

Mixing of Granular Media-Calculation of Stress at the Blade Front



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Abstract

Particulate materials are processed in various machines and equipment where it needs to ensure a motion of machines parts in a form of the flat blade through a layer of the grainy material. Conveyers, feeders, driers and especially mixers of granular materials can be mentioned as examples. Markedly, mixing elements of various sizes and shapes are moved through a bed of the granular material that resists this movement. Knowledge of interaction between the granular material and element moving through a bed of the granular media is necessary for the regular design and calculation of such machines. The paper deals with exact determination of force effects of the mixed material on moving blade submerged in a grain material. The solution is based on a yield equilibrium tensions of the particulate material described by theory of Mohr's circle. A box in form of rectangular prism will be used in experiments which enables uniform rectilinear motion of the blending elements of a simple design.

Keywords: Mohr's circle; Rectilinear motion of blade; Homogenizer; Particulate material; Yield equilibrium; Failure zone

Introduction

Mixing is wide-spread process in the various branches of industry. For example, in the chemical, food, and pharmacy industry, in the agriculture, or energetics. The special case of blending is mixing of the systems consisting of rigid particles. Mixing elements, various of sizes and shapes, are transferred through a bed of the grainy material. Knowledge of interaction between the grainy material and moving object is necessary for a construction of such machines. However, a theoretical knowledge of this problem is only little analyzed. Target of the paper is to give a theoretical and experimental basis for study of the grainy material motion in such cases. Designed experimental equipment enables to measure a force vector if grainy material acts against displacement of the blade. It makes possible to observe formation of failure zone of the particulate material in front of the moving blade. Steady state motion of the grainy material over a wide flat blade was studied in work [1]. Blade of the rectangular shape moves with constant velocity in horizontal direction through layer of the particulate material. Target of research is to explain the action of force of the granular medium on the flat blade moving through the granular medium layer. Normal of the blade can be deviated from direction of its motion.

Theoretical Analysis

Valorization results of hitherto performed investigation

Some authors attempted to describe the energetical aspect of mixing process according to relations well-known in hydrodynamics. The authors looked for analogy between flow off a solid body by liquid and the movement of the mixing element through the layer of particulate material [2-4] or mixing of the particulate materials and mixing of liquids [5,6]. However, these efforts were mostly marked by big errors due to inadequate simplification of mechanics laws of the particulate materials. It was confirmed in these works that it is not possible to continue this way. For estimation of energetical aspect of the particulate material mixing it is necessary to start with tension state of the particulate material. It is possible to find several research works in literature dealing with these problems. Their authors base the solution of the energetical aspect of the mixing process on the tension state of the particulate materials [7-15]. They used a simple geometric models in their experiments which made it possible to solve action of force between the grainy material and the mixing element in two-dimensional problem. Derived

functions, containing an influence of internal and external angles of friction of the particulate material, are complicated also for design of the mixing elements of simply form. Their determination assumes that the material is in the yield equilibrium. This state may be described by the theory of Mohr's circle. Based on the works mentioned above, it is possible to draw several conclusions. Region of the failure zone forms in front of the moving blade through a layer of the granular media (Figure 1). A course of the slip lines is possible to determine by the theory of Mohr's circle in this region. Then, a force acting on the blade can be estimated by form of the slip lines. This force is very important for calculation of the blender input power. The force acting on the blade depends on the form and the size of the plastic zone.

The form and the size of the failure zone depends especially on mechanical-physical properties of the grainy material (namely on internal (φ_i) and external (φ) angles of friction of the particulate material), geometric dimensions of the blade and depth of the blade immersion (z) in a batch. In these works, it was investigated that bulk density (ρ_s), internal (φ_i) and external (φ) angles of the friction of the particulate material belong among the most important parameters characterizing grainy material. Theoretical studies and experimental results proved that the internal (φ_i) and external (φ) angles of the friction are at movement of the granular media markedly assert, and hence it is not possible to ignore them, which is declared by some authors.

