

Buckling Analysis of Guyed Concrete Transmission Poles



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Abstract

The buckling loads of guyed, pre-stressed concrete transmission poles of various heights and classes are computed using the Gere and Carter Method (GCM) and Linear Buckling Analysis using ANSYS finite element software. Four (4) specific end conditions are investigated: fixed-free, fixed-pinned, pinned-pinned and fixed-fixed ends. The buckling loads obtained by the two methods are compared. It is observed that there is good agreement between the values for buckling loads obtained by two approaches for all the end conditions, the maximum average discrepancy being about 5.84%. Suggestions for further extension of the study are made.

Keywords: Buckling; Convergence; End conditions; Finite element; Guyed, Pole, Transmission, Pre-stressed, Concrete, Wood

Notation: d =diameter; d_a =diameter at point of guy attachment; d_g =diameter at ground line; d_e =diameter at pole butt; f'_c =compressive strength of concrete; k =effective length factor; D_e =depth of embedment; E =modulus of elasticity; I =moment of inertia; I_a =moment of inertia at A; L =length of pole or column; L_{AG} =height of pole above ground; M_g =Ultimate moment capacity at ground line; P^* = multiplier in Equation (2); P_A =critical load for a uniform column of diameter d ; P_E =critical buckling load; P_{cr} = critical buckling load; α =exponent in Equation (3); t =pole wall thickness

Introduction

Guyed transmission poles are commonly used in transmission lines at turning angle locations and at dead ends (See Figure 1). Although guying is routinely employed in wood pole systems owing to a lack of bending strength of wood, it is also occasionally used for poles of other materials too such as steel, pre-stressed concrete, laminated wood and composite [1].

As can be seen from Figure 1, the purpose of the guy wires is to back up the wire tension resultants from the conductors and the ground wires and transmit them to the anchors. Figure 1a shows a single anchor associated with all four guy wires; but Figure 1b,c show multiple anchors. Bending moments in the pole are generally minimal since each guy location acts as a partial lateral support, restraining movement in a specific direction. A single set of guy wires can be adequate for small line angles depending on the imposed wire tensions. But a 90-degree dead end shown in Figure 1d requires guys and anchors in both orthogonal directions. The most common slope for a guy wire

is 45 degrees. One result of the guy wire tensions is the vertical load component at each guy location and this component-cumulative for multiple guy wires-acts as an axial load on the pole/column. For 90-degree dead ends the axial loads come from both directions. Therefore, design involves check for pole buckling [2].

From analysis perspectives, a concrete transmission pole is basically a tapered beam-column with constantly varying cross section from base to top. Pole wall thickness is generally kept constant. Design of guyed concrete poles in USA is governed by various codes and standards [3-6,7-9]. Since concrete is a stable and not a bio-degradable material, there is no imposition of a strength reduction factor in design.

Past research included several landmark studies on guyed wood poles, including the FE Method (FEM). Non-linear stiffness matrices were developed for various shapes; but little effort was made to systematically analyse guyed poles of different

structural materials. In addition to wood, guyed poles currently include tubular steel, pre-stressed concrete, laminated wood and fiber-reinforced polymer (composite) poles. This study is

part of an undertaking towards that larger goal which comprises several papers on guyed transmission poles.

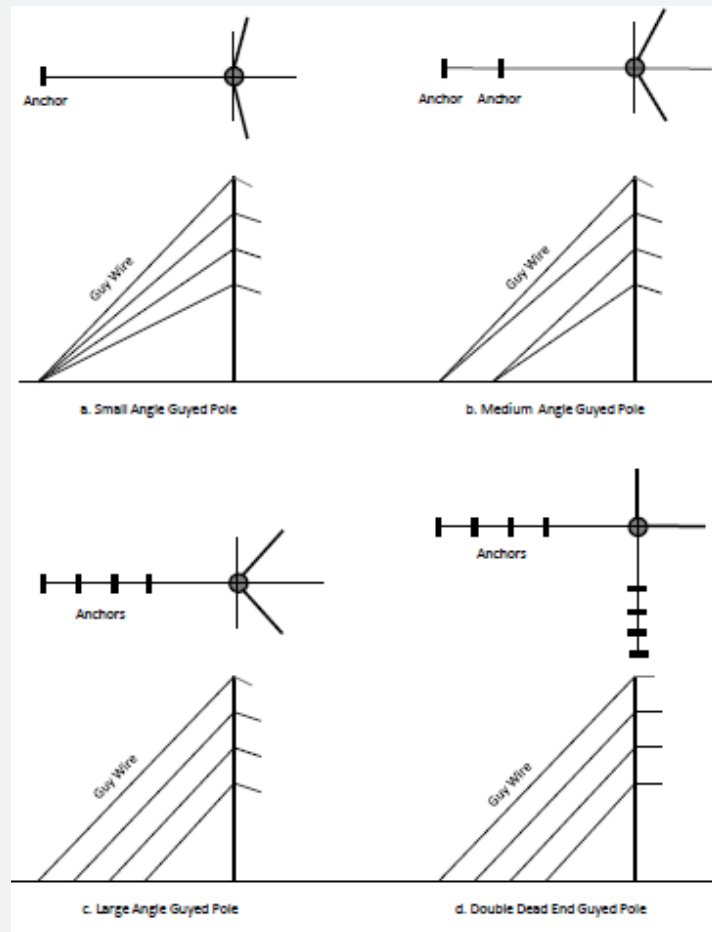


Figure 1a-d: Guyed Transmission Pole Structures - Angle and Dead End.

