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Relation between Viscosity Coefficient, Micro-Polar Parameter and Hartmann Number A Technical Note

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Abstract

A technical note on magneto-hydrodynamic Stokes flow of micro polar fluid past an axially symmetric particle with magnetic pole at center is written. It has been concluded that without magnetic effect, the quantity of drag is doubled when micro polarity parameter coincides with fluid viscosity coefficient. In the presence of magnetic effect, the numerical value of Hartmann number is calculated from the expression connecting viscosity coefficient, micro-polar parameter and Hartmann number.

Keywords: Magneto-hydrodynamic; Stokes flow; Newtonian fluid; Micro-polar fluid; Viscosity coefficient; Micro-polarity parameter; Hartmann number

Introduction

There are many fluids like plasmas, liquid metals, salt water, and electrolytes etc. that lie under the class of magneto hydrodynamics and attract the attention of mechanical engineers, scientists and chemists for a longer period of time. The main significant quantity of magneto hydrodynamic flow of micro polar fluid past an axially symmetric particle is the drag experienced by the stationary body or moving through the fluid. George Gabriel Stokes [1], a renowned mechanical engineer and physicist of UK coined first the new concept of Stokes drag on sphere by solving the Navier–Stokes equation clubbed with continuity equation under zero-slip boundary condition by neglecting the convective inertia terms in the vicinity of sphere. Then, this rule given by him is known as Stokes law. After his landmark work in continuum mechanics, a new region of work called Stokes flow, emerge out significantly and many authors have contributed and lead to generate drag value analytically and numerically solving the Navier-Stokes equations and continuity equations by applying Stokes approximation with no-slip boundary conditions. Application of Stokes flow can be explored in biochemistry, marine engineering, naval engineering, biology and others as well. This was Hartmann [2] who initiated the theory of laminar flow of an electrically conductive liquid in a homogeneous magnetic field. Chester [3] studied the effect of magnetic field on Stokes flow in a conducting fluid and modified the classical Stokes drag solution by magnetic field, which is uniform at infinity and is in the direction of flow of the fluid, given as

$$D=D_s=\left\{1+38M+7960M^2-437680M^3+O\left(M^4\right)\right\}, \quad (1.1)$$

where D_s is the classical Stokes drag and 'M' is the Hartmann number. He also proved that when the magnetic Reynolds number R_m, is small the magnetic field is essentially independent of the fluid motion. Ludford [4] discussed the effect of an aligned magnetic field on Oseen flow of a conducting fluid. Payne and Pell [5] tackled the Stokes flow problem for a class of axially symmetric bodies and found the general expression of Stokes drag on axially symmetric bodies in terms of stream function. Imai [6] has discussed the flow of conducting fluid past bodies of various shapes. Gotoh [7] has discussed the magneto hydrodynamic flow past a sphere and calculated the drag on sphere. Chang I-Dee [8] studied the problem of Stokes flow of a conducting fluid past an axially symmetric body in the presence of a uniform magnetic field and gave the formula of drag on axially symmetric body placed in the conducting fluid under the effect of uniform magnetic field. He utilizes the perturbation technique. In his Ph.D. thesis at Harvard University, Blerkom [9] studied the magneto-hydrodynamic flow of a viscous fluid past a sphere. Riley [10] considered the slow flow of a viscous, conducting fluid past a non-conducting sphere at whose center is a magnetic pole. He calculated the drag in terms of Hartmann number.

Eringen [11] introduced first the theory of micro polar fluid. The micro polar fluid differs with classical Newtonian fluid by only microstructure properties viz; micro-rotation and micro-inertia. The features of this type of fluid can easily be found in complex fluids like polymeric suspensions, animal blood, liquid crystals, lubricants, colloidal suspensions, bubbly fluids, granular fluids. Kanwal [12] obtained the drag on solid bodies moving through the viscous and electrically conducting fluids. Mathon and Ranger [13] tackled the problem of magneto-hydrodynamic streaming flow past a sphere at low Hartmann numbers. Ariman et al. [14] provided a complete review of micro polar continuum fluid mechanics. Ramkissoon and Majumdar [15] have evaluated the drag on an axially symmetric body in the Stokes flow of micro polar fluid. Bansal and Kumari [16] have studied the MHD slow motion past a sphere and calculated the drag on sphere in both Stokes and Oseen's limits. Datta and Srivastava [17] proved a new form of Stokes drag on axially symmetric bodies based on geometric variables. Shu and Lee [18] provided the fundamental solutions for micro polar fluids. Based on geometric variables, Srivastava and Srivastava [19] calculated the drag on axially symmetric body in micro polar fluid. They concluded that the drag value reduced to classical Stokes drag by taking micro polarity parameter k tends to zero. In the latest work, Srivastava [20] extended the Stokes drag to Oseen's drag in magneto hydro dynamics.

