical exchanges are not recorded - "*The ESA 2010 system records all transactions in monetary terms. The values to be recorded for non-monetary transactions must therefore be measured indirectly or otherwise estimated*" [17], but considering the scale of the decarbonisation in the coming decade, and the need for a just and fair transition where everybody contributes a fair share of the decarbonisation, personal carbon quota systems and trading schemes seem promising physical accounting to new research's developments.

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