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A Mathematical Analysis of Blind Spots within Enclosed Spaces

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Abstract

In monitoring and security scenarios, particularly within construction sites, factories, and other enclosed environments, understanding and managing blind spots is crucial for enhancing surveillance effectiveness and operational safety. This study presents a mathematical framework for analyzing blind spots in rectangular enclosed spaces, focusing specifically on how room geometry and entrance placement influence visibility. By modeling these rooms on a two-dimensional coordinate plane, we establish equations for defining blind spot regions and provide methods to accurately calculate their areas. Our findings demonstrate that positioning entrances at the center of a wall minimizes blind spot areas, significantly enhancing surveillance coverage. Conversely, placing entrances at wall edges maximizes blind spots, potentially benefiting contexts where privacy and reduced external visibility are desired. These insights offer practical strategies for architectural planning, surveillance system design, robotic monitoring, and AI-driven autonomous navigation. Further research could explore more complex scenarios involving irregular room geometries, multiple entrances, and dynamic obstacles, broadening the applicability and effectiveness of blind spot optimization in diverse real-world environments.

Keywords: Blind spot; Mathematical framework; Room geometry; Entrance placement; Surveillance; Monitoring; Architectural design

Introduction

Effective monitoring and supervision are crucial at construction sites, commercial buildings, and factories producing industrial goods to prevent accidents involving hazardous materials and unauthorized intrusions. However, continuous, real-time human patrols of large areas are impractical and inefficient. Consequently, there is increasing adoption of robotic surveillance systems equipped with visual recognition and mobility capabilities to overcome these limitations [1,2]. When deploying robotic systems for surveillance, managing blind spots-areas not visible from certain vantage points-becomes critically important. Undetected blind spots can significantly compromise the safety, efficiency, and effectiveness of monitoring operations [3]. Furthermore, attempting to cover these areas may result in longer patrol paths and increased energy consumption, reducing overall operational efficiency. Thus, a thorough understanding of the spatial factors and architectural features contributing to blind spot formation is essential.

Urban buildings and industrial facilities typically feature rectangular layouts and include structural elements such as doors, internal partitions, rooms, and columns, which inherently obstruct visibility and create blind spots [4]. This study addresses the challenge by employing mathematical framework to analyze blind spots within enclosed rectangular environments, exploring the effects of room geometry and entrance placement on their formation. The insights derived from this research have practical implications for enhancing robotic surveillance efficiency in construction and industrial settings. Additionally, the findings are valuable for architectural planning in residential areas, commercial complexes, correctional institutions, and may further inform military strategy, autonomous navigation systems, and smart infrastructure development.

This research is grounded in established visibility analysis methods. Benedikt [5] introduced the concept of “isovists,” defining them as spatial polygons representing all areas visible from an observer’s location. These isovists are typically visualized as shaded regions extending from a point on a floor plan, clearly delineating visible and hidden areas [5,6]. Turner and colleagues [7] further expanded this concept into visibility graphs, modeling spatial relationships through interconnected nodes representing specific locations. This approach identifies areas of low visibility and informs strategic design and layout decisions. Building upon these foundational methods, researchers have developed advanced mathematical tools, including visibility polygons and computational geometry algorithms, to quantify visibility precisely within enclosed environments [8,9].

This study specifically adopts an isovist-based approach, applying geometric modeling techniques to clearly define and calculate blind spot areas in relation to room geometry and entrance positioning. Through quantitative analyses, this research aims to elucidate how entrance placements influence blind spot formation within two-dimensional rectangular rooms. By establishing these relationships, the study offers practical and actionable recommendations that enhance the effectiveness of robotic surveillance, inform architectural design, and support strategic security planning. Consequently, this work bridges theoretical visibility concepts and their tangible applications in real-world scenarios.

The remainder of this article is structured as follows: First, we introduce the key variables that influence the size and shape of blind spots and establish a mathematical framework for their analysis. Next, we present strategies for minimizing or maximizing blind spot areas. Finally, we summarize the study's findings and discuss their practical applications in surveillance, monitoring, architecture, and military contexts.

Variables determining the blind spot and its shape

In this section, we examine the area and shape of the blind spot. First, we define the room's structure as follows. For simplicity, let's place the room on a coordinate plane, with the lower outer wall containing the door as the x-axis and the left wall of the entrance as the y-axis. The intersection of these two axes is labeled as point O(0,0). We will label the four corners of the room as points A, B, C, and D, while the corners where the room connects to the exterior, excluding point O, are labeled as points E, F, and G, as shown in Figure 1.

