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A Study on Comparison of Probability Distributions for Frequency Analysis of Low-flow

Vivekanandan N[1]* and Poornima RK[2]

[1] *Scientist-B, Central Water and Power Research Station, Pune, Maharashtra, India*

[2] *M.Tech. student, Department of Civil Engineering, University Visvesvaraya College of Engineering, Bangalore, India*

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***Corresponding author:** Vivekanandan N, Scientist-B, Central Water and Power Research Station, Pune, Maharashtra, India

Abstract

Assessment of low-flow is important for hydrological and water resource management, water quality regulation, reservoir storage design, hydro-electric power generation in arid periods and habitat protection. This can be achieved through Low-flow Frequency Analysis (LFA) by fitting probability distribution to the annual minimum d-day average flow series for different duration of 'd' viz., 7-, 10-, 14- and 30-days that is derived from the daily stream flow data series. This paper presents a study on comparison of four probability distributions such as Pearson Type-3, Log Pearson Type-3, 2-parameter Log Normal (LN2) and 2-parameter Weibull for frequency analysis of low-flow for river Cauveri at Kollegal gauging site. Standard parameter estimation procedures viz., method of moments, maximumlikelihood method and L-Moments (LMO) are used for determination of the parameters of the distributions wherever applicable. The adequacy of fitting four distributions adopted in LFA is evaluated by quantitative assessment using Goodness-of-Fit (viz., Chi-Square and Kolmogorov-Smirnov) and diagnostic (viz., correlation coefficient and root mean squared error) tests, and qualitative assessment using the fitted curves of the estimated low-flow. On the basis of the results of quantitative and qualitative assessments, the study indicates the LN2 (LMO) distribution is better suited for estimation of low-flow at Kollegal site of Cauvery river.

Keywords: Chi-square; Correlation coefficient; Kolmogorov-Smirnov; L-Moments; Log Normal; Low flow; Root mean squared error

Introduction

Assessment of low-flow is important for hydrological and water resource management, as it is a critical factor for basin restoration, water quality regulation, reservoir storage design, hydro electric power generation in arid periods and habitat protection [1]. Numbers of indices such as mean annual runoff, mean daily flow, median flow, Annual Minimum d-day Average Flow (AMdAF), absolute minimum flow are widely used to characterize the low-flow. Among these, AMdAF is generally adopted procedure for characterizing low-flow in a stream, which is computed by averaging the flow using moving average method for 'd' consecutive days viz., 7-, 10-, 14- and 30-days. An associated, annual event based, low-flow statistic q(d,T) gives a low-flow estimate, which is defined as the AMdAF that is expected to be occurred once in T-year return period [2].

Out of number of available probability distributions, the Pearson Type-3 (PR3), Log Pearson Type-3 (LP3), 2-parameter Log Normal (LN2) and 2-parameter Weibull (WB2) are most commonly used in LFA [3-6]. Ahn et al. [7] applied Power and SMEMAX (Small, MEdium and MAXimum) transformation, LN2, LP3 and WB2 distributions to estimate the 7-day and 30- day low-flows for different return periods at four gauged points of the Ansung stream in Korea. Bowers et al. [8] analyzed the seasonal river flow data and found both power law and LN2 distributions are relevant to dry seasons. They also found that the river flow data in wet seasons are typically better fitted using LN2 than power law. Farmer et al. [9] recommended the use of inverse moments or negative moment orders for low flow series because the positive moment orders do not effectively capture the probabilistic lower tail behaviour of flows above a certain exceedance probability. Randall & Freehafer [10] applied regression method to study on low-flow statistics at ungauged sites in the Lower Hudson River Basin, New York. They found that the logarithmic transformation yielded less accurate equations inconsistent with some conceptualized relationships. In light of the above, in the present study, the PR3, LP3, LN2 and WB2 distributions are adopted in LFA for estimation of low flow.

