

Numerical Analysis of Stressed State of an Elastic Strip Plate with Collinear Cracks and Thin-Walled Inclusions at Antiplane Shear Loading



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Abstract

In this paper, a long elastic strip plate with collinear cracks at antiplane deformation in the case that each crack tips are joined by thin-walled inclusions deformed according to the known Winkler's model is considered. The uniformly distributed shear forces causing the antiplane deformation of the plate are acting on the horizontal sides of the strip and the edges of cracks are free of inclusions. For convenience in numerical calculation the strip plate is divided to several plates so that each segment has one crack at the center. The solution of the stated problem via Fourier sine transformation is reduced to singular integral equation (SIE), and, consequently, to a system of linear equations. Numerical calculations based on the Gauss Quadratic solution are achieved. For the main characteristics of stated problem, such as the SIF, the crack opening, the shear stresses on the edges of the inclusion, and the shear stresses out-of-crack the obvious equations are obtained and the special cases considered.

Keywords: Numerical analysis; S.I.F; Tip inclusions; Anti-plane Shear; SIE

Introduction

In this paper, we calculate the stress distribution state and S.I.F of the crack tips and dislocations of edges of a long strip elastic rectangular plate (Figure 1). The stress Intensity is essentially decreasing the known strength and durability of structural members and engineering parts. For this reason, the necessity of theoretical investigation of stress concentration zones and the development of the methods which decrease the stress intensities is occurred. One of these methods was proposed in [1], the edges of linear finite crack of elastic infinite plate at the end areas are joined via thin-walled inclusion in the shape of continuously distributed linear and non-linear deformed springs, meanwhile, the plate is subjected by uniformly distributed tensile remote stress perpendicular to the central line of crack. Taking into account the above-mentioned

physical model of inclusions and based on assumptions in [2-4], the valuable decrease of stress intensity factors (SIF) at the end points of crack can be achieved by the appropriate selection of elastic and geometric characteristics of problem, and this can prevent the crack propagation. Applying Fourier finite sine transformation, the solution of stated problem can be reduced to the solution of singular integral equation (SIE), and, consequently, via the known method [5-7], the solution of singular integral equations can be reduced to the system of linear equations. For the main characteristics of stated problem, such as the SIF, the crack opening, the shear stresses on the edges of the inclusion, and the shear stresses out-of-crack on the line of its location, the obvious equations are obtained, the special cases are considered and for various materials the decreasing trend of S.I.F based on the various shear modulus were shown.

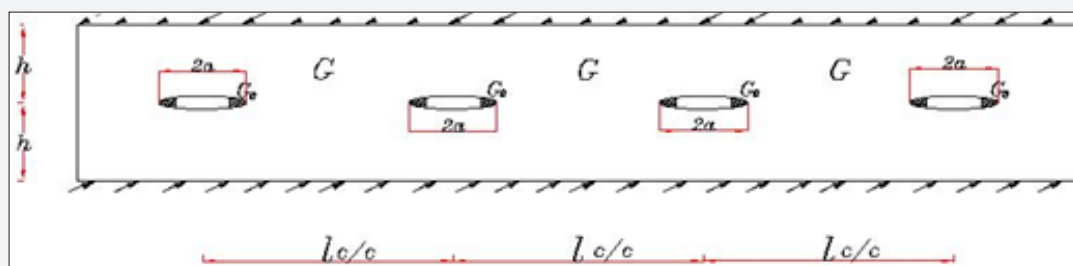


Figure 1: Elastic strip plate D1 with collinear cracks at antiplane deformation with inclusions at e tips.

Governing Equation of Boundary Value Problem

Following the approach represented in [1] consider a prismatic elastic body with a rectangular cross-section in Cartesian coordinates $Oxyz$ occupying an area $\Omega = \{-\infty \leq x \leq \infty; -h \leq y \leq h; -\infty < z < \infty\}$ and possessing a shear modulus G . The prismatic elastic body is rigidly clamped by the vertical edge $x = -\infty$ and $x = +\infty$, and loaded by the shear forces equal to $T(x)$ acting both in positive and in negative directions of Oz -axis at the horizontal $y = \pm h$. Furthermore, on the symmetry plane $y = 0$, the body Ω has several through-in-thickness cracks each in a shape of strip with length $2a$ on plane $y = 0$ and $-\infty < z < \infty$ at distances equal to ℓ ($a < \ell/2$). The shear forces of equal intensities $T_0(x)$ are acting in opposite directions of Oz -axis on the upper (+) and lower (-) areas of edges $\omega_{\pm} = \{y = \pm 0; \ell/2 - b < x < \ell/2 + b; -\infty < z < \infty\}$ ($b < a$) of the crack. Besides that at the ending areas

$$\omega_{0i}^{\pm} = \{y = \pm 0; x \in (\ell/2 - a; \ell/2 - b) \cup (\ell/2 + b; \ell/2 + a); -\infty < z < \infty\}$$