For the in-depth analysis of classical Stokes drag in Newtonian and micro polar fluids as well as drag in magneto hydrodynamic Stokes flow, author refers books of few renowned researchers like [21-28], etc. The target of present problem is to explore the relationship between viscosity coefficient, micro polar parameter and Hartmann number by combining the features of drag values described in papers of Riley [10] for magneto hydrodynamic Stokes flow, Datta and Srivastava [17] for Newtonian fluid and Srivastava and Srivastava [19] for micropolar fluid. For the specific flow configuration, numerical value of Hartmann number is also presented.

Formulation of Problem and Governing Equations

Let us consider the axially symmetric body of characteristic length L placed along its axis (x-axis, say) in a uniform stream U of micro polar viscous fluid of density ρ_1, kinematic viscosity ν, viscosity coefficient μ and micro polar parameter k. When Reynolds number UL/ν is small, the steady motion is governed by Stokes equations [15],

$$-(\mu+k)\nabla\times\nabla\times u+k\,\nabla\times\omega-\nabla p+\rho_1 F=0, \quad (2.1)$$

$$(\alpha_1+\alpha_2+\alpha_3)\nabla(\nabla.\omega)-\alpha_3\nabla\times\omega+k(\nabla\times u)-2k\omega=0, \quad (2.2)$$

$$\nabla.\boldsymbol{u}=0, \quad (2.3)$$

subject to the no-slip and no spin boundary condition. In the above equations, u is the velocity vector, ω the micro-rotation or spin vector, μ the viscosity coefficient, p the pressure, 'k' the coupling constant or micro polarity of the fluid, F the external force per unit mass and α_1, α_2, α_3 are characteristics constants of the particular fluid under consideration. It should be noted here that the general expression for Stokes drag on small axially symmetric particle placed under slow incompressible viscous micro polar fluid with small micro polarity experiencing no external body forces is proposed with the restriction of no-slip and no spin boundary conditions. The solution of these equations for described axially symmetric bodies are being discussed by Srivastava and Srivastava [19].

We consider the equation of low Reynolds number flow of an incompressible conducting micropolar fluid past an axi-symmetric body in a magnetic field which is uniform at infinity. Chester [3] proved that when the magnetic Reynolds number R_m is small the magnetic field is essentially independent of fluid motion. For the case where the body and the fluid have nearly the same permeability, a uniform magnetic field will result i.e., $\mathbf{H'}=H_0\,\mathbf{i}=$ magnetic field at infinity. This indicates from the symmetry that there is no electric field, since for all such flows the electric currents form closed circuits. The governing equations and the no-slip boundary conditions for the present problem now becomes [8]

$$-\nabla p+\nabla^2\mathbf{v}-M^2\left[\mathbf{v}-(\mathbf{v.i})\mathbf{i}\right]=\mathbf{0}, \quad (2.1a)$$

$$\nabla.\mathbf{v}=0 \quad (2.1b)$$

$$\mathbf{v}\to\mathbf{i}\ \text{ as }\ r\to\infty\ \left(r^2=x^2+y^2+z^2\right) \quad (2.2a)$$

$$\mathbf{v}=\mathbf{0}\ \text{at the body} \quad (2.2b)$$

In equations (2-3), all entities are non-dimensional, and their abbreviations are as follows;

U = free-stream velocity,

a = characteristic length of body,

$$v=\frac{v'}{U},\ p=\frac{a(p'-p'_\infty)}{(\mu+k)U},\ x=\frac{x'}{a}\ \text{, etc.,}$$

$$Re=\frac{\rho Ua}{\mu+k}=\text{Reynolds number,}$$

$$R_m=Ua(\mu+k)\sigma=\text{magnetic-micropolar Reynolds number,}$$

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$$M = \mu H_0 a\left(\frac{\sigma}{\mu+k}\right)^{\frac{1}{2}} = \text{Hartmann number}$$

$\mathbf{i}$ = unit vector along x-direction.