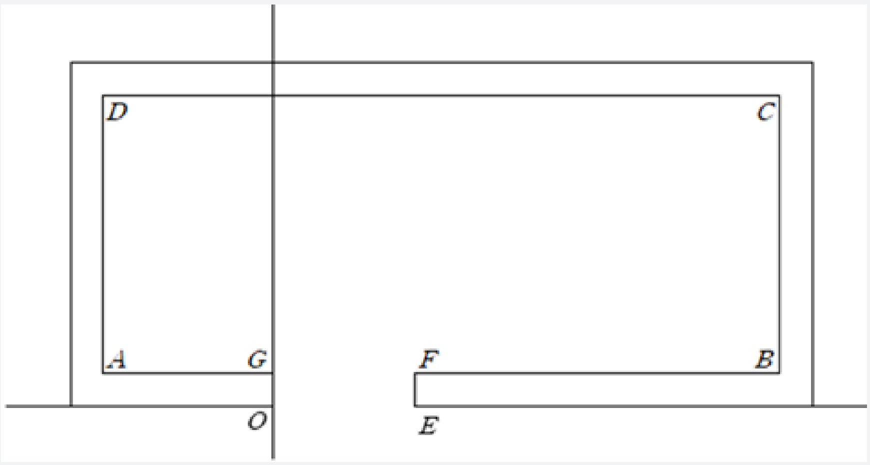

Figure 1: The room structure for blind spot analysis 1.

Next, we assign length variables. The length of $\overline{AG}$ is denoted as L_1, and the length of $\overline{FB}$ as L_2. The entrance width $\overline{GF}$ as L, the wall thickness is assumed to be uniformly w. Additionally, the vertical length of the room is defined as H, measuring that $\overline{BC}$ and $\overline{AD}$ both have a length of H. Under this setup, we can determine the coordinates of all points described in Figure 1 as the following mathematical expression (1):

$$A(-L_1,w), B(L+L_1,w), C(L+L_1,w+H), D(-L_1,w+H),$$
$$E(L,0), F(L,w), G(0,w), O(0,0) \quad (1)$$

To illustrate blind spots, we need to draw two lines: one passing through points O and F, and another passing through points G and E. The intersections of these lines with segments $\overline{BC}$ and $\overline{AD}$ are designated as points X and Y, respectively. If the room's vertical length H is too short, these lines may intersect segment $\overline{CD}$ before reaching $\overline{BC}$ and $\overline{AD}$. To ensure the validity of this construction, we assume that H is sufficiently long (Figure 2).

Thus, the blind spot consists of two triangular sections: △AGY and △BFX. Each of these is a right triangle, and to determine their side lengths, we must first establish the equations of the lines $\overleftrightarrow{OF}$ and $\overleftrightarrow{GE}$ drawn earlier.

Case 1: Equation of line $\overleftrightarrow{OF}$ and coordinate of X

Since the given points are O(0,0) and F(L, w), the slope of the line $\overleftrightarrow{OF}$ is $\frac{w}{L}$. Because $\overleftrightarrow{OF}$ passes through the origin, its equation is (2):

$$y = \frac{w}{L}x \quad (2)$$

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Since point X is the intersection of the line $\overrightarrow{OF}$ and segment $\overline{BC}$, we need to determine the equation of segment $\overline{BC}$. Since the segment $\overline{BC}$ is parallel to the y-axis and has the fixed x-coordinate L+L_2, its equation is (3):

$$x = L + L_2 \quad (3)$$

Therefore, the intersection between the following two equations, x=L+L_2 and $y = \frac{w}{L}x$ can be calculated as the following mathematical expression (4):

$$(x, y) = \left(L + L_2, \frac{w}{L}(L + L_2) \right) = \left(L + L_2, w + \frac{wL_2}{L} \right) \quad (4)$$

Thus, the coordinate of point X can be mathematically expressed as (5):

$$X\left(L + L_2, w + \frac{wL_2}{L} \right) \quad (5)$$

Therefore, the two side lengths of △BFX excluding the hypotenuse can be determined as L_2 and $\frac{wL_2}{L}$.