The aim of this paper is to investigate the buckling behaviour of guyed pre-stressed concrete transmission poles employing the FEM and verify the values of the buckling loads with theoretical estimates. A set of 15 concrete poles representing the most common sizes and classes of poles utilized in guying situations, together with four kinds of end conditions, were considered. Only linear elastic buckling is studied. Although initial out-of-straightness is often important in buckling patterns, it is not considered here. This paper is intended to be the third of a series of papers on guyed poles; others will be on wood, steel, laminated wood and composite poles.

Literature Review

Theoretically, a transmission pole of any material is a tapered beam-column with constantly varying cross section from base to top. As such, analytical modelling involves the challenge of incorporating stiffness matrices into analysis algorithms. Hence a significant amount of work had been directed towards formulating finite element stiffness matrices to handle tapered

beam-columns [10-18]. The earliest study on tapered beam-columns is contained in the classic 1968 paper by Gere & Cater [19] where expressions were derived for the buckling strength of steel beam-columns for different end conditions. This study provided the foundation for estimating the buckling capacity of wood poles.

Peabody & Wekezer [20] investigated the buckling strength of transmission and distribution poles using the FEM. Buckling capacities were determined with an Eigenvalue approach using elastic and geometric stiffness matrices of beam elements. One important conclusion drawn in this study is that assuming the unbraced length of the beam-column as the distance from the lowest guy to the ground can lead to a dangerous overestimation of the buckling strength. This implies that a more rational approach would be to consider the entire pole above the ground as a candidate for buckling.

The ASCE Manual 91 [3] is another design reference for guyed transmission structures. This Manual also refers to the Gere-

Carter Method and discusses the difficulties associated with real-life boundary conditions. Idealized end conditions rarely exist in real poles. An important point made in the Manual is that most theoretical methods, including the Gere-Carter Method, lead to large errors in situations where the capacity of the pole is controlled by bending stresses amplified by large compressive loads (i.e.) a true nonlinear beam-column behaviour. Based on a nonlinear finite element analysis of a guyed pole, including guy wires modelled as cable elements, it is shown that theoretical estimates are 1/2 to 2 1/2 times that predicted by the FEM. It is therefore left to the design engineer to adopt a reasonable factor of safety in using theoretical procedures.

The RUS Bulletin 153 [14] also contains another procedure to compute elastic buckling loads of wood poles. It is based on the American Institute of Timber Construction (AITC) approach which suggests that the critical section for a guyed wood pole is located at 2/3 the distance of from the ground to the bottom-most guy attachment. It also recommends a minimum factor of safety of 1.50 to be applied to the computed loads. Subsequent studies suggested that the RUS procedure is more conservative than the Gere-Carter Method (GCM). Commercial computer

programs such as PLS-Pole [21] include both the GCM and RUS options for users. Despite a number of theoretical and analytical investigations, no full-scale tests were performed to confirm and generalize the observations from those studies.

End Conditions of Guyed Poles

IEEE 751 [22] discusses the issue in detail. The end restraints in guyed poles are difficult to evaluate. Poles with bi-sector guys, as seen in Figure 1a-c, approximate a condition of being pinned at the point of guy attachment and fixed at the base, *in plane of the guy wire*. However, at 90 degrees to this plane, the pole is basically a free cantilever. Also, it is generally believed that the conductors also offer some kind of restraint. Therefore, the overall effective boundary conditions may be between the fixed-free ($k=2$) and fixed-pinned ($k=0.7$). For 90 degree guyed dead ends as shown in Figure 1d, the fixed-pinned condition occurs in both planes ($k=0.7$). Some engineers question whether the guy wires restrain lateral motion sufficiently to justify a $k=0.7$. Due to this, routine designs assume $k=1$ although a value of 0.80 is often suggested [6]. (See Figure (1e) for configuration of various boundary conditions).