Other symbols have their usual meanings in electro-hydrodynamics and magneto-hydrodynamics. Primed entities are in physical units (as per [3, 8]). The solution of these equations is given by Riley [10].

Methods

Srivastava [20] has provided the axial Stokes drag on axially symmetric body Figure 1 placed in axial flow of micro polar fluid as

$$F_x = \frac{1}{2}\frac{\lambda b^2}{h_x} = \frac{4}{3}\frac{\lambda b^2}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha}, \text{ where } \lambda = 6\pi(\mu+k)\,U_x, \quad = \frac{8\pi(\mu+k)b^2 U_x}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha}. \qquad (3.1)$$

where the subscript 'x' is inserted to indicate that the force is calculated in the axial flow direction. While using (3.1), it should be kept in mind that 'b' denotes intercept between the meridian curve and the axis of the normal perpendicular to the axis i.e., $b = R$ at $\alpha = \pi/2$. This expression of drag may be split in two parts as

$$F_x = \frac{8\pi(\mu+k)b^2 U_x}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha} = \frac{8\pi\mu b^2 U_x}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha} + \frac{8\pi k b^2 U_x}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha} = F_s + k\frac{8\pi b^2 U_x}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha} \qquad (3.2)$$

where F_s is axial Stokes drag. This expression of total drag (3.2) may further be written as

$$F_x - F_s = k\frac{8\pi b^2 U_x}{\int_0^{\pi} R\,\sin^3\alpha\, d\alpha} \Rightarrow \frac{F_x - F_s}{F_s} = \frac{k}{\mu} \qquad (3.3)$$

Now, on considering the effect of magnetic field in the present flow configuration, the requisite drag was calculated by Riley [10] in terms of Hartmann number as

$$\frac{F_x - F_s}{F_s} = \frac{37}{210}M^2 + O\left(M^2\right) \rightarrow 0.7205M \text{ as } M \rightarrow \infty \qquad (3.4)$$

where 'M' is Hartmann number.

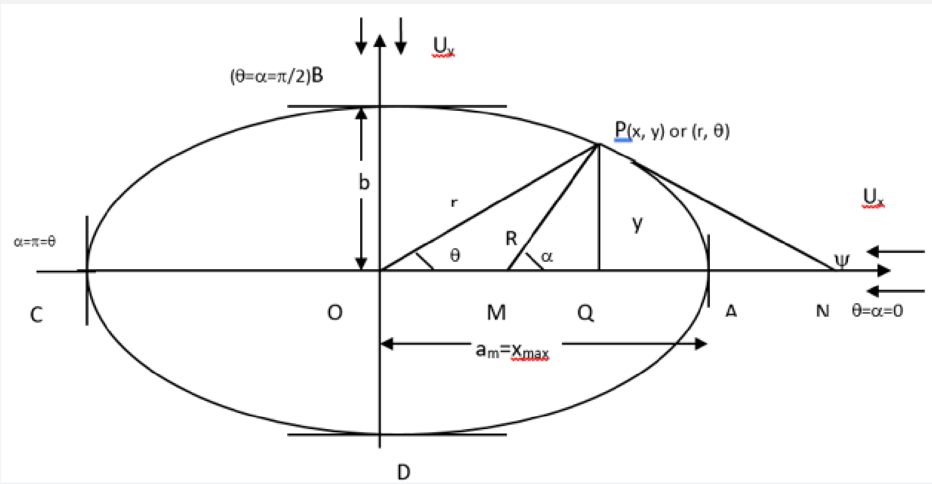

Figure 1: Geometry of axially symmetric body.

Solution

For micro polar fluid with small micro polarity effect permits us to consider $k \neq 1, 2, 3, \ldots$ etc. which lead us to the point where we get drag value $F_x = 2F_s, 4F_s, 6F_s \ldots$ etc. Which gives an idea that on fixing the micro polarity parameter equal to viscosity coefficient provide the drag value doubled to the corresponding Stokes drag on body placed under axial flow of micro polar fluid. Further, on taking the effect of magnetic field in the present situation of existing fluid phenomenon, expression of drag (3.4) given by Riley, N. can be clubbed with (3.3) leads us to the relationship.

$$\frac{k}{i} = \frac{37}{210}M^2 + O\left(M^2\right) \qquad (4.1)$$

For $k/\mu = 1$, this relationship provides the approximate numerical value at the level of second degree of Hartmann number 'M' as 2.38. The other values of Hartman number can be evaluated by using (4.1) by using the ratio $k/\mu = 2, 3, \ldots$ etc. By (3.4), for $k/\mu = 1$, we can have

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$$\frac{F_X - F_S}{F_S} = \frac{37}{210} M^2 + O\left(M^2\right) \rightarrow 1.71479 \text{ as } M \rightarrow \infty \quad (4.2)$$

The relation (4.1) and the limiting value of drag given in (4.2) can be verified very easily for various axially symmetric bodies like sphere, spheroid, deformed sphere etc. The same can also be verified experimentally.

