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Constrained Bayesian Methods for Testing Directional Hypotheses Restricted False Discovery Rates

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Abstract

Constrained Bayesian method (CBM) and the concept of false discovery rates (FDR) for testing directional hypotheses is considered in the paper. Here is shown that the direct application of CBM allows us to control FDR on the desired level. Theoretically it is proved that mixed directional false discovery rates (mdFDR) are restricted on the desired levels at the suitable choice of restriction levels at different statements of CBM. The correctness of the obtained theoretical results is confirmed by computation results of concrete examples.

Subject Classifications: 62F15; 62F03.

Keywords: Directional hypotheses; Constrained Bayesian method; False discovery rate; Mixed directional false discovery rates; False acceptance rate

Abbreviations: CBM: Constrained Bayesian method; DFDR: Directional False Discovery Rate; FDR: False Discovery Rates; MDFDR: Mixed Directional False Discovery Rates; FAR: False Acceptance Rate

Introduction

The traditional formulation of testing simple basic hypothesis versus composite alternative is a well studied problem in many scientific works [1-8]. The problem of making the sense about direction of difference between parameter values, defined by basic and alternative hypotheses, is important in many applications [9-17]. Here the decision whether the parameter outstrips or falls behind of the value defined by basic hypothesis is meaningful. For parametrical models, this problem can be stated as

$$H_0:\theta=\theta_0 \text{ vs. } H_-:\theta<\theta_0 \text{ or } H_+:\theta>\theta_0 \quad (1)$$

Where θ is the parameter of the model, θ_0 is known. These alternatives are called skewed or directional alternatives. The consideration of directional hypotheses found their applications in different realms. Among them are biology, medicine, technique and so on [17,18]. The appropriate tests “has just begun to stir up some interests in the educational and behavioral literature” [19-22].

Directional false discovery rate (DFDR) or mixed directional false discovery rate (mdFDR) are used when alternatives are skewed [17]. The optimal procedures controlling DFDR (or mdFDR) use two-tailed procedures assuming that directional alternatives are symmetrically distributed. Therefore, decision rule is symmetric in relation with the parameter’s value defined by basic hypothesis [14,23]. For the experiments where the distribution of the alternative hypotheses is skewed, the asymmetric decision rule, based on skew normal priors and used Bayesian methodology for testing when minimizing mdFDR, is offered in Bansal et al. [17]. There theoretically is proved “that a skewed prior permits a higher power in number of correct discoveries than if the prior is symmetric”. This result is confirmed by simulation study comparing the proposed rule with a frequentist’s rule and the rule offered in Benjamini, et al. [23]. Because CBM allows us to foresee the skewness by not only a prior probabilities but also by restriction levels in the constraints, it is expected that it will give more powerful decision rule in number of correct discoveries than existed symmetric or asymmetric in the prior decision rules. Therefore, different statements of CBM, for testing skewed hypotheses with restricted mdFDR, are considered below.

In Section 2 some possible statements of CBM for testing directional hypotheses are considered and the fact that FDR could be controlled on the desired level for each statement of CBM is proved. Concretization of the proposed theoretical results for the normally distributed directional hypotheses is given in Section 3. Computation results of concrete example for normal basic and truncated normal alternative hypotheses by simulation of the appropriate samples are given in section 4. Discussion of the obtained results and made conclusions are presented in sections 5 and 6, respectively.

CBM for testing directional hypotheses

Different statements of CBM for testing a set of hypotheses are given in Kachiashvili, et al. [24]; Kachiashvili, [25]; Kachiashvili et al. [26]. They differ from each other by the kind of restrictions put on the Type I or Type II errors made at testing. Let's introduce the following denotations for statement of the problem of testing hypotheses [27]. Let the sample $x^T=(x_1,...,x_n)$ be generated from $p(x;\theta)$, and the problem of interest is to test $H_i:\theta_i\in\Theta_i$ $i=1,2,\ldots,S$, where $\Theta_i\subset R^m$ $i=1,2,...,S$ are disjoint subsets with $\cup\Theta_i=R^m$. The number of tested hypotheses is S. Let the prior on θ be denoted by $\sum_{i=1}^{S}\pi(\theta|H_i)p(H_i)$, where for each $i=1,2,...,S$, $p(H_i)$ is the a priori probability of hypothesis H_i and $\pi(\theta|H_i)$ is a prior density with support Θ_i; $p(x|H_i)$ denotes the marginal density of x given H_i, i.e., $p(x|H_i)=\int_{\Theta_i}p(x|\theta)\pi(\theta|H_i)d\theta$ and $D=\{d\}$ is the set of solutions, where $d=\{d_1,...,d_S\}$, it being so that

$$d_i=\begin{cases}1, & if\ \ hypothesis\ \ H_i\ \ is\ \ accepted\ ,\\ 0, & otherwise;\end{cases}$$