Generally, Method of Moments (MoM) is applied for determination of parameters of the probability distributions [11]. But, the studies carried out by various researchers indicated that the estimated parameters of distributions fitted by the MoM are often less accurate than those obtained by other parameter estimation procedures viz., Maximum Likelihood Method (MLM) and L-Moments (LMO) [12-14]. Therefore, for the present study, the MoM, MLM and LMO are applied for determination of the parameters of the distributions that are adopted in LFA. The adequacy of fitting four probability distributions adopted in LFA is evaluated by quantitative assessment using non-parametric Goodness-of-Fit (viz., Chi-Square (χ^2) and Kolmogorov-Smirnov (*KS*)) and diagnostic (i.e., Correlation Coefficient (*CC*) and Root Mean Squared Error (*RMSE*)) tests, and qualitative assessment using the fitted curves of the estimated low-flow. This paper presents the procedures applied in determining the parameters of PR3, LP3, LN2 and WB2 distributions using MoM, MLM and LMO for estimation of low flow at Kollegal gauging site of river Cauvery and the results obtained thereof.

Methodology

Table 1: CDF with quantile estimator of four probability distributions.

Distribution	CDF	Quantile Estimator
PR3	$F(q)=\begin{cases} G\left(\beta,\frac{q-\xi}{\alpha}\right) & ,\alpha>0 \\ 1-G\left(\beta,\frac{q-\xi}{\alpha}\right) & ,\alpha<0 \end{cases}$	No explicit expression of the quantile function is available.
LP3	$F(q)=\begin{cases} G\left(\beta,\frac{ln(q)-\xi}{\alpha}\right) & ,\alpha>0 \\ 1-G\left(\beta,\frac{ln(q)-\xi}{\alpha}\right) & ,\alpha<0 \end{cases}$	No explicit expression of the quantile function is available.
LN2	$F(q)=\phi\left(\frac{ln(q)-\alpha}{\beta}\right), q>0, \beta>0$	$q(T)=exp(\alpha+K_T\beta)$ Wherein $K_T=4.91\left(\left(\frac{1}{T}\right)^{0.14}-\left(1-\left(\frac{1}{T}\right)\right)^{0.14}\right)$
WB2	$F(q)=1-e^{-\left(\frac{q}{\alpha}\right)^{\beta}}$, q>0, α>0, β>0	$q(T)=\alpha\left(-h\left(1-\left(1/T\right)\right)\right)^{1/\beta}$

Table 2: Theoretical descriptions of GoF tests statistic.

GoF Test	Test Statistic	Description of Symbols		
χ^2	$\chi_c^2=\sum_{j=1}^{NC}\frac{\left(O_j(q)-E_j(q)\right)^2}{E_j(q)}$	m	=	Number of parameters of the distribution
		χ_c^2	=	Computed value of χ^2 statistic by probability distribution
		$O_j(q)$	=	Observed frequency value (q) of j^{th} class
		$E_j(q)$	=	*Expected frequency value (q) of j^{th} class*
		NC	=	Number of frequency classes
KS	$KS=Max\sum_{i=1}^{N}\left\lvert F_e(q_i)-F_D(q_i)\right\rvert$	$F_e(q_i)$	=	Empirical CDF of q_i using Weibull plotting position formula $\left(P=\frac{r}{N+1}\right)$ wherein r is the rank assigned to the samples arranged in ascending order in such a way that $q_1<q_2<q_3<......<q_N$
		$F_D(q_i)$	=	Derived CDF of q_i using probability distribution
		N	=	Sample size

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In this paper, the AMdAF series for different duration of 'd' viz., 7-, 10-, 14-, and 30-days are extracted from the daily stream flow data series and also used in LFA. The Cumulative Distribution Function (CDF) with quantile estimator (q(d,T)) of four distributions [15,16] adopted in LFA are presented in Table 1. In Table 1, q is the variable (i.e., low-flow), ξ is the location parameter, α is the scale parameter, β is the shape parameter, $F(q)$ is the CDF of q, is $G(...)$ is the incomplete Gamma integral and $\varphi(...)$ is the CDF of the standard normal distribution. The estimators of the parameters (viz., location, scale and shape) are determined by MoM, MLM and LMO; and are used to estimate the q(d,T) for a duration (d) and return period (T). The procedures involved in determining the parameters of the distributions using MoM, MLM and MLM are briefly described in the text book titled 'Flood Frequency Analysis' by [17].