Numerical Discussion

We consider the micro polar fluid with small micro polarity parameter k and fix k=2ì , 3ì , 4ì , 5ì etc., then from Riley's expression of drag $(F_x - F_s)/F_s$ and F_x/F_s may be calculated numerically by using (4.2). For ratio k≠1, 2, 3, 4, 5 etc., the numerical values of Hartmann number may be calculated from relationship (4.1). All the numerical values are presented in Table 1. It is clear from table that drag value increases with increase in ratio k/μ due to the linear relationship between the two variants [19]. For fix value of ratio k/μ, we can always have an approximate value of Hartmann number, up to second power of M, responsible for creating magnetic effect on Stokes flow of micro polar fluid. We receive increasing trend in Hartmann number with respect to increasing value of ratio k/μ due to parabolic relation given in (4.1). This variation has been depicted in (Figure 2).

Table 1: For axially symmetric bodies, numerical values are presented.

For axially symmetric bodies [17]							
$(F_x - F_s)/F_s$	1	2	3	4	5	n	Stokes flow of Micro polar Fluid past axially symmetric bodies
F_x/F_s	2	3	4	5	6	n+1	
k/	1	2	3	4	5	$n = 37M^2/210$, positive integer	
M	2.38	3.3689	4.1263	4.764	5.3271	$(210n/37)^{1/2}$	Micro polar Fluid under the effect of magnetic field with magnetic pole at the centre of body

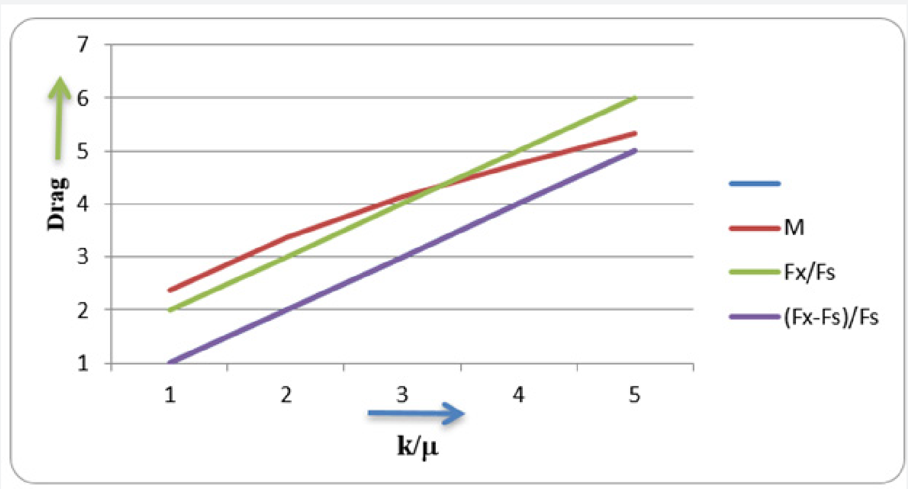

Figure 2: Variation of drag and Hartmann number with ratio .

Conclusion

A technical note on magneto-hydrodynamic Stokes flow of micro polar fluid past an axially symmetric particle with magnetic pole at the center is tackled without solving the governing equations of considered flow phenomenon. The expression of drag [19] in terms of micro polarity parameter ‘k’ and viscosity coefficient ‘ì’ is matched with the expression of drag [10] in terms of Hartmann number ‘M’ providing the required relationship

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between micro polarity parameter 'k', viscosity coefficient 'ì ' and Hartmann number 'M'. This relationship, $M=\sqrt{\frac{210n}{37}}, n=\frac{k}{}$, must be satisfied by all axially symmetric bodies which falls under the class described by Datta and Srivastava [17] for all fluid-structure problem of magneto hydrodynamic Stokes flow of micro polar fluid with small effect of magnetic field and micro polarity parameter. The author claims that this idea may open the door to study the peculiar features of Newtonian fluid and micro polar fluid in the presence of magnetic field.

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