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Transformation of Buckling Shapes Obtained from Shell Finite Elements in GBT (Generalized Beam Theory) Modes

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Abstract

This study presents the procedure in the transformation of the displacement fields obtained from shell finite elements into Generalized Beam Theory (GBT) modes. Such transformation is useful for geometrically linear buckling analysis, especially if one desires to convert a generic critical buckling shape, typically obtained in shell finite element analysis, into orthogonal deformation mode shapes, which is the main feature of GBT. Therefore, it is possible to reach a buckling modal classification: global, distortional and local, which is re-quired in Eurocode 3 to reach the amplitude of initial equivalent imperfections. A special attention is attributed to the matrix transformation between shell and GBT models, which is developed based on GBT kinematic as-assumptions. Finally, a detailed example of a thin-walled hollow circular cross-section is presented to clarify the present approach.

Keywords: Generalized Beam Theory; Coupling Equations; Shell Elements; Mixed GBT-shell model; Buck-ling

Introduction

GBT, Generalized Beam Theory, is a structural theory, which condenses kinematic properties of shell structures into beam structures. Initially, it was developed to describe open thin-walled beams by Richard Schardt [1], who in worked in linear analysis and non-linear analysis [2]. Later, GBT had a notorious development in Lisbon, where a group of researchers have extended the theory for a wider range of analysis and applications, such as branched cross-section, numerical implementation, nonlinear analysis and shear deformation, [3-13].

Among the GBT's applications, the buckling analysis of columns and beams is a highlight. Similar to finite Strip Method, GBT has the capacity to clearly express and classify a buckling mode as global, distortion and local modes. The simple fact to reach critical loads for non-trivial buckling shapes would be enough to put GBT as a relevant theory in practical applications of structural design, especially in Direct Strength Method (DSM) found in codes as AISI and NBR. But the applications of GBT can go much further. As presented here, GBT is also useful in the setup of initial equivalent imperfections, found in codes like Eurocode 3. In fact, the amount of initial equivalent imperfections to be applied in the structural models is totally based in the classification of buckling shape. This classification is not possible to be achieved from traditional beam or shell finite element analysis.

Since shell finite elements not only provide a powerful tool in buckling analysis, but also it is a disseminated technique, the transformation from the buckling shapes obtained in this type of models into the modal classification of GBT can enrich shell finite element analysis based on GBT features. The current study develops a novel application of GBT kinematic assumptions to reach the desired transformation above.

Transformation from Shell buckling shapes into GBT mode shapes

The transformation matrices between shell and GBT elements are based on the superposition property of GBT's deformation modes, which sets up the relationship between the degrees of freedom (DoF) of GBT and shell. In matrix form one can write: $[\vartheta]_{shell} = [T_c][\vartheta]_{GBT}$,, where: $[\vartheta]_{shell}$ and $[\vartheta]_{GBT}$ are the displacement vectors of shell and GBT's DoF's, respectively; and $[T_c]$ is the transformation matrix, which is based on GBT's description of displacement field as a summation of the modal displacement:

$$u(x,s) = \sum_{i=1}^{n} {}^{i}u(s)\,{}^{i}V_{,x}(x) \quad (1)$$

$$v(x,s) = \sum_{i=1}^{n} {}^{i}v(s)\,{}^{i}V(x) \quad (2)$$

$$w(x,s) = \sum_{i=1}^{n} {}^{i}w(s)\,{}^{i}V(x) \quad (3)$$

Here, *u*, *v* and *w* are the displacements in longitudinal, transversal tangential and transversal perpendicular