Goodness-of-fit tests

A non-parametric GoF tests viz., Chi-square (χ^2) and Kolmogorov-Smirnov *(KS)* are applied for checking the adequacy of fitting PR3, LP3, LN2 and WB2 distributions adopted in LFA. The theoretical descriptions of GoF tests statistic are presented in Table 2. The rejection region of χ^2 test statistic at the desired significance level (η) is given by $\chi_C^2 \geq \chi^2_{1-\eta,\ \mathrm{NC\text{-}m\text{-}1}}$. The theoretical values of GoF tests statistic for different significance level (η) are available in the technical note on 'Goodness-of-Fit Tests for Statistical Distributions' [18].

Test criteria: If the computed values of GoF tests statistic given by the distribution are less than that of its theoretical value at the desired significance level then the selected distribution is considered to be adequate for LFA.

Diagnostic test

The selection of most suitable probability distribution with parameter estimation method for estimation of low-flow is made through diagnostic test using *CC* and *RMSE*. The theoretical expressions of *CC* and *RMSE* [19] are given as below:

$$CC = \frac{\sum_{i=1}^{N}\left(q_i(o)-\overline{q(o)}\right)\left(q_i(e)-\overline{q(e)}\right)}{\sqrt{\sum_{i=1}^{N}\left(q_i(o)-\overline{q(o)}\right)^2 \sum_{i=1}^{N}\left(q_i(e)-\overline{q(e)}\right)^2}} \quad (1)$$

$$RMSE = \left(\frac{1}{N}\sum_{i=1}^{N}\left(q_i(o)-q_i(e)\right)^2\right)^{1/2} \quad (2)$$

where, $q_i(o)$ is the observed low-flow (q) of i^{th} sample and $q_e(o)$ is the estimated low-flow (q) of i^{th} sample, $\overline{q(o)}$ is the average of the observed low-flows and $\overline{q(e)}$ is the average of the estimated low-flows. The probability distribution with high *CC* (say, *CC*>0.9) and minimum *RMSE* is identified as a better-suited for estimation of low-flow.

Application

In this paper, a study on comparison of four probability distributions viz., PR3, LP3, LN2 and WB2 using MoM, MLM and MLM for estimation of low-flow at Kollegal gauging site of river Cauvery is carried out. The Cauvery river rises at an elevation of 1341 m at Talakaveri on the Brahmagiri range near Cherangala village of Kodagu district of Karnataka [20]. The river basin lies between 75° 27' to 79° 54' east longitudes and 10° 9' to 13° 30' north latitudes. The total length of the river from origin to outfall is 800 km and drains into the Bay of Bengal. The river basin extends over states of Tamil Nadu, Karnataka, Kerala and Union Territory of-Puducherry draining an area of 81155 km^2, which is nearly 2.7% of the total geographical area of the country. The Kollegal gauging site is located between 77° 06' 00'' east longitude and 12° 11' 21'' north latitude in Chamarajanagar district of Karnataka state, which is one of the tributary of Cauvery river. The catchment area of the Kollegal site is 21082 km^2. Figure 1 shows the location map of the study area. In the present study, the daily stream flow data observed at Kollegal gauging site for the period 1990 to 2018 is used.

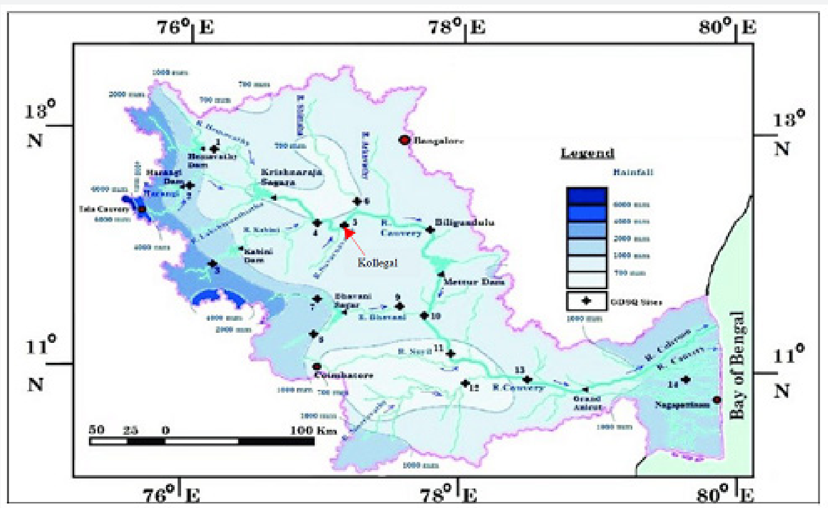

Figure 1: Location map of the study area (Source: Ghosh [20]).