The edges of the crack are joined by the thin-walled inclusions with the shear modulus G deforming by the Winkler model. Let's assume that the prismatic body Ω subjected to the above-mentioned shear forces is in a state of anti plane deformation in the direction of Oz -axis on the basic plane Oxy . The main rectangle $D_1 = \{-\infty \leq x \leq +\infty; -h \leq y \leq h\}$ with several cracks $\omega = \{y = 0; \ell/2 - a < x < \ell/2 + a\}$ ($0 < a < \ell/2$) is cross-section of the body Ω with the plane located on this plane Oxy (Figure 1).

It is necessary to determine the dislocation density on the crack edges, SIF, the crack edges opening, the shear contact stresses on the edges of the inclusion, and the shear stresses outside the crack on the line of its location. For convenience in numerical calculation the strip plate is divided to several plates so that each segment has one crack at the center (Figure 2).

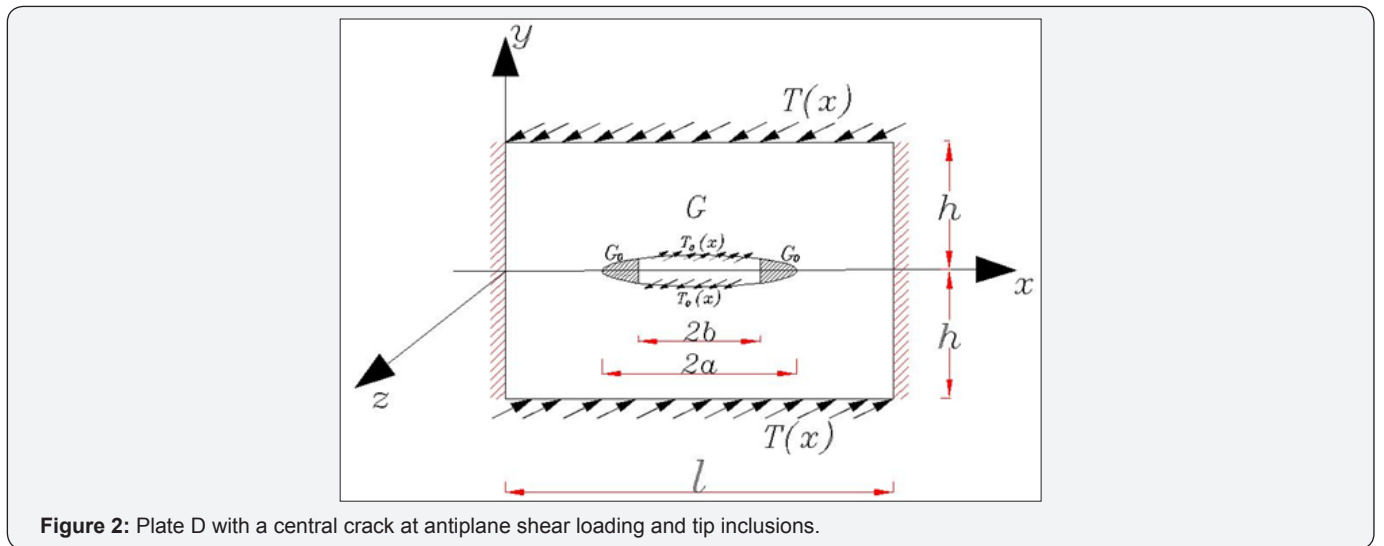


Figure 2: Plate D with a central crack at antiplane shear loading and tip inclusions.