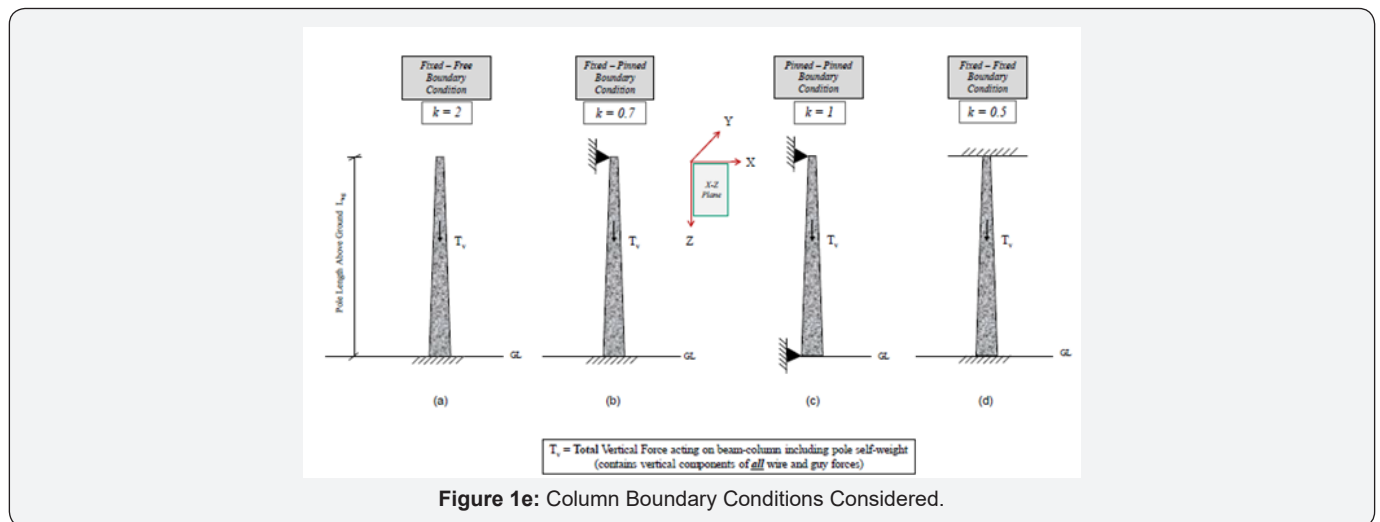


Figure 1e: Column Boundary Conditions Considered.

Euler's Elastic Buckling Theory

A column is considered stable only if the critical elastic buckling load (or Euler load) is not exceeded. The elastic buckling load of a column is defined as the axial load that is just sufficient to keep an initially straight, perfectly elastic bar in a slightly bent form. This load is usually taken as the ultimate load that the column can support, assuming the column is relatively slender and buckling occurs in the elastic range. The Euler's critical buckling load for long slender columns of uniform section is given by:

$$P_E = \frac{\pi^2 EI}{(kL)^2} \quad (1)$$

where

P_E = critical buckling load

k = effective length factor

L = actual length of column

E = modulus of elasticity of column material

I = least moment of inertia of the column cross-section

Equation (1) is obviously applicable only to uniform, homogenous columns where the axial load is concentrically applied at the end of the column. This assumption is not valid for a guyed *tapered* pole: constantly varying cross section aside, a typical guyed transmission pole has axial loads from guy wires applied intermittently at various heights along the pole. Hence Euler theory cannot be directly used for such poles.

Gere and Carter Method

The Gere & Carter Method (GCM) [19] is commonly used in practice for calculating critical buckling loads for guyed tapered

transmission poles. This method proposes modification to the Euler’s critical buckling equation as follows:

$$P_{cr} = P_A P^* \tag{2}$$

where:

P_A =Critical load for a uniform column with circular sections having diameter d (at guy attachment)

P^* =A multiplier dependent on the end conditions of the column

$$= \left\{ \frac{d_g}{d_a} \right\}^\alpha \tag{3}$$

where

α = An exponent that is a function of shape of the column

d_a = Diameter at point of guy attachment

d_g = Diameter at the ground line

L = Distance from ground line to the point of guy attachment

Equations for P_A and P^* for various end conditions [22] are summarized in Table 1.

Table 1: Equations for various end conditions for a tapered pole.

End Conditions	PA	P*
Fixed-Free	$\frac{\pi^2 EI_A}{(2.0L)^2}$	$\left\{ \frac{d_g}{d_a} \right\}^{2.7}$
Fixed-Pinned	$\frac{\pi^2 EI_A}{(0.7L)^2}$	$\left\{ \frac{d_g}{d_a} \right\}^{2.0}$
Pinned-Pinned	$\frac{\pi^2 EI_A}{(1.0L)^2}$	$\left\{ \frac{d_g}{d_a} \right\}^{2.0}$
Fixed - Fixed	$\frac{\pi^2 EI_A}{(0.5L)^2}$	$\left\{ \frac{d_g}{d_a} \right\}^{2.0}$

(Ref: IEEE 751)

The Gere and Carter Method assumed that the column is initially perfectly straight, elastic and compressed by an axial load. It is a theoretical method verified by limited tests on steel shapes for tapered poles of various open and closed cross sections subject to only axial load. When using this method for NESC ice and wind loads with specific load factors, it is recommended to use a factor of safety between 2.50 and 3.0 [9]. For extreme wind loads, where all load factors are 1.0, the resulting factors

of safety will be between 1.5 and 2.0. For pure dead ends, higher safety factors must be used due to larger axial loads.

Application to Selected Concrete Poles

As opposed to round wood poles where the cross section is solid and round, all pre-stressed concrete transmission poles are circular with a specific wall thickness. This thickness ranges from 3 inches to 6 inches (7.6cm to 15.2cm). All concrete poles contain facility for steel tendons in the longitudinal direction (post-tensioned). Five (5) concrete poles ranging in length from 50ft. to 70ft. (15.2m to 21.3m), involving three (3) standard pole classes and four (4) specific boundary conditions are studied. This set of 5x3=15 poles represent common sizes of poles utilized in guying situations. The definition of a pole class is illustrated in Figure 2 while Table 2 shows the numerical values of lateral loads associated with RUS concrete pole classes [7,8]. Other design properties of these poles are shown in Table 3 [23]. Effect of pre-stressing tendons is not considered in this paper since the focus is on axial response and not flexure. Also, calculations for section properties use only gross area and gross moment inertia [6]. Concrete is assumed *un-cracked* for all calculations.

Table 2: Standard Spun Concrete Pole Classes.