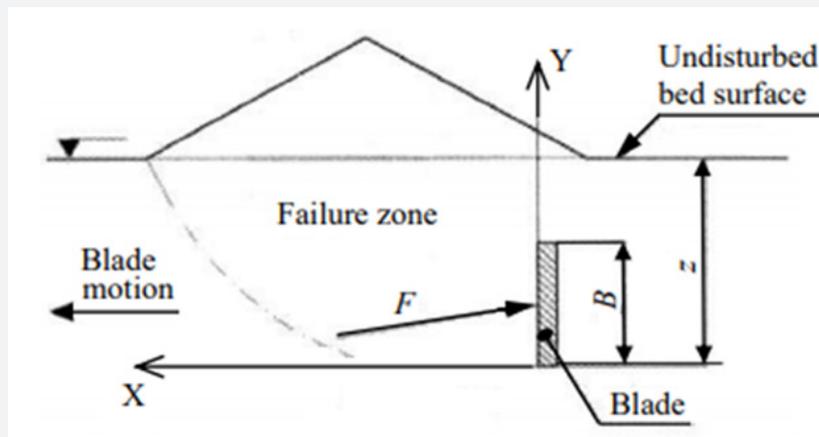


Figure 1: The blade motion through the layer of the granular material.

Analytical calculation of a stress acting on the blade

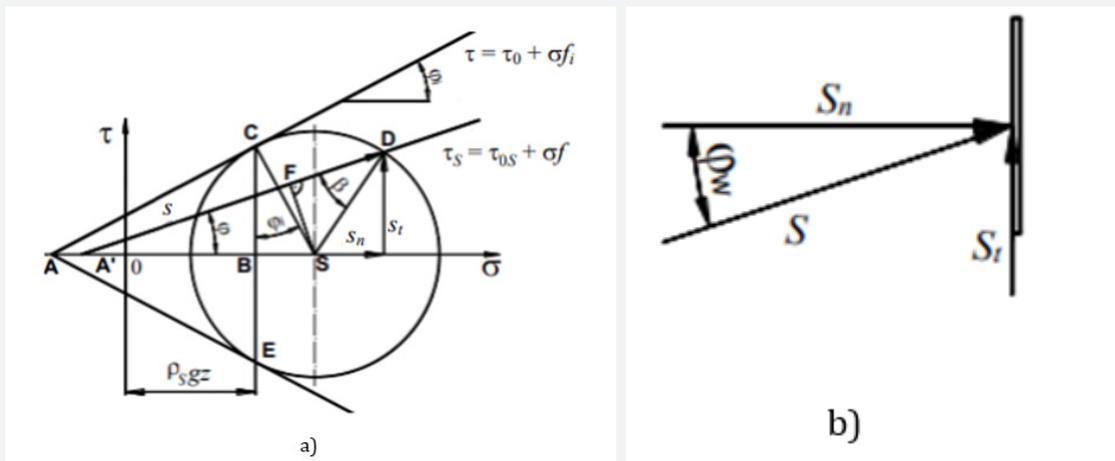


Figure 2: Mohr's diagram.

Theoretical determination of the stress acting on blade was performed for a following case. The blade of rectangular shape is situated so that its shorter edge (B) is vertical and perpendicular to a horizontal plane (Figure 1). The blade width (L) is considerably

bigger than its height (B). Problem is to define a relationship for calculation of the normal stress component (s_n) acting on the blade front (Figure 2) (Figure 2b). The stress distribution between

the mixing element and cohesion particulate material is simply plotted by Mohr's circle (Figure 2a). In this case the circle is Mohr's circle of stresses, the straight-line OS is an axis of normal

stresses σ and the straight line, perpendicular to the line OS and passing through point O, is the axis of tangential stresses τ .