Now, let's derive the governing equations of the stated problem. For this purpose, it must be initially mentioned that the component $u_z = w(x, y)$ in the direction Ox -axis is the only non-zero component of displacement in the case of anti-plane deformation, and harmonic function in the area $D \setminus \omega_0$. The components of shear stress τ_{xz} and τ_{yz} are the only non-zero components of stresses. Therefore, the problem can be mathematically stated as a boundary value problem in the following way:

$$\begin{cases} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 & ((x, y) \in D \setminus \omega_0) \\ w(x, y)|_{x=0} = w(x, y)|_{x=\ell} = 0 & (-h < y < h); \\ \tau_{yz}|_{y=h} = \tau_{yz}|_{y=-h} = G \frac{\partial w}{\partial y}|_{y=\pm h} = T(x) & (0 < x < \ell); \\ \tau_{yz}|_{y=0} = G \frac{\partial w}{\partial y}|_{y=0} = T_0(x) & (x \in (\ell/2 - b; \ell/2 + b); b < a); \\ \tau_{yz}|_{y=\pm 0} = \pm G_0 w(x, y)|_{y=\pm 0} = \pm k G w(x, y)|_{y=\pm 0} & (x \in (\ell/2 - a; \ell/2 - b) \cup (\ell/2 + b; \ell/2 + a); k = G_0 / G) \end{cases} \quad (1)$$

For of the determination of boundary value problem (1), the rectangle D is divided by Ox -axis onto upper $D_+ = \{0 \leq x \leq \ell; 0 \leq y \leq h\}$ and lower $D_- = \{0 \leq x \leq \ell; -h \leq y \leq 0\}$ rectangles. The following supporting boundary value problems are considered for them.

$$\begin{cases} \frac{\partial^2 w_{\pm}}{\partial x^2} + \frac{\partial^2 w_{\pm}}{\partial y^2} = 0 & (0 < x < \ell; 0 < |y| < h) \\ w_{\pm}(x, y)|_{x=0} = w_{\pm}(x, y)|_{x=\ell} = 0 & (0 < |y| < h); \\ \tau_{yz}|_{y=\pm h} = G \frac{\partial w_{\pm}}{\partial y}|_{y=\pm h} = T(x) & (0 < x < \ell); \\ \tau_{yz}|_{y=\pm 0} = \tau_{\pm}(x) & (0 < x < \ell); \end{cases} \quad (2)$$

Where, the sign "+" and "-" are related to the rectangles D_+ and D_- , correspondingly.

$\tau_{\pm}(x)$ is the unknown fracture shear stress outside the crack ω_0 on the its line.

$$\tau_{\pm}(x) = \begin{cases} T_0(x) & x \in (\ell/2 - b; \ell/2 + b); \\ \tau_{yz}|_{y=\pm 0} & (x \in (\ell/2 - a; \ell/2 - b) \cup (\ell/2 + b; \ell/2 + a)); \\ \tau(x) & (x \in (0; \ell/2 - a) \cup (\ell/2 + a; \ell)). \end{cases} \quad (3)$$

Taking into account the symmetry of stated problem with respect to $-y$ -axis

$$w_+(x, y) = -w_-(x, y) \quad (0 < x < \ell); \quad \tau_+(x) = \tau_-(x) \quad (0 < x < \ell); \quad (4)$$

Consequently, the determination of supporting problem (2) for the rectangle D_+ can be considered only. Based on the reference [8], the above-mentioned problem can be determined via Fourier finite sine transformation on the variable X .

$$\bar{w}_+(n, y) = \int_0^\ell w_+(x, y) \sin\left(\frac{\pi nx}{\ell}\right) dx \quad (n=1, 2, \dots, 0 < y < h) \quad (5)$$

Therefore, the Fourier inverse transformation has the following expression:

$$w_+(x, y) = \frac{2}{\ell} \sum_{n=1}^{\infty} \bar{w}_+(n, y) \sin\left(\frac{\pi nx}{\ell}\right) \quad (0 < x < \ell; 0 < y < h) \quad (6)$$

Multiplying by $\sin(\pi nx/\ell)$ both sides of the differential equation and the border conditions of (2), and integrating it from 0 to ℓ , Fourier finite transformation (5) can be applied to the boundary value problem (2) for D_+ . The boundary value problem in Fourier transformations can be obtained after the simple reductions.

$$\begin{cases} \frac{d^2 \bar{w}_+}{dy^2} - \frac{\pi^2 n^2}{\ell^2} \bar{w}_+ = 0 & (0 < y < h) \\ G \frac{d\bar{w}_+}{dy} \Big|_{y=0} = \bar{\tau}_+(n); \quad G \frac{d\bar{w}_+}{dy} \Big|_{y=h} = \bar{T}(n) & (n=1, 2, \dots) \end{cases} \quad (7)$$