$\delta(x)=\{\delta_1(x)\ \delta_2(x),...,\delta_S(x)\}$ is the decision function that associates each observation vector x

with a certain decision

$$x\xrightarrow{\delta(x)}d\in D;$$

Γ_j is the region of acceptance of hypothesis H_j, i.e., $\Gamma_j=\{x:\delta_j(x)=1\}$. It is obvious that $\delta(x)$ is completely determined by the Γ_j regions, i.e. $\delta(x)=\{\Gamma_1,\Gamma_2,...,\Gamma_S\}$. Let's $L_1(H_i,\delta_j(x)=1)$ and $L_2(H_i,\delta_j(x)=0)$ be the losses of incorrectly accepted and incorrectly rejected hypotheses. Then the total loss of incorrectly accepted and incorrectly rejected hypotheses $L(H_i,\delta(x)$ is the following:

$$L(H_i,\delta(x)\ =\sum_{j=1}^{S}L_1(H_i,\delta_j(x)=1)+\sum_{j=1}^{S}L_2(H_i,\delta_j(x)=0)\quad(2)$$

Adapting the made denotations to skewed hypotheses, let's consider some kind of CBM, from all possible statements, for testing directional hypotheses (1). (Notation 1: numbering of the tasks, described below, is preserved from Kachiashvili, et al. [27] 2.1. Restrictions on the averaged probability of acceptance of true hypotheses (Task 1). In this case, CBM has the following form Kachiashvili, et al. [28]: to minimize the averaged loss of incorrectly accepted hypotheses

$$r_\delta=\min_{\{\Gamma_j\}}\left\{\sum_{i=1}^{S}p(H_i)\sum_{j=1}^{S}\int_{\Gamma_j}L_1(H_i,\delta_j(x)=1)\,p(x|H_i)\,dx\right\}\quad(3)$$

subject to the averaged loss of incorrectly rejected hypotheses

$$\sum_{i=1}^{S}p(H_i)\sum_{j=1}^{S}\int_{R^n-\Gamma_j}L_2(H_i,\delta_j(x)=0)p(x|H_i)dx\ =$$

$$=\sum_{i=1}^{S}p(H_i)\sum_{j=1}^{S}\int_{R^n}L_2(H_i,\delta_j(x)=0)p(x|H_i)dx\ -$$

$$-\sum_{i=1}^{S}p(H_i)\sum_{j=1}^{S}\int_{\Gamma_j}L_2(H_i,\delta_j(x)=0)p(x|H_i)dx\ \le r_1\quad(4)$$

Where r_1 is some real number determining the level of the averaged loss of incorrectly rejected hypotheses. For directional hypotheses (1) and loss functions

$$L_1(H_i,\delta_j(x)=1)=\begin{cases}0 & at\ \ i=j,\\ K_1 & at\ \ i\ne j;\end{cases}\ \text{and}\ \ L_2(H_i,\delta_j(x)=0)=\begin{cases}K_0 & at\ \ i=j,\\ 0 & at\ \ i\ne j;\end{cases}\quad(5)$$

using concepts of posterior probabilities, the problem (3), (4) transforms in the following form Kachiashvili et al. [28]

$$r_\delta=\min_{\{\Gamma_-,\Gamma_0,\Gamma_+\}}\{p(H_-)\cdot K_1[P(x\in\Gamma_0\,|\,H_-)+P(x\in\Gamma_+\,|\,H_-)]+$$
$$+p(H_0)\cdot K_1[P(x\in\Gamma_-\,|\,H_0)+P(x\in\Gamma_+\,|\,H_0)]+$$
$$+p(H_+)\cdot K_1[P(x\in\Gamma_-\,|\,H_+)+P(x\in\Gamma_0\,|\,H_+)]\},\quad(6)$$

subject to

$$p(H_-)P(x\in\Gamma_-\,|\,H_-)+p(H_0)P(x\in\Gamma_0\,|\,H_0)+p(H_+)P(x\in\Gamma_+\,|\,H_+)\ge 1-\frac{r_1}{K_0}.\quad(7)$$

The solution of the problem (6) and (7) by Lagrange undetermined multiplier method gives

$$\Gamma_-=\{x:K_1\cdot(p(H_0\,|\,x)+p(H_+\,|\,x))<\lambda_1\cdot K_0\cdot p(H_-\,|\,x)\},$$
$$\Gamma_0=\{x:K_1\cdot(p(H_-\,|\,x)+p(H_+\,|\,x))<\lambda_1\cdot K_0\cdot p(H_0\,|\,x)\},$$
$$\Gamma_+=\{x:K_1\cdot(p(H_-\,|\,x)+p(H_0\,|\,x))<\lambda_1\cdot K_0\cdot p(H_+\,|\,x)\},\quad(8)$$

where Lagrange multiplier λ_1 is determined so that in condition (7) equality takes place.