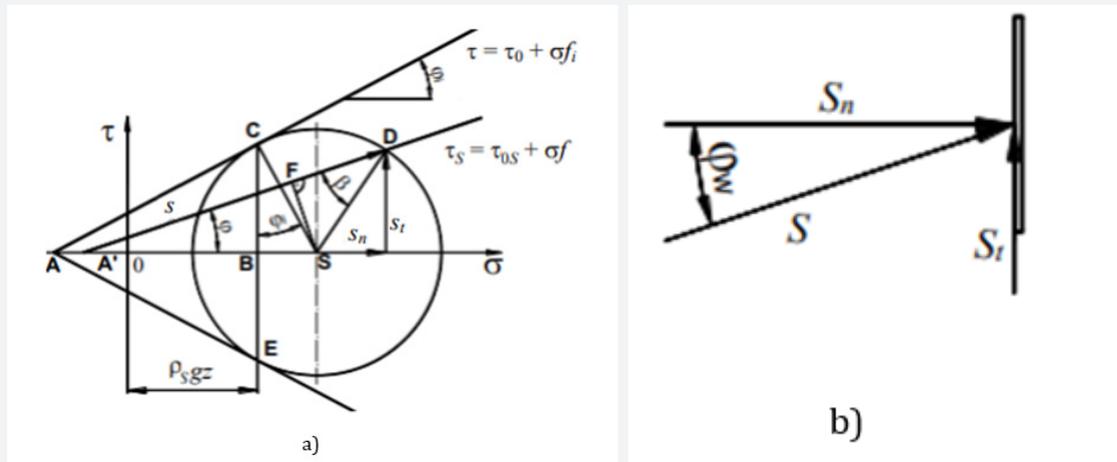


Figure 2: Mohr's diagram.

For triangle BSC, at the identity $tg(\varphi_i) = f_i$, it is possible to write:

$$SC = SD = \frac{BC}{\cos(\varphi_i)} = \left(\tau_0 + \rho_s g z \operatorname{tg}(\varphi_i) \right) \frac{1}{\cos(\varphi_i)} \quad (1)$$

and from triangle SAF follows that:

$$SF = A'S \sin(\varphi) = \left[\rho_s g z \left(1 + (\operatorname{tg}(\varphi_i))^2 \right) + \tau_0 \operatorname{tg}(\varphi_i) + \frac{\tau_{0s}}{\operatorname{tg}(\varphi)} \right] \sin(\varphi) \quad (2)$$

where φ_i is the angle of internal friction of the granular material and φ is the angle of external friction between the granular material and the blade surface. It is possible to suppose an identity $\varphi_w = \varphi$ for the larger ratios of $z/B \geq 2.5$, see experimental work [1].

If the angle $\angle SDF$ is designated by β then:

$$\sin(\beta) = \frac{SF}{SD} = \frac{\left[\rho_s g z \left(1 + (\operatorname{tg}(\varphi_i))^2 \right) + \tau_0 \operatorname{tg}(\varphi_i) + \frac{\tau_{0s}}{\operatorname{tg}(\varphi)} \right] \sin(\varphi)}{\left[\tau_0 + \rho_s g z \operatorname{tg}(\varphi_i) \right] \frac{1}{\cos(\varphi_i)}} \quad (3)$$

and:

$$\beta = \arcsin \left[\frac{\left[\rho_s g z \left(1 + (\operatorname{tg}(\varphi_i))^2 \right) + \tau_0 \operatorname{tg}(\varphi_i) + \frac{\tau_{0s}}{\operatorname{tg}(\varphi)} \right] \sin(\varphi)}{\left[\tau_0 + \rho_s g z \operatorname{tg}(\varphi_i) \right] \frac{1}{\cos(\varphi_i)}} \right] \quad (4)$$

Eq. (4) it is possible to rewrite by goniometric function:

$$\left(1 + (\operatorname{tg}(\varphi_i))^2 \right) = \frac{1}{(\cos(\varphi_i))^2} \quad (5)$$

in the following form:

$$\beta = \arcsin \left[\frac{\left[\frac{\rho_s g z}{(\cos(\varphi_i))^2} + \tau_0 \operatorname{tg}(\varphi_i) + \frac{\tau_{0s}}{\operatorname{tg}(\varphi)} \right] \sin(\varphi)}{\left[\frac{\rho_s g z}{(\cos(\varphi_i))^2} + \frac{\tau_0}{\sin(\varphi_i) \cos(\varphi_i)} \right] \sin(\varphi_i)} \right] \quad (6)$$