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Results and Discussions

In the present study, the AMdAF series for different duration of 'd' viz., 7-, 10-, 14-, and 30- days was extracted from the daily stream flow data series and are used for LFA. The descriptive statistics of the AMdAF series for different duration of 'd' viz., 7-, 10-, 14- and 30- days are presented in Table 3.

Table 3: Descriptive statistics of the AMdAF series

AMdAF Series	Average (m^3/s)	Standard Deviation (m^3/s)	Coefficient of Skewness	Coefficient of Kurtosis
d=7	23.632	7.853	0.22	-1.215
	(3.106)	(0.348)	(-0.242)	(-0.925)
d=10	26.014	9.274	0.768	0.907
	(3.197)	(0.360)	(-0.158)	(-0.261)
d=14	27.996	10.305	0.96	1.858
	(3.268)	(0.368)	(-0.165)	(-0.012)
d=30	33.864	11.929	0.809	1.581
	(3.462)	(0.359)	(-0.266)	(-0.081)
Figures given within the brackets indicates the descriptive statistics of the log-transformed data				

Estimation of q(d,T) using probability distributions

Table 4: Estimated 7-day low-flow for different return periods using PR3, LP3, LN2 and WB2 distributions.

Return Period (year)	q(7,T)(m^3/s)									
	PR3		LP3		LN2			WB2		
	MoM	MLM	MoM	MLM	MoM	MLM	LMO	MoM	MLM	LMO
1.01	38.12	43.23	53.40	47.65	47.67	49.52	51.23	41.72	41.33	42.23
2	23.34	23.24	22.03	22.04	22.43	22.34	22.33	23.59	23.69	23.55
5	16.95	17.09	16.62	16.63	17.08	16.76	16.55	16.77	16.99	16.61
10	13.77	14.10	14.45	14.46	14.81	14.42	14.15	13.38	13.63	13.18
20	11.22	11.74	12.93	12.93	13.17	12.73	12.43	10.78	11.03	10.56
25	10.5	11.07	12.52	12.53	12.73	12.28	11.97	10.06	10.32	9.84
50	8.44	9.20	11.45	11.45	11.54	11.07	10.75	8.14	8.39	7.93
100	6.64	7.57	10.59	10.58	10.56	10.09	9.76	6.60	6.84	6.39

Table 5: Estimated 10-day low-flow for different return periods using PR3, LP3, LN2 and WB2 distributions.

Return Period (year)	q(10,T) (m^3/s)									
	PR3		LP3		LN2			WB2		
	MoM	MLM	MoM	MLM	MoM	MLM	LMO	MoM	MLM	LMO
1.01	52.68	52.58	56.6	56.01	54.86	55.78	57.49	47.87	48.35	47.63
2	24.84	24.78	24.47	24.47	24.50	24.47	24.47	25.83	25.79	25.85
5	18.07	18.20	18.08	18.14	18.31	18.17	17.98	17.86	17.71	17.93
10	15.15	15.40	15.43	15.52	15.73	15.55	15.30	13.99	13.81	14.08
20	13.03	13.40	13.54	13.64	13.87	13.68	13.40	11.07	10.88	11.16
25	12.46	12.86	13.03	13.14	13.37	13.18	12.89	10.27	10.09	10.37
50	10.95	11.45	11.69	11.79	12.04	11.84	11.53	8.17	7.99	8.27
100	9.73	10.32	10.59	10.71	10.96	10.75	10.44	6.51	6.34	6.60