Where, notation $(n=1, 2, \dots)$ is accepted.

$$\{\bar{\tau}_+(n); \bar{T}(n)\} = \int_0^\ell \{\tau_+(x); T(x)\} \sin\left(\frac{\pi nx}{\ell}\right) dx \quad (8)$$

The boundary value problem (7) can be defined by the following equation:

$$\bar{w}_+(n, y) = \frac{\ell}{\pi n G \operatorname{sh}(\pi n h / \ell)} \{ \bar{T}(n) \operatorname{ch}(\pi n y / \ell) - \bar{\tau}_+(n) \operatorname{ch}[\pi n(y-h)/\ell] \} \quad (n=1, 2, \dots, 0 \leq y \leq h);$$

Therefore, the following equation can be concluded from the previous one

$$\bar{w}_+(n, 0) = \frac{\ell}{\pi n G \operatorname{sh}(\pi n h / \ell)} [\bar{T}(n) - \bar{\tau}_+(n) \operatorname{ch}(\pi n h / \ell)] \quad (n=1, 2, \dots) \quad (9)$$

The Singular Integral Equation

After some simple transformations and calculations according to [1] and [9-11] the following equations can be derived:

$$\begin{aligned} & \frac{1}{\pi} \int_{-1}^1 \frac{\omega_0(\rho) d\rho}{\rho-r} + \frac{1}{\pi} \int_{-1}^1 K_0(r, \rho) \omega_0(\rho) d\rho - \frac{1}{\pi} \int_{-1}^1 K(r, \rho) \omega_0(\rho) d\rho = \\ & \begin{cases} f(r) - g(r) & (r \in (-c, c)); \\ f(r) - \lambda_0 \psi_0(r) & (r \in (-1, -c) \cup (c, 1)); \quad (c = \sin\beta / \sin\alpha) \end{cases} \end{aligned} \quad (10)$$

Denoting

$$\omega_0(r) = \phi_0(\arccos(r \cdot \sin\alpha)); \quad f(r) = \tilde{f}(\arccos(r \cdot \sin\alpha)); \quad g(r) = \tilde{T}_0(\arccos(r \cdot \sin\alpha)); \quad \psi_0(r) = \tilde{\phi}_0(\arccos(r \cdot \sin\alpha));$$

$$f(r) = \frac{2}{\pi} \sqrt{1 - \sin^2 \alpha \cdot r^2} \int_{-1}^1 \left[\sum_{n=1}^{\infty} \frac{U_{n-1}(r \cdot \sin\alpha) U_{n-1}(u)}{\operatorname{ch}(nh_0)} \right] \tilde{T}(\arccos u) du; \quad (11)$$

$$K_0(r, \rho) = \frac{(r + \rho) \sin^2 \alpha}{\sqrt{1 - \sin^2 \alpha \cdot \rho^2} (\sqrt{1 - \sin^2 \alpha \cdot r^2} + \sqrt{1 - \sin^2 \alpha \cdot \rho^2})};$$

$$(-1 < r, \rho < 1)$$

$$K(r, \rho) = 2 \sin\alpha \frac{\sqrt{1 - \sin^2 \alpha \cdot r^2}}{\sqrt{1 - \sin^2 \alpha \cdot \rho^2}} \sum_{n=1}^{\infty} \frac{e^{-nh_0}}{\operatorname{ch}(nh_0)} U_{n-1}(r \cdot \sin\alpha) T_n(\rho \cdot \sin\alpha);$$

The first integral of equation (10) for $\rho = r$ is assumed as a main value of the Cauchy's relations. Meanwhile, $T_n(x)$ and $U_{n-1}(x)$ are Chebyshev polynomials of the first and the second kind,

Finally, the stress intensity factors at the end points $(a_1; b_1)$ of the crack can be expressed by the following equations:

$$K_{III}(a_1) = \lim_{x \rightarrow a_1-0} \left[\sqrt{2\pi(a_1-x)} \tau_{yz}(x, 0) \right];$$

$$K_{III}(a_1) = \lim_{x \rightarrow a_1-0} \left[\sqrt{2\pi(a_1-x)} \tau_{yz}(x, 0) \right];$$

$$K_{III}(b_1) = \lim_{x \rightarrow b_1+0} \left[\sqrt{2\pi(x-b_1)} \tau_{yz}(x, 0) \right]; \quad (a_1 = \ell/2 - a; \quad b_1 = \ell/2 + a)$$