The stress s in the Mohr's diagram (Figure 2a) specifies the stress acting on the blade front. The values of the normal (s_n) and

tangential stress (s_t) acting on the blade (Figure 2b) are given by point D in the Mohr's diagram (Figure 2a). For calculation of the normal stress component (s_n), the first it is needed to determine a relationship for the calculation stress S , that is possible to write in the following form, see (Figure 2a):

$$s = A'D = A'F + FD \quad (7)$$

Abscissa size $A'F$ can be determined from triangle $A'FS$, then:

$$A'F = A'S \cos(\varphi) = \left[\rho_s g z \left(1 + (\operatorname{tg}(\varphi_i))^2 \right) + \tau_0 \operatorname{tg}(\varphi_i) + \frac{\tau_{0s}}{\operatorname{tg}(\varphi)} \right] \cos(\varphi) \quad (8)$$

and abscissa size FD from the triangle FDS :

$$FD = SD \cos(\beta) = \left(\tau_0 + \rho_s g z \operatorname{tg}(\varphi_i) \right) \frac{1}{\cos(\varphi_i)} \cos(\beta) \quad (9)$$

Combining (8), (9) and (7) we obtain the relationship for the calculation stress S :

$$s = \left[\rho_s g z \left(1 + (\operatorname{tg}(\varphi_i))^2 \right) + \tau_0 \operatorname{tg}(\varphi_i) + \frac{\tau_{0s}}{\operatorname{tg}(\varphi)} \right] \cos(\varphi) + \left(\tau_0 + \rho_s g z \operatorname{tg}(\varphi_i) \right) \frac{1}{\cos(\varphi_i)} \cos(\beta) \quad (10)$$

Modification of the relationship (10) by the relationship (5) gives the next relation for the calculation of stress S acting on the front of the blade (Figure 2b):

$$s = \frac{\rho_s g z}{(\cos(\varphi_i))^2} \left[\cos(\varphi) + \sin(\varphi_i) \cos(\beta) \right] +$$

$$\frac{\tau_0}{\cos(\varphi_i)} \left[\sin(\varphi_i) \cos(\varphi) + \cos(\beta) \right] + \frac{\tau_{0s} (\cos(\varphi))^2}{\sin(\varphi)} \quad (11)$$

The normal stress (s_n) acting on the blade is expressed by function:

$$s_n = s \cos(\varphi) \quad (12)$$

constant height of the blade in projection (B) (Figure 9). This means that when the blade is inclined under the angle $\alpha < 90^\circ$, then the value of the horizontal force ($H_{(\alpha < 90^\circ)}$) is bigger than the value of the force (H) for $\alpha = 90^\circ$. On the contrary, when the blade is inclined under the angle $\alpha > 90^\circ$, then the value of the horizontal force ($H_{(\alpha > 90^\circ)}$) is smaller than the value of the force (H) for $\alpha = 90^\circ$. Generally, it is possible to suppose that total force (F) is diverted from the normal of the blade by external angle of the friction (φ)

. For various angles of the blade inclination (α) it is necessary to consider an orientation of the total force (F) acting on the blade. Orientation of the resistance force (F), as is plotted in (Figure 10), was determined by experimental measurements [1]. The horizontal force (H), or $H_{(\alpha < 90^\circ)}$, or $H_{(\alpha > 90^\circ)}$, necessary to the movement of the blade through the granular material, was estimated by the resistance force (F) (Figure 10).

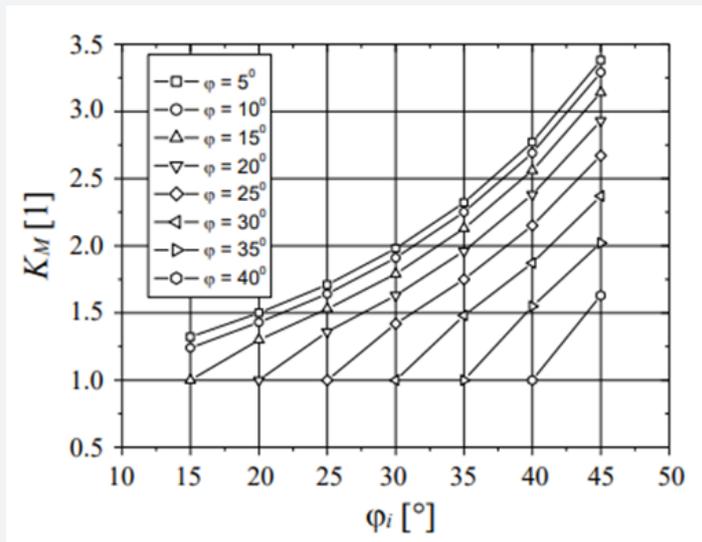


Figure 4: Dependence of K_M on angle of internal friction (φ_i) for cohesionless particulate material. Parameter of curves is angle of external friction (φ).