Concrete Class Designation	Minimum Ultimate Moment Capacity at 5 feet from Pole Tip (kip-ft)	Lateral Load at Tip (lbs.)*
C-12.0	96	12,000
C-11.0	88	11,000
C-10.0	80	10,000
C-09.0	72	9,000
C-08.0	64	8,000
C-07.4	57	7,410
C-06.5	50	6,500
C-05.7	44	5,655
C-04.9	38	4,875
C-04.2	32	4,160
C-03.5	27	3,510
C-02.9	23	2,925
C-02.4	19	2,405
C-02.0	15	1,950

(Courtesy: RUS/USDA)

1kip-ft=1.356kN-m, 1 lb.=4.45N, 1ft.=30.48cm

*Applied 2ft. from the tip of the pole

Table 3: Properties of Prestressed Concrete Poles: Standard Classes.

Pole Length, L(ft)	Pole Class*	Ground Line or Embedment D_e (ft)	Pole Height Above Ground, $L_{AG}=L-D_e$ (ft)	Pole Wall Thickness 't' (in)	Outside Diameter at Top d_a (in.)	Outside Diameter at Ground Line d_g (in.)	GL Moment Capacity M_g (kip-ft)
50	C-03.5	7	43	3.25	8.25	15.99	151.7
50	C-02.9	7	43	3.125	8.25	15.99	123
50	C-02.4	7	43	3.25	6.5	14.24	98.4

55	C-03.5	7.5	47.5	3.25	8.25	16.8	168.35
55	C-02.9	7.5	47.5	3.125	8.25	16.8	136.5
55	C-02.4	7.5	47.5	3.25	6.5	15.05	109.2
60	C-03.5	8	52	3.25	8.25	17.61	185
60	C-02.9	8	52	3.125	8.25	17.61	150
60	C-02.4	8	52	3.25	6.5	15.86	120
65	C-03.5	8.5	56.5	3.25	8.25	18.42	201.65
65	C-02.9	8.5	56.5	3.125	8.25	18.42	163.5
65	C-02.4	8.5	56.5	3.25	6.5	16.67	130.8
70	C-03.5	9	61	3.25	8.25	19.23	218.3
70	C-02.9	9	61	3.125	8.25	19.23	177
70	C-02.4	9	61	3.25	6.5	17.48	141.6

Note: 1. Pole Embedment D_e is equal to 10% of Pole Length + 2.0ft. 1in.=25.4mm, 1 ft.=0.305 m, 1kip-ft=1.356kN-m

2. Minimum 28-day Concrete Strength $f'_c = 10000$ psi 1 psi = 6.895kPa

3. Modulus of Elasticity based on ACI 318 = $57000 * (\text{sqrt}(f'_c))$ GL=Ground Line

* Based on RUS/USDA and StressCrete standard concrete pole classes

4. Pole Taper=0.180in/ft

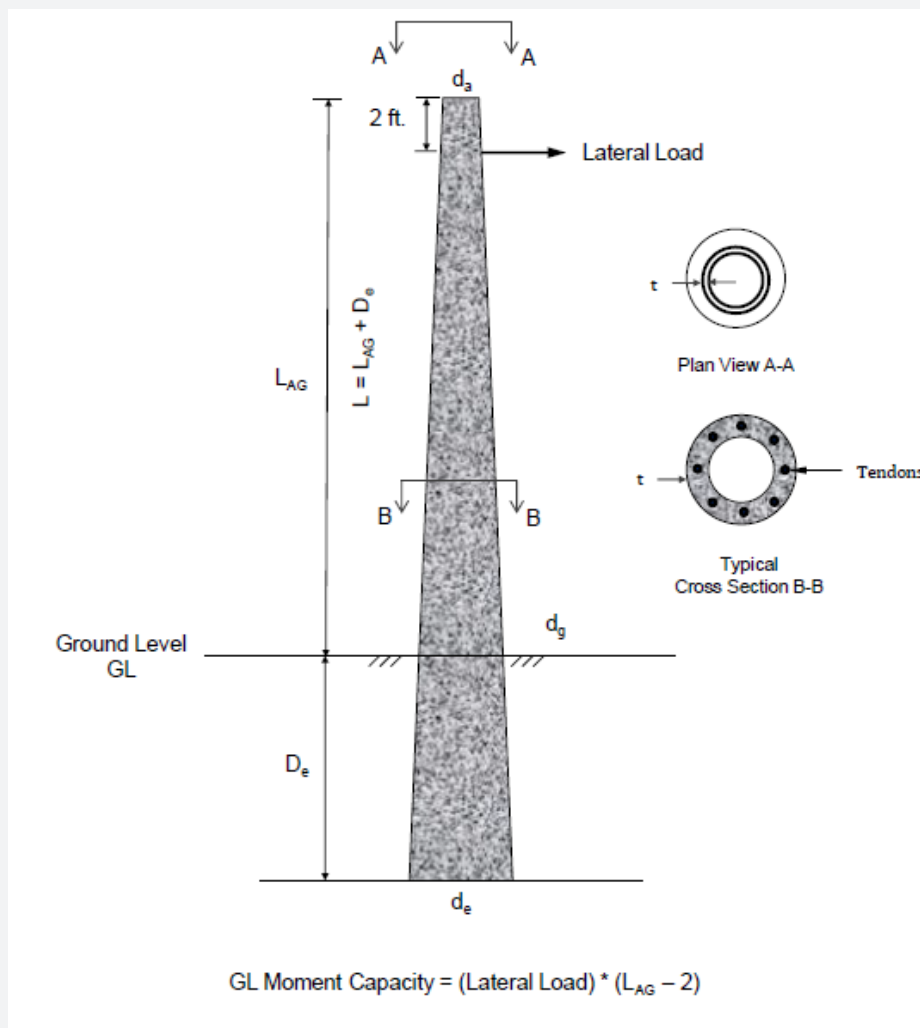


Figure 2: Definition of Concrete Pole Class.