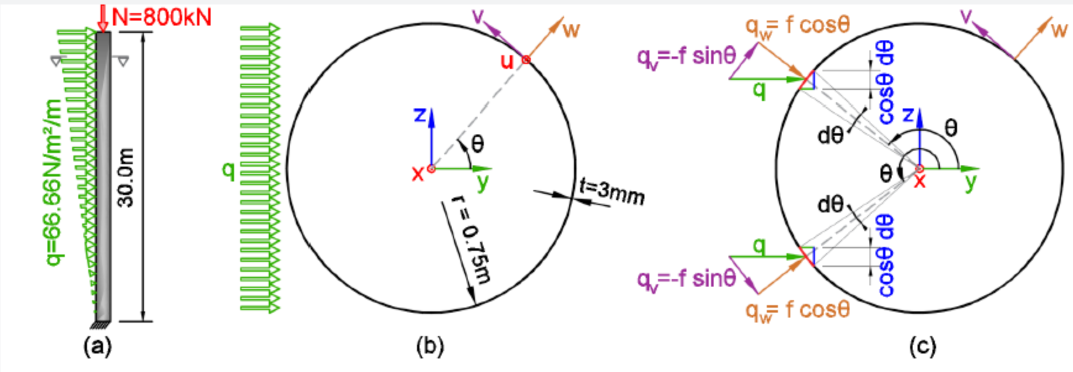

Figure 1: Thin-walled circular hollow section under a normal and linear projected force. a) elevation, b) cross section, c) force and projected area in a local coordinate system.

directions respectively, as shown in Figure 1; V is the amplification function of these displacements, which

describes the amount of each *i* GBT's mode shapes, indicated as an upper-left index. Hence, these equations

are used to express the shell's nodal displacement, which already leads to the transformation. For instance,

the summation of the transversal displacement w can be represented in a matrix form: $[w]_{shell} = [T]_w [V]_{GBT}$ .... (4)

Where the above vectors and the transformation matrix are:

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix}_{shell} = \begin{bmatrix} {}^2w_1 & {}^3w_1 & {}^4w_1 & \cdots & {}^mw_1 \\ {}^2w_2 & {}^3w_2 & {}^4w_2 & \cdots & {}^mw_2 \\ {}^2w_3 & {}^3w_3 & {}^4w_3 & \cdots & {}^mw_3 \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ {}^2w_n & {}^3w_3 & {}^4w_n & \cdots & {}^mw_n \end{bmatrix} \begin{bmatrix} {}^2V \\ {}^3V \\ {}^4V \\ \vdots \\ {}^mV \end{bmatrix}_{GBT} \quad \text{.................... (5)}$$

The indexes *n* and *m* indicate the number of shell node and GBT's modes, respectively. One can observe that the first GBT mode *i*=1 is not used. This mode represents the longitudinal compression/tension, which has

no transversal displacement and, consequently, there are no application here. In equation 5, the transformation matrix $\left[T\right]_W$ must be square. I.e, the number of GBT's modes, in which the shell buckling shape will be decomposed, must be the same number of nodes from which the displacements are extracted. In fact, to obtain the vector of GBT's amplification of each mode,

$[V]_{GBT}$ $\left[V\right]_{GBT} = [T]_w^{-1} [w]_{shell}$ .... (6)

A final remark is concerning the choice of the shell nodes to be used as base of the transformation. One must be aware to avoid linear dependency (or near to linear dependency) among the displacement. If two shell nodes have almost the same displacements, then numerical instabilities can occur in the inverse procedure in equation 6.

Numerical Example

As a detailed numerical example of the application of the transformation between the buckling shapes of shell finite elements into GBT modes, let us consider the thin-walled circular hollow steel cross-section shown in Figure 1. This cross-section is applied in a vertical cantilever structure subjected to a linear projected surface load, i.e. the total load applied in the structure is not a product of the surface load and the area of the surface, but actually the product of the surface load and the project area on the global coordinate direction z. Also, a vertical normal load of 800kN is applied at the top of the structure. The material parameters are Young Modulus $E = 205,\ 000 / mm^2$, Poisson's ratio $\mu = 0.3$ and Shear Modulus $G = 78,\ 846.2\text{N} / mm^2$.

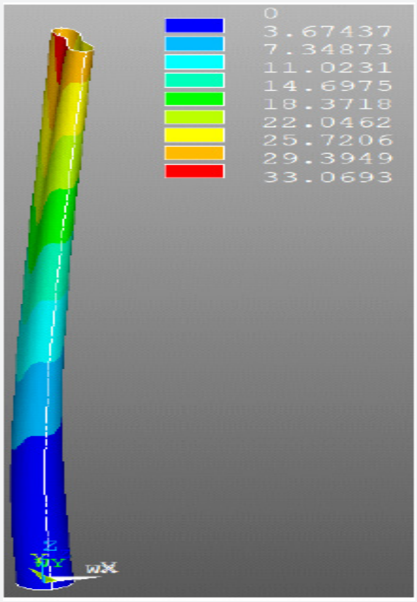

Figure 2: First buckling shape.