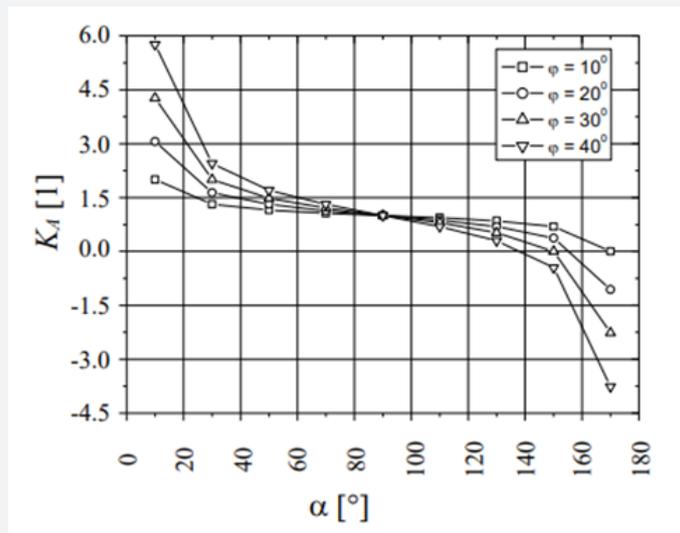


Figure 5: Dependence of K_A on angle of blade inclination (α). Parameter of curves is angle of external friction (φ).

Value of the stress s_n is calculated by the relationship (13). The horizontal component of the force (H) acting on the blade plate with length L and height B for the case of $\alpha = 90^\circ$ is

calculated by the value of the stress s_n :

$$H = s_n A = s_n B L = K_M \rho_s g z B L \quad (18)$$

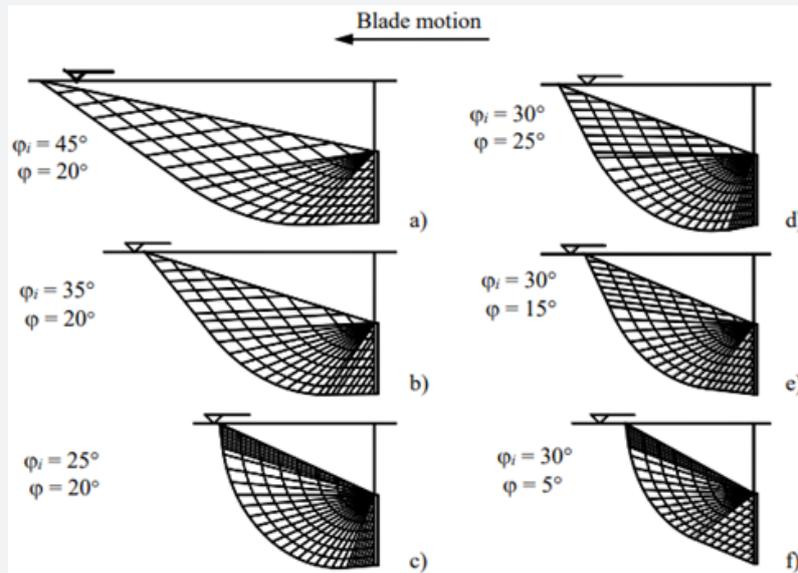


Figure 6: Influence of the angles of internal friction (φ_i) and external friction (φ) on shear region in front of the blade ($B = 40$ mm, $z=80$ mm)