(12)

and through the half of dislocation density on the edges of the crack

$$K_{III}(b_1) = G \lim_{x \rightarrow b_1+0} \left[\sqrt{2\pi(b_1-x)} \phi'(x) \right] \quad K_{III}(a_1) = G \lim_{x \rightarrow a_1-0} \left[\sqrt{2\pi(x-a_1)} \phi'(x) \right]$$

(13)

Gauss Quadrature Method

Now, as it was mentioned above, the determinative singular integral equations (S.I.E) (10) can be reduced to a system of linear equations and following to the approach represented in [1], the determinative S.I.E (10) can be reduced to the system of Algebraic linear equations as follows:

$$\sum_{p=1}^M K_{mp} X_p = a_m \quad (m = \overline{1, M})$$

$$K_{mp} = \begin{cases} \frac{1}{M} \left\{ \frac{1}{\rho_p - r_m} + K_0(r_m, \rho_p) - K(r_m, \rho_p) + \frac{\lambda \sin\alpha \operatorname{sign} r_m}{2\sqrt{1 - \rho_p^2 \sin^2 \alpha}} [H(c+r_m) - H(c-r_m)] \operatorname{sign}(r_m - \rho_p) \right\}; \\ \quad (m = \overline{1, M-1}; \quad p = \overline{1, M}) \\ \frac{1}{\sqrt{1 - \rho_p^2 \sin^2 \alpha}} \quad (m = M; \quad p = \overline{1, M}); \end{cases} \quad (14)$$

$$a_m = \begin{cases} f(r_m) - [H(r_m + c) - H(r_m - c)] g(r_m); & (m = \overline{1, M-1}) \\ 0 & (m = M); \end{cases}$$

$$X_p = X_0(\rho_p) \quad (p = \overline{1, M}); \quad \rho_p = \cos\left(\frac{2p-1}{2M} \pi\right) \quad (p = \overline{1, M});$$

$$r_m = \cos\left(\frac{\pi m}{M}\right) \quad (m = \overline{1, M-1}).$$

Where M is an arbitrary natural number, r_m and ρ_p are the roots of Chebyshev's polynomials of the first kind $T_m(\rho)$ and the second kind $U_{M-1}(r)$.

The main physical characteristics of the stated problem can be expressed by the determination of the system of linear equations (14). The plate shear stress out of the crack at the interval $y=0, x \in (0, l/2-a) \cup (l/2+a, l)$ can be obtained as below:

$$\tilde{\tau}_0(t) = -\frac{\sin \alpha}{M} \sum_{p=1}^M \left\{ \frac{1}{\rho_p \sin \alpha - t} + \frac{t + \rho_p \cdot \sin \alpha}{\sqrt{1-\rho_p^2 \cdot \sin^2 \alpha} \left(\sqrt{1-t^2} + \sqrt{1-\rho_p^2 \cdot \sin^2 \alpha} \right)} \right. \\ \left. - \frac{2\sqrt{1-t^2}}{\sqrt{1-\rho_p^2 \cdot \sin^2 \alpha}} \sum_{n=1}^{\infty} \frac{e^{-nh_0}}{ch(nh_0)} U_{n-1}(t) T_n(\rho_p \cdot \sin \alpha) \right\} X_p + \frac{2\sqrt{1-t^2}}{\pi} \int_{-1}^1 \left[\sum_{n=1}^{\infty} \frac{U_{n-1}(t) U_{n-1}(u)}{ch(nh_0)} \right] \tilde{f}(\arccos u) du. \tag{15}$$

For the dimensionless crack opening, the following equation can be obtained ($-1 \leq r \leq 1$):

$$\tilde{\psi}_0(r_m) = 2\psi_0(r_m) = \frac{\pi \sin \alpha}{M} \sum_{p=1}^M \frac{\text{sign}(\rho_p - r_m) X_p}{\sqrt{1-\rho_p^2 \cdot \sin^2 \alpha}} \quad (m = \overline{1, M-1}); \tag{16}$$

For the dimensionless shear stresses on the edges of inclusion applying equation (20), the following equation can be derived

$$\bar{\tau}_1(r_m) = \frac{\pi \lambda_0 \sin \alpha}{2M} \sum_{p=1}^M \frac{\text{sign}(\rho_p - r_m) X_p}{\sqrt{1-\rho_p^2 \cdot \sin^2 \alpha}} \quad (r \in (-1; -c) \cup (c, 1)). \tag{17}$$