Initially, an eigenvalue buckling analysis was carried out in the commercial package ANSYS®. The discretization of the structure involves 100 nodes in each cross-section, in a total of 601 sections. 60.000 shell elements of type SHELL-181 (based on Mindlin-Reissner theory with linear interpolation functions) are used. After running the analysis, the first buckling shape, presented in Figures 2 & 3, was obtained. It is clear that this buckling mode has components of global (bending) and local/distortion (ovalization) displacements. The question is how much of each effect exist. This answer is obtained by the transformation

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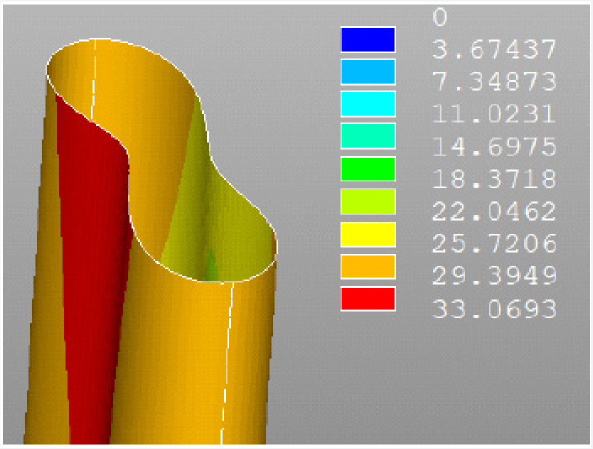

Figure 3: Detail of buckling shape at the top of the structure.

given in eq. 6.

To setup this equation, one must introduce the GBT's mode shapes. Usually, it leads to a quadratic eigenvalue problem, which has a non-trivial setup [14,15]. But, in the case of circular hollow sections this step is replaced by orthogonal deformation shapes based on Fourier series [1,16].

In Figure 4 some of these deformation shapes are presented and their respective values are given by:

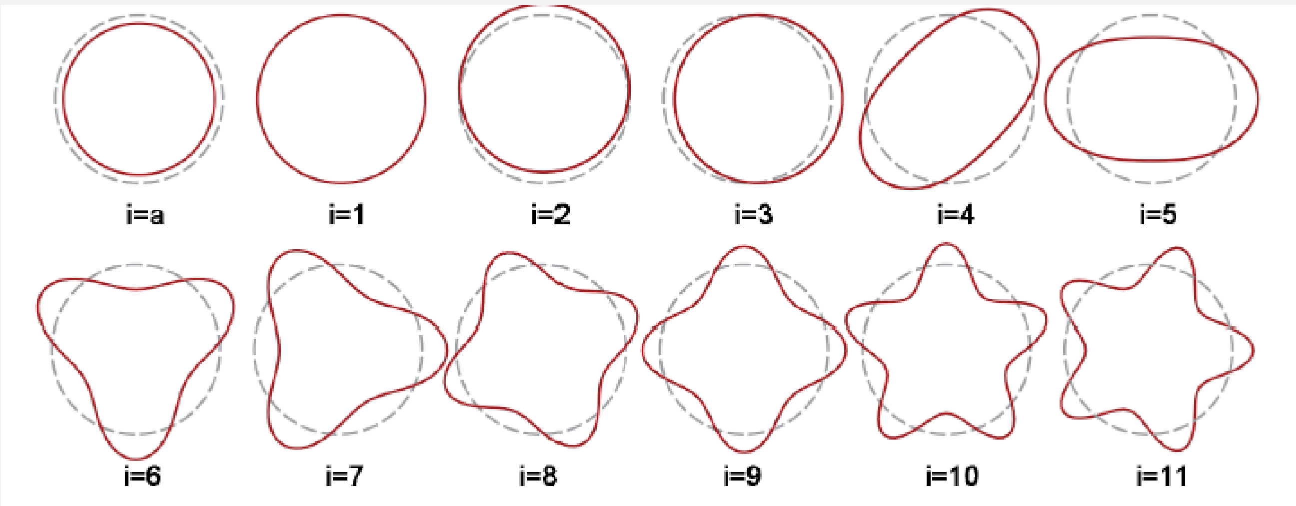

Figure 4: Transverse deformation shape modes of a thin-walled circular hollow section according to GBT.