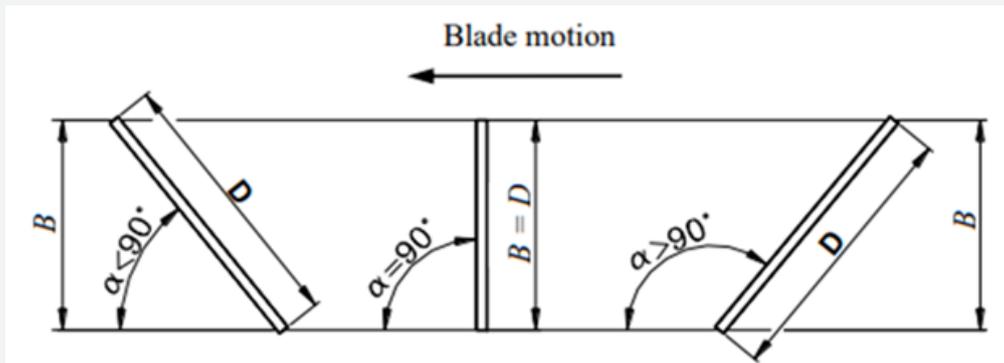


Figure 7: Orientation of the blade at the angle $\alpha < 90^\circ$, $\alpha = 90^\circ$, and $\alpha > 90^\circ$.

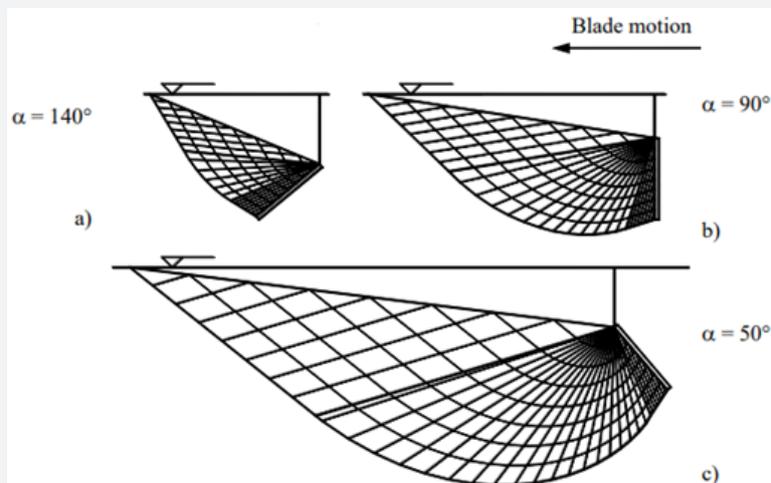


Figure 8: Influence of the angle of the blade inclination (α) on formation of the shear zone ($\varphi_i = 35^\circ$, $\varphi = 30^\circ$, $z=60$ mm, $D=40$ mm).

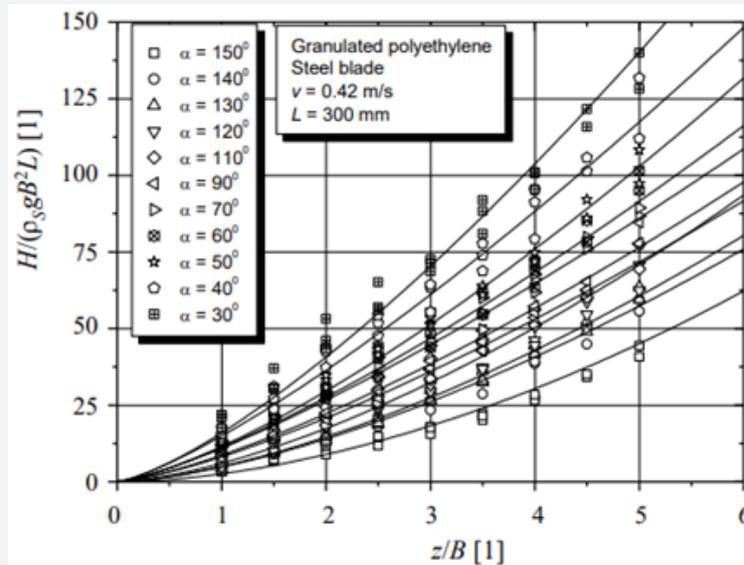


Figure 9: Dependence of the dimensionless horizontal component of the force $H/(\rho_s \cdot g \cdot B^2 \cdot L)$ on the dimensionless immersion of the blade z/B .