In the present study, the parameters of PR3, LP3, LN2 and WB2 distributions were determined by MoM, MLM and LMO for the AMdAF series for different duration of 'd' viz., 7-, 10-, 14- and 30-days. These parameters were used to estimate q(d,T) for different duration (d) and for different return period (T); and the results are presented in Tables 4 to 7. From LFA results, it is noted that the q(d,T) estimates for different duration of 'd' viz., 10-, 14- and 30-days obtained from WB2 (MLM) distribution are low-

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er than those values of other probability distributions adopted in LFA for the return periods from 10-year to 100-year. From the data analysis, it is found that the determination of parameters of PR3 and LP3 distributions using LMO is not feasible and hence the LFA results of PR3 (LMO) and LP3 (LMO) are not presented in Tables 4 to 7. Also, the GoF and diagnostic tests results of PR3 (LMO) and LP3 (LMO) are not presented in Tables 8 and 9.

Table 6: Estimated 14-day low-flow for different return periods using PR3, LP3, LN2 and WB2 distributions.

Return Period (year)	*q*(14,T) (m³/s)									
	PR3		LP3		LN2			WB2		
	MoM	MLM	MoM	MLM	MoM	MLM	LMO	MoM	MLM	LMO
1.01	58.95	60.74	61.09	61.33	60.29	61.03	62.67	52.58	53.28	51.82
2	26.37	27.16	25.99	26.26	26.27	26.26	26.26	27.72	27.64	27.77
5	19.21	20.73	19.21	19.33	19.46	19.36	19.18	18.90	18.67	19.13
10	16.29	18.34	16.49	16.46	16.64	16.51	16.28	14.67	14.40	14.95
20	14.28	16.81	14.46	14.42	14.62	14.48	14.21	11.50	11.22	11.79
25	13.75	16.43	14.07	13.88	14.08	13.93	13.66	10.65	10.37	10.94
50	12.39	15.49	12.73	12.43	12.63	12.48	12.20	8.40	8.13	8.68
100	11.34	14.81	11.66	11.25	11.46	11.31	11.02	6.63	6.38	6.90

Table 7: Estimated 30-day low-flow for different return periods using PR3, LP3, LN2 and WB2 distributions.

Return Period (year)	*q*(30,T) (m³/s)									
	PR3		LP3		LN2			WB2		
	MoM	MLM	MoM	MLM	MoM	MLM	LMO	MoM	MLM	LMO
1.01	68.49	68.66	78.97	78.3	70.86	72.58	74.41	61.85	62.72	61.27
2	32.27	32.21	31.38	31.44	31.94	31.88	31.88	33.66	33.55	33.70
5	23.65	23.70	23.47	23.48	23.95	23.68	23.48	23.39	23.08	23.57
10	19.98	20.10	20.35	20.31	20.61	20.28	20.01	18.38	18.02	18.60
20	17.34	17.54	18.16	18.10	18.20	17.84	17.53	14.59	14.21	14.82
25	16.63	16.86	17.58	17.51	17.55	17.18	16.87	13.56	13.18	13.79
50	14.77	15.07	16.06	15.95	15.82	15.44	15.11	10.82	10.45	11.05
100	13.27	13.65	14.83	14.70	14.42	14.02	13.70	8.65	8.30	8.87

Table 8: Computed values of GoF tests statistic by P3, LP3, LN2 and WB2 distributions.

GoF Tests	Computed Values of GoF Tests Statistic									
	PR3		LP3		LN2			WB2		
	MoM	MLM	MoM	MLM	MoM	MLM	LMO	MoM	MLM	LMO
χ^2										
d=7	9.103	9.125	4.621	4.625	4.621	4.621	4.624	9.103	9.125	8.069
d=10	2.552	2.565	4.621	4.625	4.621	4.621	4.624	1.517	1.520	1.517
d=14	0.828	0.828	0.828	0.828	0.828	0.828	0.840	2.552	1.862	1.172
d=30	2.552	2.552	0.483	0.483	0.483	0.483	0.485	2.552	2.552	2.552
KS										
d=7	0.117	0.12	0.116	0.119	0.124	0.125	0.128	0.121	0.122	0.118
d=10	0.098	0.105	0.102	0.105	0.110	0.112	0.115	0.082	0.085	0.082
d=14	0.076	0.081	0.081	0.084	0.085	0.085	0.090	0.080	0.081	0.083
d=30	0.067	0.069	0.083	0.085	0.073	0.073	0.075	0.089	0.087	0.086

Theoretical values of $\chi^2_{1,0.05}=3.84$, $\chi^2_{2,0.05}=5.99$ and $KS_{29,0.05}=0.246$; Number of frequency classes considered in computation of χ^2 test statistic is five; Significance level (η) is considered in GoF tests is 0.05. Degrees of freedom is considered as one for PR3 and LP3 distributions whereas two for LN2 and WB2 distributions.