Finally, the dimensionless S.I.F $K_{III}^{(0)}$ is obtained in the following way:

$$K_{III}^{(0)} = K_{III}(b_i) / G\sqrt{\ell} = \sqrt{tg\alpha} X_0(1) = \frac{\sqrt{tg\alpha}}{M} \sum_{p=1}^M (-1)^{p+1} X_p \text{ctg} \left(\frac{2p-1}{4M} \pi \right). \tag{18}$$

Where, $X_0(1)$ the value is applied through the Lagrange

interpolation coefficients based on Chebyshev's polynomials. Therefore, solving of the system of linear equations (14), the main physical characteristics of the stated problem can be expressed by equations (15)-(18).

Numerical Calculation

For numerical calculations we consider a special case of the loading of the rectangular plate. For this case, the crack edges are free of shear forces, and shear concentrated force acting on the horizontal sides of rectangular plate:

$$T_0(x) \equiv 0, \quad T(x) = P\delta(x - \ell/2),$$

Where, $\delta(x)$ is a certain Dirac Delta function. In this case $g(r) \equiv 0$, in (11), as well as, with respect to equation (8), the following equation has been obtained

$$\bar{T}(n) = \int_0^{\ell} P\delta(x - \ell/2) \sin(\pi nx/\ell) dx = P \sin(\pi n/2) = \begin{cases} 0, & i \text{ } \delta \delta \quad n = 2q; \\ (-1)^{q+1} P \delta \quad n = 2q-1 & (q = 1, 2, \dots) \end{cases} \tag{19}$$

Taking into account the above-mentioned consideration, the function $\tilde{f}(\xi)$ can be expressed in the following way [1]:

$$\tilde{f}(\xi) = 2P_0 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin[(2n-1)\xi]}{ch[(2n-1)h_0]}; \quad P_0 = P/\ell G; \quad (0 < \xi < \pi) \tag{20}$$

And the function $f(r)$ from equation (11) can be obtained in the following way.

$$f(r) = \tilde{f}(\arccos(r \cdot \sin \alpha)) = 2P_0 \sqrt{1-r^2 \cdot \sin^2 \alpha} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{U_{2n-2}(r \cdot \sin \alpha)}{ch[(2n-1)h_0]} \quad (-1 < r < 1). \tag{21}$$

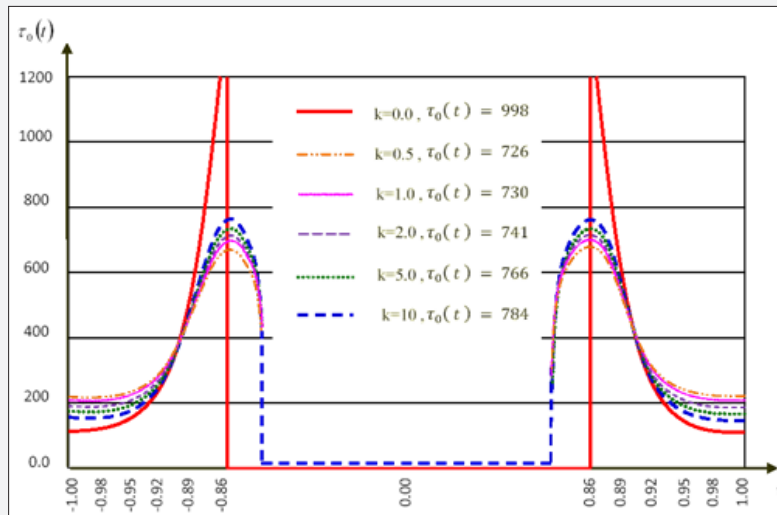


Figure 3 : Antiplane shear stresses (t) on the edges of inclusions and out-of-crack based on the various shear modulus G_0 .