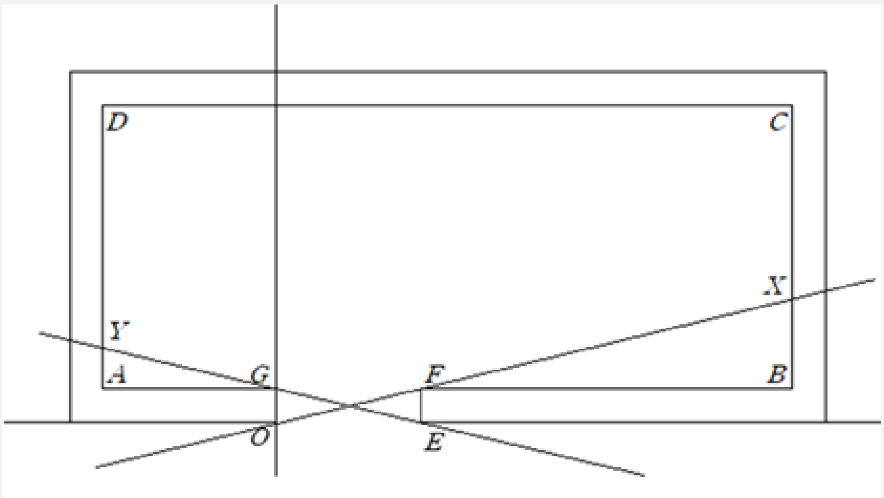

Figure 2: The room structure for blind spot analysis 2.

Case 2: Equation of line $\overrightarrow{GE}$ and coordinate of Y

Since the given points are E(L, 0) and G(0, w), the slope of the line $\overline{GE}$ is: $-\frac{w}{L}$. Thus, the equation of this line can be expressed by the by the equation (6):

$$y = -\frac{w}{L}x + c \text{, for some constant c} \quad (6)$$

Since this line passes through E(L, 0), by substituting this point into the above equation we get c=w. Therefore, the equation of the line can be found as the equation (7):

$$y = -\frac{w}{L}x + w \quad (7)$$

Similarly, the equation of the line containing segment $\overline{AD}$ is given by: x=-L_1. By solving a system of linear equations (8):

$$y = -\frac{w}{L}x + w \text{ and } x = -L_1 \quad (8)$$

we can determine the intersection between the two lines (point Y) as the following mathematical expression (9):

$$y\left(-L_1, \frac{wL_1}{L} + w \right) \quad (9)$$

Therefore, the two side lengths of triangle △AGY, excluding the hypotenuse, can be calculated as L_1 and $\frac{wL_1}{L}$.

Methods to Minimize or Maximize the Blind Spot

In the previous section, we established that the blind spot consists of two right triangles and determined the lengths of their non-hypotenuse sides. In this section, we analyze how to minimize or maximize the area of the blind spot. Minimizing the blind spot is crucial for those monitoring the room's interior from the outside-such as in a prison setting, where surveillance of confined individuals is necessary. Conversely, maximizing the blind spot benefits individuals seeking to remain hidden within the room.

The blind spot consists of two triangles, △AGY and △BFX. Recalling that the lengths of the two non-hypotenuse sides of △BFX can be calculated as L_2 and $\frac{wL_2}{L}$, while those of △AGY can be calculated as L_1 and $\frac{wL_1}{L}$, thus the areas of two triangles can be expressed as (10):

$$\frac{wL_2^{\,2}}{2L} \text{ and } \frac{wL_1^{\,2}}{2L} \quad (10)$$

and the total area of the blind spot is defined by the following mathematical expression (11):

$$\frac{w}{2L}\left(L_1^{\,2} + L_2^{\,2} \right) \quad (11)$$

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Now, we discuss how to maximize or minimize the total blind spot area. First, certain parameters must remain unchanged. The wall thickness w cannot be altered. Additionally, we assume that both the vertical length of the room H and the total horizontal length ($L+L_1+L_2$) are fixed.

If the entrance width L was adjustable, the problem would become trivial. For instance, setting L equal to the entire horizontal length would eliminate the blind spot entirely, while reducing L to zero would cause the entire room to become a blind spot. To maintain a meaningful analysis, we keep L constant.

Since both L and the horizontal length ($L+L_1+L_2$) are fixed, it follows that the sum L_1+L_2 is also fixed. Mathematically, we define this as $L_1+L_2=L_0$, for some constant L_0. Here, L_0 represents the horizontal length of the whole room (L_1+L_2+L) minus the entrance width L. Since $\frac{w}{2L}$ is a constant, the problem reduces to maximizing or minimizing $L_1^2+L_2^2$. This can be formally stated as follows.