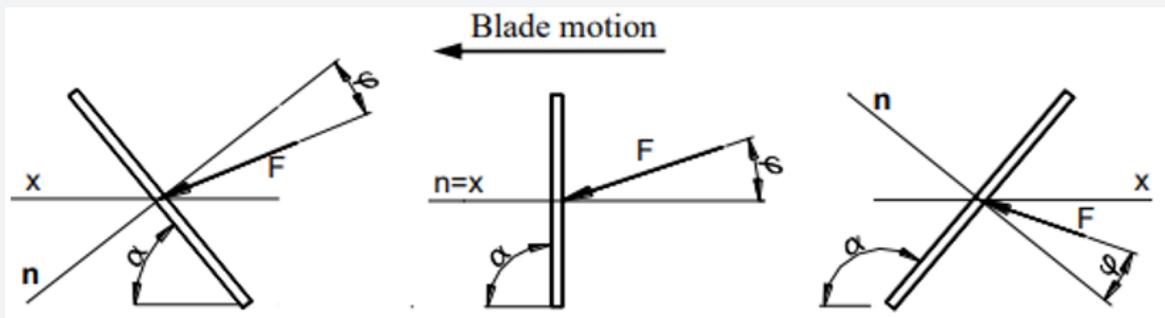


Figure 10: Orientation of the resistance force (F) for the various angles of the blade inclination (α) .

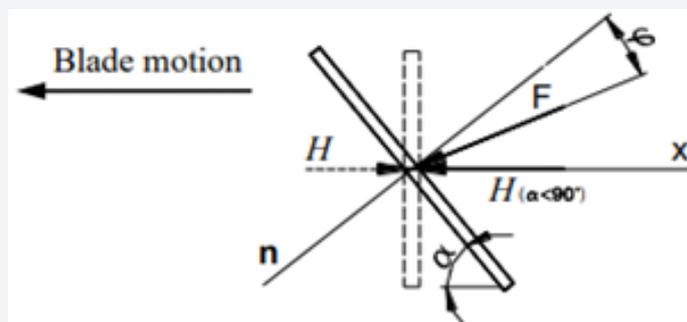


Figure 11: Forces acting on the blade.

From the balances of the forces (Figure 11) in direction of the blade normal, the following is valid:

$$\frac{H}{\cos(90 - \alpha)} = F \cos(\varphi) \quad (19)$$

Magnitude of the horizontal component of the force $(H_{(\alpha < 90^\circ)})$, for a case of the angle of the blade inclination $\alpha < 90^\circ$, is given by

expression for a component of the total force in direction x:

$$H_{(\alpha < 90^\circ)} = F \sin(\alpha + \varphi) \quad (20)$$

Combination of (19) and (20) gives a relationship for calculation of the horizontal component of the force $(H_{(\alpha < 90^\circ)})$:

$$H_{(\alpha < 90^\circ)} = \frac{H \sin(\alpha + \varphi)}{\cos(90 - \alpha) \cos(\varphi)} \quad (21)$$

Considering the geometric function $\cos(90 - \alpha) = \sin(\alpha)$, the relationship (21) can be rewritten as:

$$H_{(\alpha < 90^\circ)} = \frac{H \sin(\alpha + \varphi)}{\sin(\alpha) \cos(\varphi)} = HK_A \quad (22)$$

where K_A is the dimensionless function, whose values depend only on the angle of the blade inclination (α) and on the angle of the external friction (φ) (Figure 5). This represents the following substitution in relationship (22):

$$K_A = \frac{\sin(\alpha + \varphi)}{\sin(\alpha) \cos(\varphi)} \quad (23)$$

By the same procedure, it is possible to obtain the same relationship (22) for the angle of the blade inclination $\alpha > 90^\circ$. Then we can write for the horizontal component of the force ($H_{(\alpha < 90^\circ)}$) the following relationship:

$$H_{(\alpha > 90^\circ)} = H_{(\alpha < 90^\circ)} = \frac{H \sin(\alpha + \varphi)}{\sin(\alpha) \cos(\varphi)} = HK_A \quad (24)$$