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Table 9: C C and values given by P3, LP3, LN2 and WB2 distributions.

AMdAF series	PR3		LP3		LN2			WB2		
	MoM	MLM	MoM	MLM	MoM	MLM	LMO	MoM	MLM	LMO
CC										
d=7	0.978	0.979	0.961	0.962	0.971	0.970	0.969	0.979	0.978	0.979
d=10	0.984	0.984	0.985	0.985	0.985	0.985	0.985	0.973	0.974	0.973
d=14	0.981	0.981	0.982	0.983	0.982	0.982	0.983	0.967	0.968	0.966
d=30	0.981	0.981	0.980	0.981	0.982	0.982	0.982	0.971	0.972	0.971
RMSE										
d=7	1.674	1.726	2.138	2.124	1.964	1.909	1.902	1.638	1.687	1.597
d=10	1.792	1.844	1.698	1.743	1.851	1.763	1.652	2.132	2.092	2.156
d=14	2.176	2.418	2.138	2.046	2.196	2.123	1.955	2.616	2.541	2.689
d=30	2.476	2.493	2.363	2.350	2.530	2.391	2.301	2.837	2.776	2.890

Low-flow frequency curves (LFCs)

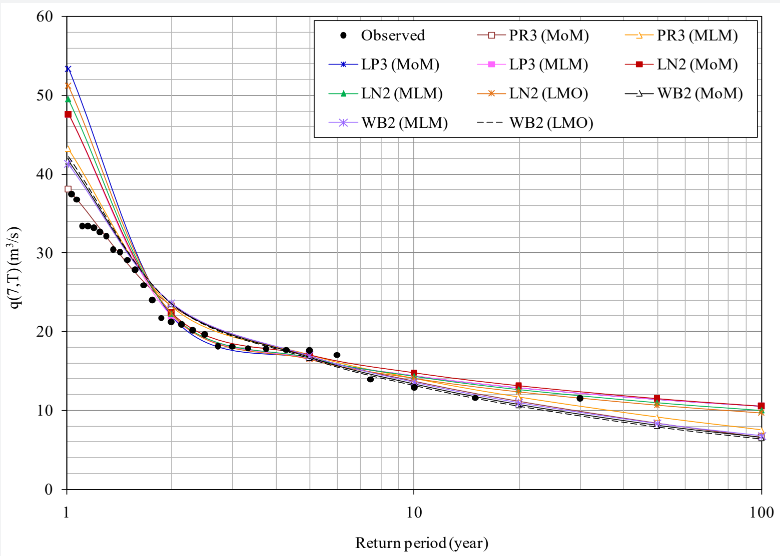

Figure 2: Plots of estimated 7-day low-flow using PR3, LP3, LN2 and WB2 distributions with observed data.

The estimated values of q(d,T) for different duration of 'd' viz., 7-, 10-, 14-, and 30-days and for the return periods from 1.01-year to 100-year were used to develop LFCs and are presented in Figures 2 to 5. From LFCs, it can be seen that there is a line of agreement between the observed and estimated low-flows using LN2 when compared with PR3, LP3 and WB2. Also, from LFCs, it is noted that the observed low-flows are nearer to the estimated low-flows by LN2 (LMO).

Analysis of results based on GoF tests

GoF tests statistic values for the AMdAF series for different duration of 'd' viz., 7-, 10-, 14-, and 30-days were computed and are presented in Table 8. From GoF tests results, it is noted that:

a) χ^2 test results didn't confirm the applicability of PR3, LP3 and WB2 distributions for modelling the AM7AF data.

b) χ^2 test results didn't support the use of LP3 for modelling the AM10AF data.

c) χ^2 test results confirmed the applicability of PR3, LP3, LN2 and WB2 for LFA using the AM14AF and AM30AF data series.

d) *KS* test results supported the use of PR3, LP3, LN2 and WB2 distributions for LFA using the AMdAF data series for different duration of 'd' viz., 7-, 10-, 14- and 30- days.