(Question) How can we maximize or minimize $L_1^2+L_2^2$ when $L_1+L_2=L_0$?

To answer this question, we first reduce the number of variables. By substituting $L_2=L_0-L_1$, we can express $L_1^2+L_2^2$ as a function of single variable L_1 as the following mathematical expression (12):

$$L_1^2+L_2^2=L_1^2+(L_0-L_1)^2=2L_1^2-2L_0L_1+L_0^2=2\left(L_1-\frac{L_0}{2}\right)^2+\frac{L_0^2}{2} \quad (12)$$

To minimize the above, we should take $L_1=\frac{L_0}{2}$, which leads to the equation (13):

$$L_1=L_2=\frac{L_0}{2} \quad (13)$$

This configuration balances the two lengths equally, minimizing their squared sum. Now, how can we maximize $L_1^2+L_2^2$? If L_1 were to approach infinity or negative infinity, then $L_1^2+L_2^2$ would tend toward infinity. However, this is not possible because both L_1 and L_2 represent physical lengths and cannot be negative. Thus, the feasible range of L_1 is constrained by $0 \le L_1 \le L_0$.

Under this condition, we seek to maximize $L_1^2+L_2^2$. Let $L_1=x$, so we can express the function as (14):

$$f(x)=2\left(x-\frac{L_0}{2}\right)^2+\frac{L_0^2}{2} \quad (14)$$

Given the constraint $0 \le x \le L_0$, we analyze the function's behavior. Since the expression is a quadratic function that opens upwards, its minimum occurs at $x=\frac{L_0}{2}$, while the maximum occurs at the endpoints of the given range. Thus, to maximize f(x), we set x=0 or $x=L_0$, meaning that the area of the blind spot is maximized when either L_1 or L_2 is zero.

We can summarize the results as follows. First, the blind spot is minimized when $(L_1, L_2)=\left(\frac{L_0}{2},\frac{L_0}{2}\right)$. This occurs when the entrance is positioned at the center of the wall, allowing for maximum visibility into the room. Next, the blind spot is maximized when set $(L_1, L_2)=(0, L_0)$ or $(L_0, 0)$. This configuration corresponds to placing the entrance at one end of the wall, creating the largest possible blind spot within the room.

Discussion and Conclusion

This study analyzed and evaluated how entrance placement influences the formation and extent of blind spots within rectangular enclosed spaces, using a mathematically grounded isovist-based approach [5-9]. The results demonstrated that positioning entrances at the center of a wall significantly minimizes blind spot areas, optimizing visibility and enhancing surveillance capabilities. Conversely, placing entrances at wall edges maximizes blind spots, potentially beneficial in contexts where privacy is desired or external visibility should be limited.

Understanding blind spot dynamics has important implications beyond theoretical analysis. Autonomous systems operating in indoor environments must account for areas where objects or people may be obscured or hidden. Optimizing room layouts to reduce blind spots can improve the performance of mobile robots, surveillance drones, and AI-driven monitoring systems [10-13].

Architecturally, the study's findings suggest practical guidelines: in environments prioritizing surveillance and security, such as hospitals, correctional facilities, and high-security installations, centrally positioned entrances enhance visibility and reduce hidden zones [14]. Conversely, residential and certain commercial spaces may strategically employ edge-located entrances to enhance privacy and minimize disturbances.

In contexts such as urban warfare and tactical military operations, blind spots can be exploited by both defenders and attackers [15]. Understanding how blind spots form based on room geometry can help military forces optimize positioning and breaching strategies. Soldiers clearing a room may use blind spot calculations to anticipate enemy hiding positions, improving their approach for minimizing risk [16,17].

While this research focused specifically on rectangular rooms with single entrances, future investigations could expand the analysis to include irregularly shaped rooms, multiple entrances, and dynamic obstacles like furniture or internal partitions. Addressing these complexities would further enrich the applicability and effectiveness of blind spot analysis across diverse real-world scenarios.

This study provides insights into how blind spots are influenced by room geometry and entrance placement. By applying these findings to architecture, security, military tactics, and AI-driven systems, we can improve surveillance efficiency, enhance tactical strategies, and design safer, more functional spaces. Future research will further expand the scope of this analysis, incorporating more complex variables to refine our understanding of blind spot optimization.

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