If we suppose that $H_{(\alpha > 90^\circ)} = H_{(\alpha < 90^\circ)} = H(\alpha)$ then from the relationships (24) and (18) we obtain:

$$H(\alpha) = K_M K_A \rho_s g z B L \quad (25)$$

By substitution $K = K_M K_A$ we obtain finally a common expression for calculation of the horizontal component of the force $H(\alpha)$ for the various cases of the angle of blade inclination (α):

$$H(\alpha) = K \rho_s g z B L \quad (26)$$

Equation (26) may be adapted in the relationship for calculation of the stress ($s_{n(\alpha)}$) acting on the front of the blade for various cases of the angle of the blade inclination (α):

$$s_{n(\alpha)} = \frac{H(\alpha)}{A} = K \rho_s g z \quad (27)$$

Conclusion

The introduction of article deals with available results of the investigation, from those some conclusions were deduced. It was confirmed that for calculation of the energetical aspect of the grainy material mixing it is needed to start from tension state of the particulate material. The bulk density, internal and external angles of the material friction, geometric dimensions of the blade, and depth of the blade immersion belong among the most important parameters characterizing the grainy matters which have an influence on the magnitude of the horizontal component of the force acting on the blade. On the basis of these information's, in this paper, the relationship including the influence of these above-mentioned parameters, was designed. It is written in the part of analytical calculation of the stress acting on the blade. Next

part of the article considers the influence of the blade inclination angle on the form and largeness of the shear zone in front of the moving blade, which significantly affects the magnitude of the force acting on the moving blade through the layer of the particular matters. The given relationship specifies theoretical value of the horizontal component of the force. If this relationship would give a total value of the force horizontal component, so then a ratio of the measured and theoretically calculated values would be close to one. If this ratio differs significantly from the value 1, then a computing model will be dependent on another, so far the unknown parameters. These ideal results will be the basis for formation of the dimensionless arguments by dimensional analysis of the homogenization process. Design of the criterion dependency for the calculation of the dimensionless horizontal component of the force acting on the blending element of the various form and for different types of the grainy material will be main goal of the next research. Knowledge of real force interaction between homogenized material and mixing element facilitates qualified design of the homogenizer drive.

Nomenclature

- A - projection of the blade area to the motion direction [m²]
- A - blade area [m²]
- B - mixing element height (blade height) [m]
- B - blade height in projection [m]
- D - blade height [m]
- F - total force acting on blending element [N]
- H - horizontal component of the force [N]
- $H_{(\alpha)}$ - horizontal force for the various angle of blade inclination [N]
- $H_{(\alpha < 90^\circ)}$ - horizontal force for $\alpha < 90^\circ$ [N]
- $H_{(\alpha > 90^\circ)}$ - horizontal force for $\alpha > 90^\circ$ [N]
- K_M - dimensionless function [1]
- K_A - dimensionless function [1]
- (L) - mixing element width (blade width) [m]
- α - angle of the blade inclination [°]
- β - angle [°]
- ρ_s - bulk density of the particulate material [kg.m⁻³]
- σ - normal stress [Pa]
- τ - shear stress [Pa]
- τ_s - shear strength between particulate material and construction materials [Pa]

S - force acting on the blade [N]
 S_n - normal component of the force acting on the blade [N]
 S_t - tangential component of the force acting on the blade [N]
 f - coefficient of external friction [1]
 f_i - coefficient of internal friction [1]
 g - acceleration due to gravity [m.s⁻²]
 s - stress [Pa]
 s_n - normal component of the stress [Pa]
 s_t - tangential component of the stress [Pa]
 $s_{n(\alpha)}$ - normal stress for various angle of the blade inclination [Pa]
 v - mixing element velocity [m.s⁻¹]
 z - depth of the mixing element immersion [m]
 τ_0 - initial shear strength [Pa]
 τ_{0s} - initial shear strength between particulate material and construction materials [Pa]
 φ - external angle of friction of the particulate material [°]
 φ_i - internal angle of friction of the particulate material [°]
 φ_w - external angle of friction of the particulate material [°]

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