Analysis of results based on diagnostic test

In addition to GoF tests, the performance of PR3, LP3, LN2 and WB2 distributions adopted in LFA using the AMdAF data series for different duration of 'd' viz., 7-, 10-, 14-, and 30-days

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was evaluated by *CC* and *RMSE*. The *CC* and *RMSE* values for the AMdAF data series for different durations of 'd' were computed by the aforesaid probability distributions and are presented in Table 9. From Table 9, it is noticed that the *RMSE* given by WB2 (LMO) is minimum for AM7AF data while LN2 (LMO) for AM10AF, AM14AF and AM30AF data when compared with those values of other probability distributions adopted in LFA while MoM, MLM and LMO are applied for determining the parameters of the distributions. But, for AM7AF series, it is noted that the fitted line by using the estimated low-flow of LN2 (LMO) is closer to the observed data though the *RMSE* of WB2 (LMO) is lower than those value of LN2 (LMO). From the *CC* values, it is noted that there is a very good correlation between the observed and estimated low-flows using PR3, LP3, LN2 and WB2 distributions, and these values vary between 0.961 and 0.985. Based on the results of the quantitative and qualitative assessments, it is identified that the LN2 (LMO) is better suited distribution for estimation of low-flow at Kollegal site.

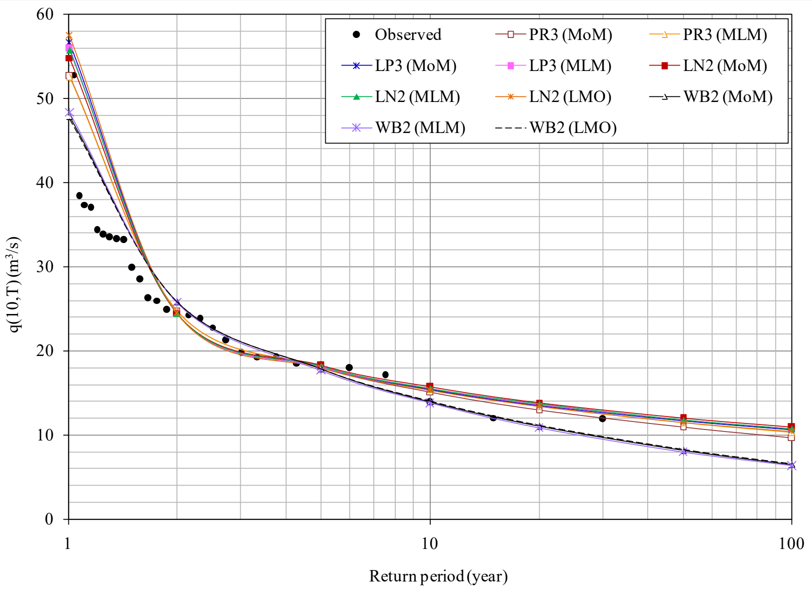

Figure 3: Plots of estimated 10-day low-flow using PR3, LP3, LN2 and WB2 distributions with observed data.