Pre-stressed concrete poles in USA are manufactured using high strength concrete. Therefore, a 28-day strength of $f'_c = 10,000$ psi (68.95 GPa) is used for concrete in this paper. Modulus of Elasticity E is computed on the basis of ACI-318 [24] as $E = 57000 (f'_c)^{1/2}$. The values of the critical buckling load obtained

by the Gere and Carter Method are given in Table 4. These values are computed using a custom spreadsheet developed for the purpose. In all cases, the effective length for buckling is taken as the pole height above ground LAG.

Table 4: Failure Loads by Gere-Carter Method (kips).

Pole No	Pole Length, $L_{AG}/L^*(ft.)$	Pole Class	Fixed-Free	Fixed-Pinned	Pinned-Pinned	Fixed-Fixed
1	43/50	C-03.5	65.8	348.2	166.8	653.7
2		C-02.9	65.8	344.8	165.1	647.1
3		C-02.4	65	170	82.4	207.5
4	47.5/55	C-03.5	35.9	311	148.9	590.7
5		C-02.9	61.8	307.4	147.5	586.7
6		C-02.4	61.1	154.7	76.3	303.3
7	52/60	C-03.5	33.8	280.2	135	558.5
8		C-02.9	57.1	277.8	133.9	555
9		C-02.4	56.5	146.9	70.6	279.9
10	56.5/65	C-03.5	31.4	256.6	126.3	496.4
11		C-02.9	53.9	254.6	125.3	492.3
12		C-02.4	53.4	136.7	65.6	262.4
13	61/70	C-03.5	30.9	233.8	115.5	459.2
14		C-02.9	50.4	232	114.6	455.7
15		C-02.4	49.9	126.7	60.8	243.5

* See Figure 2 and Table 3 for definition for L_{AG} .
1ft.=0.3048m, 1kip=4.45kN

Buckling Analysis using FEM

In the present study, ANSYS software [25] is used to perform linear or classical finite element buckling analysis of tapered concrete poles. The finite element used is a Beam-188 element that has two end nodes with 3 translational and 3 rotational degrees of freedom at each node. This element is based on Timoshenko beam theory, includes shear deformation effects, well-suited for both linear/non-linear applications and flexural, lateral and torsional stability (using Eigenvalue buckling) problems. All FE models are planar beam-columns in the X-Z plane with movement out-of-plane restrained (see Figure 1e).

Convergence study

A convergence study was made to determine the number of elements which yields accurate result. A tapered concrete pole was discretized using uniform two-node Beam 188 elements. The results are provided in Table 5 for fixed-free end conditions, indicating that 12 elements would give sufficiently accurate results for buckling load. Hence, in all further work in this paper, a finite element model having 12 equal-length elements is used. (In comparison, Peabody & Wekezer [20] used 14 equal length beam elements in their FE studies).

Table 5: Results of convergence study.

No.	No. of Elements	Critical Load (kips)
1	1	53.158
2	2	62.11
3	3	65.877
4	4	67.364
5	5	68.083
6	6	68.474
7	7	68.795
8	8	68.856
9	9	68.938
10	10	69.023
11	11	69.08

Note: 50ft. Class C-03.5 Pole

$L_{AG} = 43ft. (13.11m)$; $d_a = 8.25in. (20.96cm)$; $d_g = 15.99in. (40.61cm)$

$t = 3.25in. (8.26cm)$

$f'_c = 1.0 \times 10^4$ psi (6.89 $\times 10^4$ kPa)

$E_c = 5.7 \times 10^6$ psi (39.27 $\times 10^6$ kPa)

1 inch=2.54cm, 1ft.=0.3048m, 1kip=4.45kN, 1psi=6.89kPa

Buckling loads from FEM

The adopted FE idealization is shown in Figure 3. To maintain consistency with the GCM, the effective length of the pole

FE model is taken as the pole height above ground, L_{AG} . Table 6 gives the critical buckling loads obtained by ANSYS for the 15 concrete poles.