- For pure axial extension mode, $i = a : {}^{i}u(\theta)=0\ {}^{i}v(\theta) = 0\ {}^{i}w(\theta)=1$ .... (7)

- For pure torsion mode, $i = t : {}^{i}u(\theta)=0\ {}^{i}v(\theta) = 1\ {}^{i}w(\theta)=0$ .... (8)

- For pure longitudinal extension mode, $i = 1 : {}^{i}u(\theta)=1\ {}^{i}v(\theta) = 0\ {}^{i}w(\theta)=0$ .... (9)

- For odd modes, *i*= 3,5,7, ... Where $m=(i-1)/2 \begin{cases} {}^{i}u(\theta) = -r\cos(m\theta) \\ {}^{i}v(\theta) = -m\sin(m\theta) \\ {}^{i}w(\theta) = m^2\cos(m\theta) \end{cases}$ .... (10)

For even modes, *i* = 2;4; 6; ::: where $m=i/2 \begin{cases} {}^{i}u(\theta) = -r\sin(m\theta) \\ {}^{i}v(\theta) = -m\cos(m\theta) \\ {}^{i}w(\theta) = m^2\sin(m\theta) \end{cases}$ .... (11)

Once one has the functions for *w* in orthogonal modes, it is possible to setup the transformation matrix $[T]_w$ by selecting a few arbitrary nodes in the cross-section. Here, the chosen cross-section is at the free end of the structure, from which the nodes at the angles: 0, 18, 36, 72, 144 and 180˚are analyzed (s 4). Adopting the first six odd GBT's mode: 3, 5, 7, 9, 11 and 13, one can reach the transformation matrix with the help of the last equation in eq. 10:

$$[T]_w = \begin{bmatrix} 1 & 4 & 9 & 16 & 25 & 36 \\ 0:9510 & 3:2361 & 5:2901 & 4:9443 & 0 & -11:1246 \\ 0:8090 & 1:2361 & -2:7811 & -2:9442 & -25 & -29:1246 \\ 0:3090 & -3:2361 & -7:2811 & 4:9443 & 25 & 11:1246 \\ -0:8090 & 1:2361 & 2:7811 & -12:9442 & 25 & -29:1246 \\ -1 & 4 & -9 & 16 & -25 & 36 \end{bmatrix}$$ .... (12)

need to use GBT's even nodes. In fact, these modes are related to antisymmetric deformations. Concerning

the shell nodal displacement vector, $[w]^{T}_{shell}$ the values obtained from ANSYS® (in polar radial direction)

are: $[w]^{T}_{shell} = [w_{0^\circ} w_{18^\circ} w_{36^\circ} w_{72^\circ} w_{144^\circ} w_{180^\circ}] = [224.6242\ \ 220.3074\ 209.5505\ 126.0137\ -238.2702\ -330.6898]$ ... (10)

$[v]_{GBT} = [{}^{3}V\ {}^{5}V\ {}^{7}V\ {}^{9}V\ {}^{1}1V\ {}^{1}3V] = [277.0277\ -12.8046\ 0.0674\ -0.2671\ 0.00089\ 0.06836]$ .... (11)

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Table 1: Final percentages of the GBT's modes in the shell finite element buckling Analysis.

GBT's modes	m	Weight Factor	Amplification ⁱV	Absolute participation	% f
3	1	1	277.0277	277.0277	82.54%
5	2	4	-12.8046	51.2187	15.26%
7	3	9	0.0674	0.607	0.18%
9	4	16	-0.2671	4.2751	1.27%
11	5	25	0.00089	0.0222	0.01%
13	6	36	0.06836	2.461	0.73%
Total				335.612	

The above results stand out that mode 3 (global buckling) is the major effect, followed by the first ovalization mode (mode 5). Among the high modes, the highlight is mode 9, especially if it is compared to modes 7 and 11. Finally, in order to achieve a percentage of each mode, it is important to keep in mind that each amplification term, presented in eq. 14, must be weighted by the respectively modal value m^2: (Table 1)

Conclusion

This study presents and evaluates a transformation approach from shell to GBT buckling shapes. The transformation matrix is based on GBT's assumptions of transversal displacements, which requires the same amount of GBT's modes and nodal shell displacements. As presented in the numerical example, buckling shapes obtained from shell analysis have a mixed behavior of global, distortional and local displacements. By the present formulation is possible to decompose and quantify each one of these buckling behaviors, which are relevant in structural analysis, especially concerning the evaluation of initial equivalent imperfections.

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