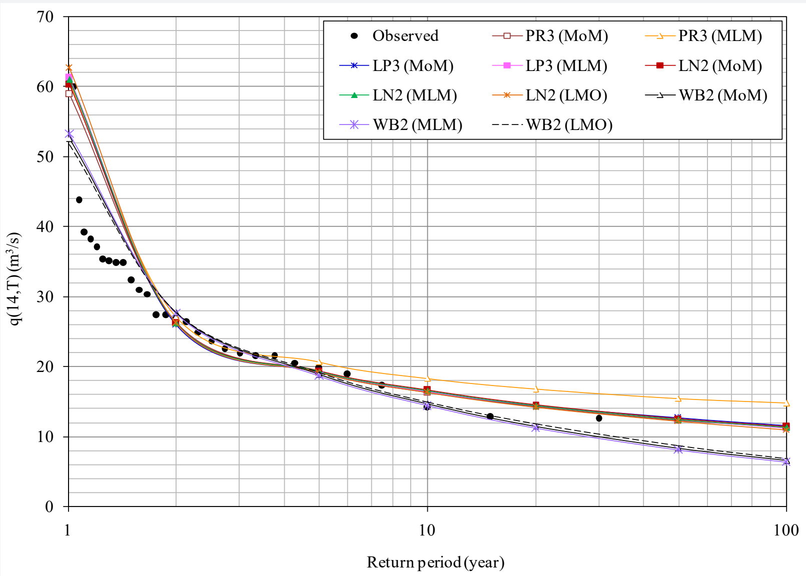

Figure 4: Plots of estimated 14-day low-flow using PR3, LP3, LN2 and WB2 distributions with observed data.

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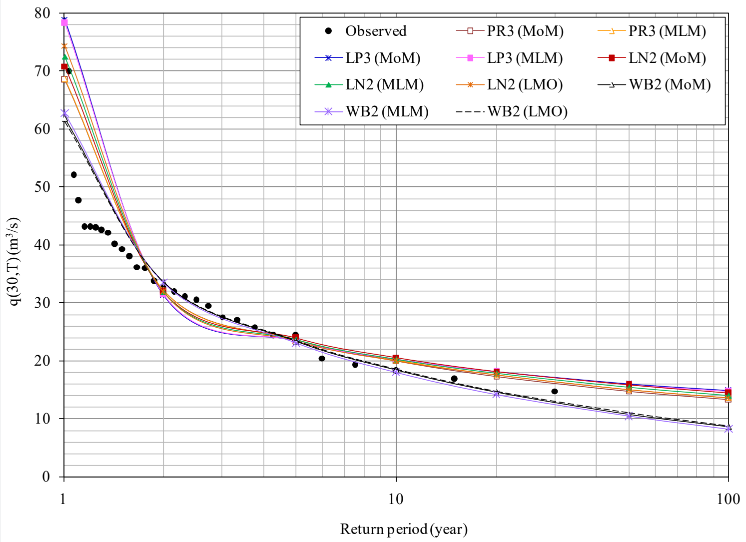

Figure 5: Plots of estimated 30-day low-flow using PR3, LP3, LN2 and WB2 distributions with observed data.

Conclusion

The paper presented a study on comparison of four probability distributions viz., PR3, LP3, LN2 and WB2 adopted in frequency analysis of low-flow for river Cauvery at Kollegal gauging site. The parameters of the distributions were determined by MoM, MLM and LMO, and are used for estimation of *q*(d,T) for different duration of 'd' viz., 7-, 10-, 14- and 30-days and for different return period vary from 1.01-year to 100-year. From the LFA results, the following conclusions were drawn from the study.

a) χ^2 test results supported the use of LN2 distribution for modelling the AM7AF and AM10AF while PR3 and WB2 for AM10AF.

b) χ^2 test results confirmed the applicability of PR3, LP3, LN2 and WB2 distributions for modelling the AM14AF and AM30AF.

c) *KS* test results confirmed the applicability of PR3, LP3, LN2 and WB2 distributions for modelling the AM7AF, AM10AF, AM14AF and AM30AF data series.

d) The *q*(d,T) estimates for different duration of 'd' viz., 10-, 14- and 30-days obtained from WB2 (MLM) distribution are lower than those values of other probability distributions adopted in LFA for the return periods from 10-year to 100-year.

e) *CC* values given by PR3, LP3, LN2 and WB2 distributions indicated that there is a good correlation between the observed and estimated low-flows and these values vary between 0.961 and 0.985.

f) Qualitative assessment (plots of the LFA results) of the outcomes was weighed with *RMSE* and accordingly LN2 (LMO) distribution is found to be better suited for estimation of low-flow.

g) The LFCs indicated that the estimated low-flows using LN2 (LMO) are nearer to the observed low-flows.

The study suggested that the estimated low-flows by LN2 (LMO) distribution could be adopted for various applications such as water resource management, water quality regulation, reservoir storage design, hydro electric power generation in arid periods and habitat protection.

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