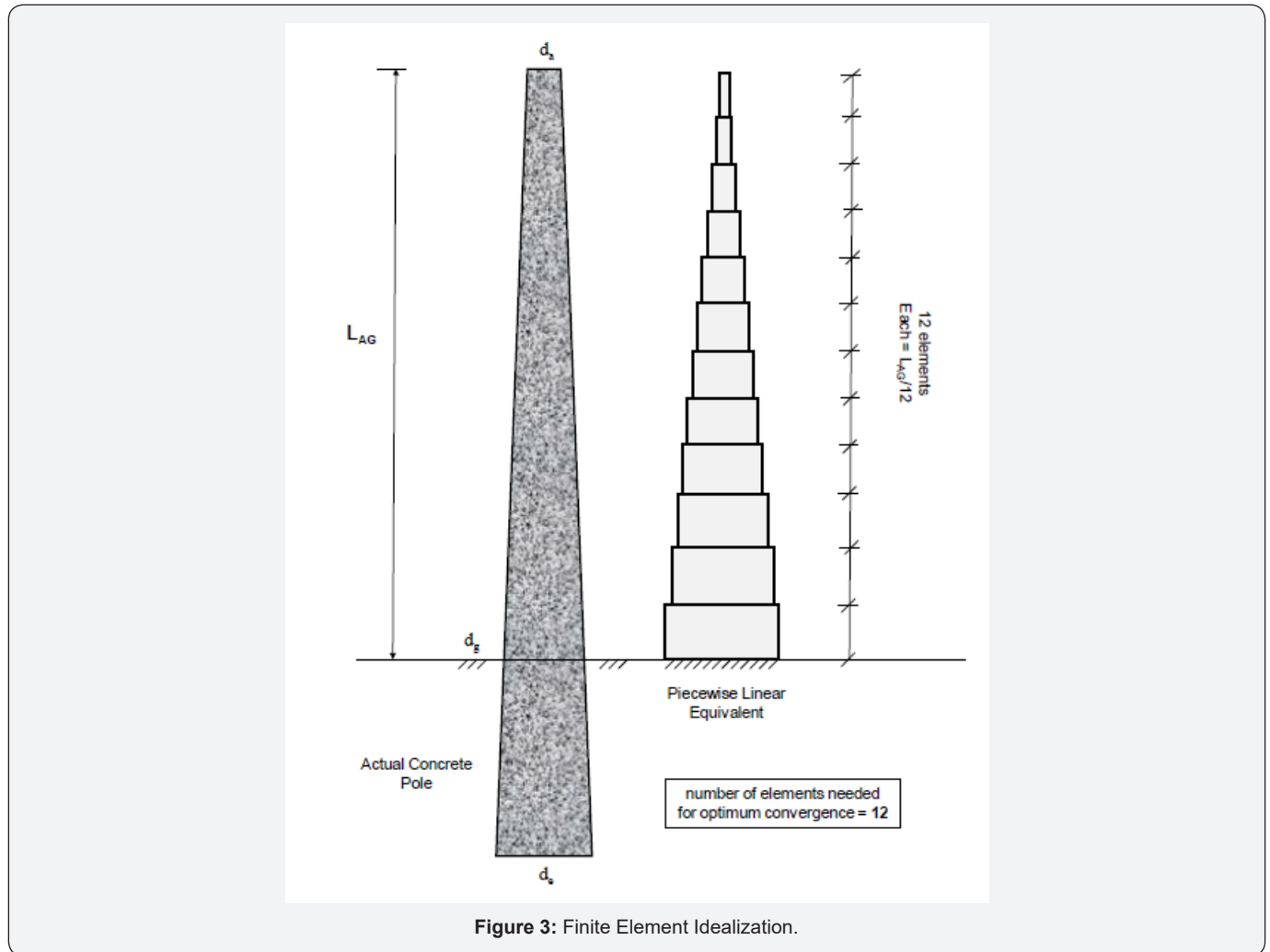


Figure 3: Finite Element Idealization.

Table 6: Failure loads by linear buckling analysis (kips).

Pole No	Pole Length, L_{AG}/L^* (ft.)	Pole Class	Fixed-Free	Fixed-Pinned	Pinned-Pinned	Fixed-Fixed
1	43/50	C-03.5	69.1	350.5	173.5	689.8
2		C-02.9	68.4	348.3	172.3	685.3
3		C-02.4	38	176.3	87.2	207.5
4	47.5/55	C-03.5	64.1	316.8	156.3	622.4
5		C-02.9	63.4	314.6	155.1	615.7
6		C-02.4	35.8	160.8	79.7	319.1
7	52/60	C-03.5	60.2	287.6	142.8	568.5
8		C-02.9	59.5	285.4	141.7	561.8
9		C-02.4	34	148.3	73.6	294.4
10	56.5/65	C-03.5	57	265.1	131.8	523.6
11		C-02.9	56.3	262.9	130.7	519.1
12		C-02.4	32.6	138.2	68.7	274.1

13	61/70	C-03.5	54.3	247.2	122.8	487.6
14		C-02.9	53.7	244.9	121.7	483.1
15		C-02.4	31.5	129.7	19.8	258.4

* See Figure 2 and Table 3 for definition for L_{AG} .

1ft. = 0.3048m, 1kip = 4.45kN

Comparison of Results and Discussion

The critical buckling loads obtained by Gere and Carter Method (Table 4) and linear buckling FE analysis (Table 6) are compared and the computed percentage discrepancy is shown in Table 7. There is excellent agreement between the two sets

of values. From the table, it is observed that the discrepancy is minimum in the case of fixed-fixed end condition and maximum for fixed-free end condition. The maximum value of percentage discrepancy is 8.20 for the fixed-free end condition. The average % discrepancy values for the four boundary conditions investigated were +5.84, +2.75, +5.12 and +4.89, respectively.

Table 7: Comparison of Failure Loads (Gere-Carter Method vs Linear Buckling Analysis).