It is obvious that the function $\omega_0(r)$ with respect to the symmetry of line $x = \ell/2$ in this special case, and, consequently, the function $X_0(r)$ according from equation (14) are odd functions, therefore, the components of the second integrals in equations (10) and (15) containing polynomials r or t in

arguments tend to zero, so that, the above-mentioned equations and the kernel-matrix K_{mp} of system of equation (14) are simplified and the expressions of functions $\tilde{f}(\xi)$ and $f(r)$ are from equations (20)-(21). The numerical analysis of the main characteristics of stated problem can be carried out for

the considered special case. Antiplaine shear stresses $\tau_0(t)$ on the edges of inclusions and out of crack based on the various shear modulus G_0 were calculated and shown in (Figure 3) also

decreasing dislocation of crack edges due to inclusions tip repair are shown in (Figure 4).

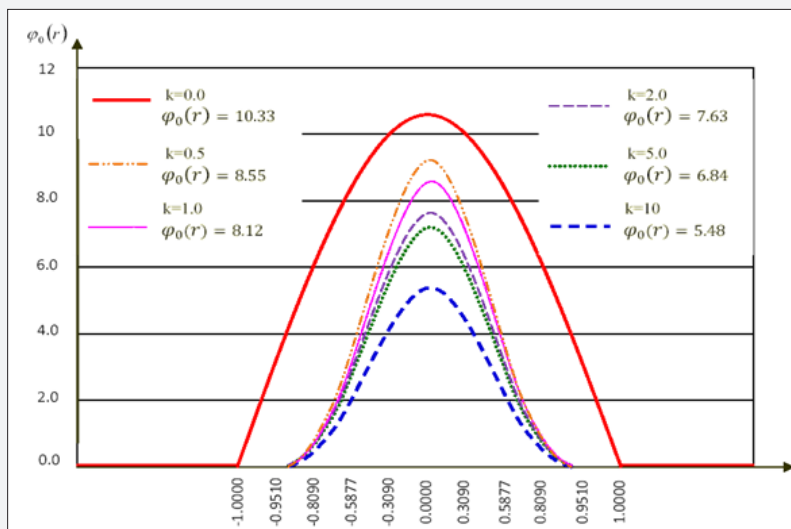


Figure 4 : Decreasing dislocation of crack edges due to inclusion repair at the tips.

Conclusion

Numerical calculations show that the repair of crack tips causes avoiding the singularities and reduces the anti-plane S.I.F K_{III} , about 50 percent, also for strengthen and stop the crack propagation near to region at the tips, it is not need to use a material with very high shear rigidity value G_0 [12]. Meanwhile the crack opening C.O.D decreases about 50 percent, and in addition the shear stresses at the crack tips fall down near to 30 percent.

The linear Algebraic system of equation (14) were solved regarding to relations (19)-(21) for the special case of anti-plane shear loading for several metals on a base metal steel for the

main plate [13]. The shear moduli of the metals over the steel shear modulus are represented as a ratio on the horizontal axis in (Figure 5).

The stress intensity factors S.I.F K_{III} that are very important and show the intensity or index of upper limit of shear stresses magnitude calculated through the equation (18) that are shown in (Figure 5), which presents the decreasing trend of S.I.F curve when the ratio of $k=G_0/G$ goes up. It also shows that the crack tip repairing by adding another material at the tips to prevent crack propagation, can reduce the S.I.F about 50 percent, means this approach is very effective to avoid the crack propagations in cracked plates, mathematically is a treatment for well-known singularities at the crack tips, defects and holes.

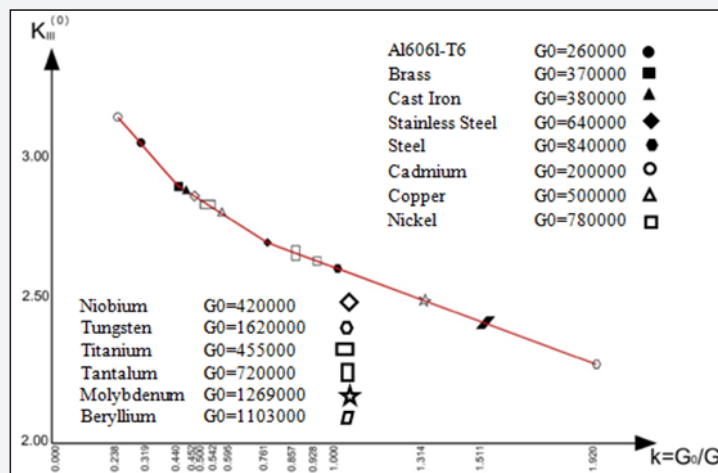


Figure 5 : Curve of decreasing S.I.F for various inclusion materials at the crack tips on the base metal steel.

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