Pole No	Pole Length, L_{AG}/L *(ft.)	Pole Class	Percentage Discrepancy			
			Fixed-Free	Fixed-Pinned	Pinned-Pinned	Fixed-Fixed
1	43/50	C-03.5	5.03	0.67	4.02	5.52
2		C-02.9	5.31	1.02	4.33	5.92
3		C-02.4	5.58	3.75	5.85	4.56
4	47.5/55	C-03.5	3.8	1.87	5	5.37
5		C-02.9	3.86	2.35	5.19	4.93
6		C-02.4	5.91	3.93	4.42	5.2
7	52/60	C-03.5	5.38	2.63	5.76	1.8
8		C-02.9	5.29	2.71	5.85	1.21
9		C-02.4	8.2	0.93	4.17	5.16
10	56.5/65	C-03.5	5.62	3.31	4.36	5.48
11		C-02.9	5.38	3.27	4.34	5.43
12		C-02.4	5.51	1.07	4.71	4.47
13	61/70	C-03.5	7.81	5.73	6.28	6.19
14		C-02.9	7.46	5.57	6.17	6.02
15		C-02.4	7.43	2.4	6.28	6.12
		Average	+ 5.84	+ 2.75	+ 5.12	+ 4.89

The drop in the buckling capacities for all poles of Class C-02.4 relative to the other poles is due to the values of section properties used for that class. Table 3 shows that the tip diameters of C-02.4 poles are 79% of those of C-03.5 and C-02.9; similarly, the ground line diameters of C-02.4 poles are about 90% of those of the other two classes. Since the moment of inertia 'I' of a circular cross-section is proportional to the 4th power of the diameter, any reduction in diameter will lead to much reduced buckling capacities (see Equations 1 and Table 2).

The main inference that can be drawn – pending validation with full-scale testing – is that the Gere and Carter Method is adequate for calculating nominal buckling loads, subject to application of factor of safety. Laboratory testing will help establish what rational factors of safety are needed for various loading situations. Peabody & Wekezer [20] obtained similar results for their FE analysis of an 80ft. (24.4m) - long wood pole where they compared FE results with those of Gere and Carter. The % discrepancy values for the same four boundary conditions were +4.43, -1.74, -0.35 and +0.36, respectively. The

paper recommended GCM for calculating buckling strength of transmission and distribution poles; but suggested performing a series of full-scale tests to validate GCM.

Upper and lower bounds of buckling capacities

As stated earlier, it is generally believed that the overall effective boundary conditions for a guyed pole may be between the fixed-free ($k=2$) and fixed-pinned ($k=0.7$). Therefore, these two situations may be treated as lower and upper bounds related to the buckling capacity. It is possible to obtain a mean value in-between the two bounds as a design reference.

Figures 4-8 show the buckling capacities bounds – and their mean values – for the 15 poles considered in the study. The values refer to GCM. For example, the mean value of buckling strength for a 60 ft. Class C-02.9 pole is 167.2kips (743.8kN). Assuming a factor of safety of 2.0 and extreme wind loads, the allowable buckling load on the pole is $167.2/2=83.6$ kips (372kN). This value may be considered as a design reference while checking buckling.

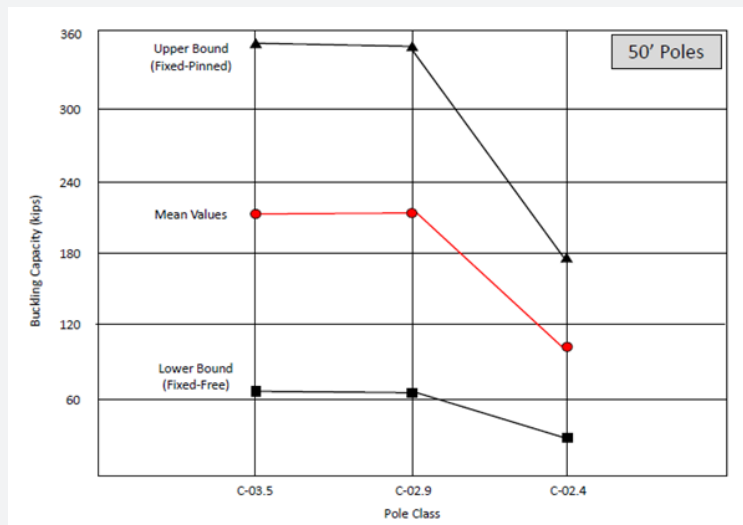


Figure 4: Upper and Lower Bounds of Buckling Capacity.

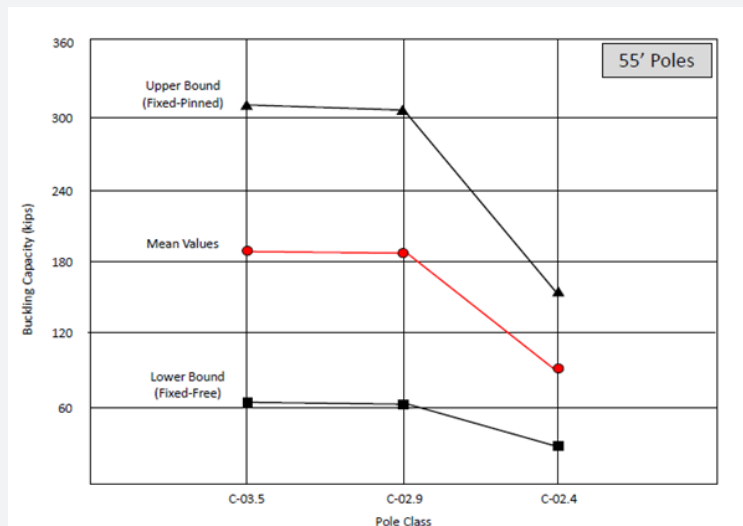


Figure 5: Upper and Lower Bounds of Buckling Capacity.

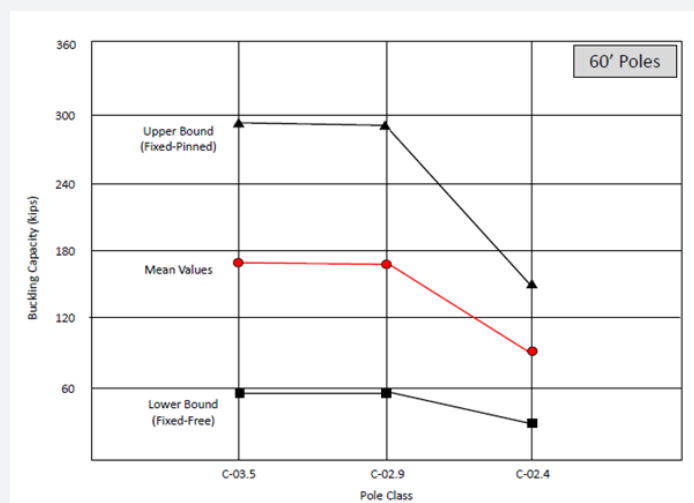


Figure 6: Upper and Lower Bounds of Buckling Capacity.

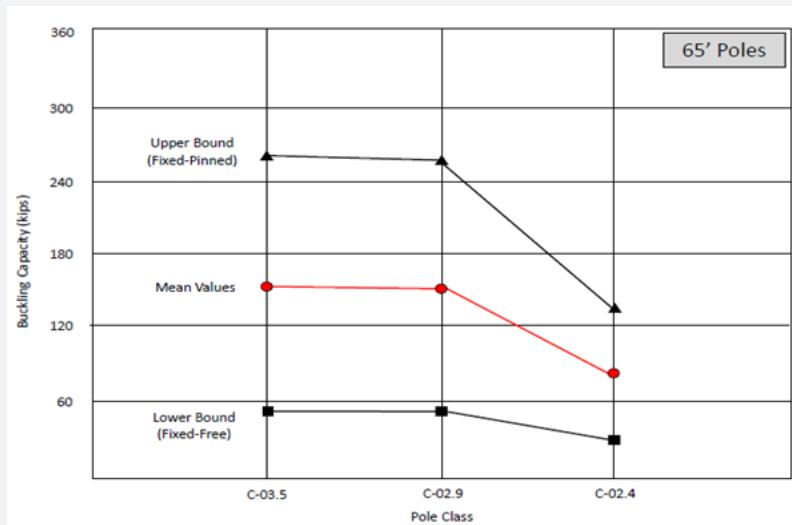


Figure 7: Upper and Lower Bounds of Buckling Capacity.

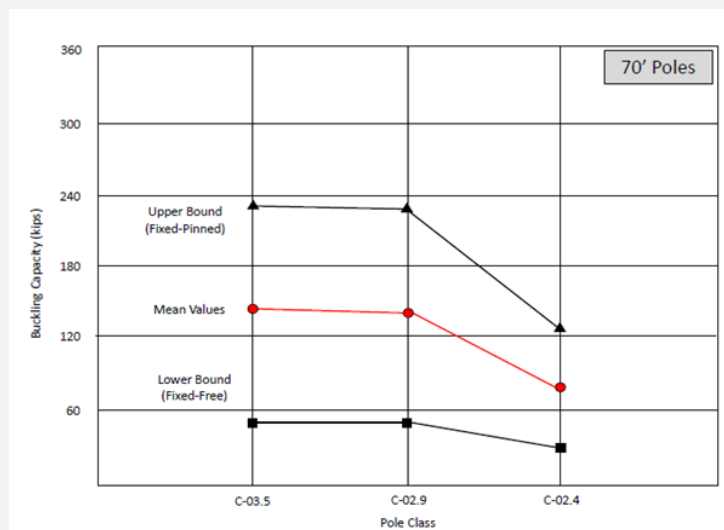


Figure 8: Upper and Lower Bounds of Buckling Capacity.

Conclusion

Fifteen concrete poles were analysed by a 12-element advanced FE model to determine critical buckling loads which were compared with theoretical estimates using the well-known Gere and Carter Method. The percentage of discrepancy between the buckling loads given by GCM and FEM is minimum for fixed-pinned end condition and maximum for fixed-free end condition. For all the cases studied in this paper, the average maximum discrepancy is about 5.84%. Similar results were obtained by Peabody & Wekezer [20] in their 1994 studies. For the cases considered it is seen that the Gere and Carter Method gives buckling loads that are quite close to that given by FEM and hence may be used conveniently in practice. To assist in routine design, the upper and lower bounds – and their mean value – of the buckling load can be considered. Given the uncertainty associated with correctly modelling the end restraints of a guyed pole, it is sug-

gested that a series of full-scale tests be performed to rationalize on the findings of this and previous studies.

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