(Notation 2: for the statement (3), (4) as well as for other statements (see Tasks 2, 4 and 5, below), depending upon the choice of x, there is a possibility that $\delta_j(x)=1$ for more than one j or $\delta_j(x)=0$ for all $j\in(-,0,+)$.

Let's introduce denotations

$$r_\delta^-=K_1\cdot\left[P(x\in\Gamma_0\,|\,H_-)+P(x\in\Gamma_+\,|\,H_-)\right]$$
$$r_\delta^0=K_1\cdot\left[P(x\in\Gamma_-\,|\,H_0)+P(x\in\Gamma_+\,|\,H_0)\right],\quad(9)$$
$$r_\delta^+=K_1\cdot\left[P(x\in\Gamma_-\,|\,H_+)+P(x\in\Gamma_0\,|\,H_+)\right]$$

and let's call them individual average risks. Then for the average risk (6) we

have

$$r_\delta=\min_{\{H_-,H_0,H_+\}}\{p(H_-)\cdot r_\delta^-+p(H_0)\cdot r_\delta^0+p(H_+)\cdot r_\delta^+\}.\quad(10)$$

At testing directional hypotheses, it is possible to make a false statement about choice among alternative hypotheses, i.e. to make a directional error, or a type III error [23]. For recognition of directional errors in the terms of false discovery rate (FDR) two variants of false discovery rate (FDR) were introduced in Benjamini et al. [29]: pure directional false discovery rate (pdFDR) and mixed directional false discovery rate (mdFDR), which are the following

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$$pdFDR = P(x \in \Gamma_- \mid H_+) + P(x \in \Gamma_+ \mid H_-) \quad (11)$$

and

$$mdFDR = P(x \in \Gamma_- \mid H_+) + P(x \in \Gamma_- \mid H_0) + P(x \in \Gamma_+ \mid H_-) + P(x \in \Gamma_+ \mid H_0). \quad (12)$$

It is obvious that $pdFDR \le mdFDR$

It is shown that the FDR is an effective model selection criterion, as it can be translated into a penalty function. Therefore, FDR gives the opportunity to increase the power of the test in general case [30]. Both variants of FDR for directional hypotheses: pdFDR and mdFDR can be expressed by Type III error rates (ERRIII):

$$pdFDR = ERR_{III}^{K} \text{ and } mdFDR = SERR_{III}, \quad (13)$$

where

$$ERR_{III}^{T} = P(x \in \Gamma_- \mid H_0) + P(x \in \Gamma_+ \mid H_0), \quad (14)$$

$$ERR_{III}^{K} = P(x \in \Gamma_- \mid H_+) + P(x \in \Gamma_+ \mid H_-), \quad (15)$$

and

$$SERR_{III} = ERR_{III}^{T} + ERR_{III}^{K}. \quad (16)$$

Here ERR_{III}^{T} and ERR_{III}^{K} are two different forms of Type III error rates, considered by different authors Mosteller, et al. [31]; Kaiser, [9]; Jones, et al. [13] and Shaffer, [14]) and $SERR_{III}$ is the summary type III error rate $(SERR_{III})$ [25].

Here in after, if necessary, let's ascribe the number of the task related to the considered CBM directly to this abbreviation.

Theorem 1. CBM 1 with restriction level of (7), at satisfying a condition $\frac{1}{P_{\min}} \cdot \frac{r_1}{K_0} = q$ where $0 < q < 1$, $P_{\min} = \min\{p(H_-), p(H_0), p(H_+)\}$ ensures a decision rule with $mdFDR$ (i.e. with $SERR_{III}$) less or equal to q i.e. with the condition $mdFDR = SERR_{III} \le q$.

Proof. Because of the peculiarity of decision making rule of CBM, alongside of hypotheses acceptance regions there exist the regions of impossibility of making a decision [26,32]. Therefore, instead of condition

$$P(x \in \Gamma_- \mid H_i) + P(x \in \Gamma_0 \mid H_i) + P(x \in \Gamma_+ \mid H_i) = 1, i \in \Psi, \Psi \equiv \{-,0,+\},$$

of the classical decision making procedures, the following condition is fulfilled in CBM

$$P(x \in \Gamma_- \mid H_i) + P(x \in \Gamma_0 \mid H_i) + P(x \in \Gamma_+ \mid H_i) + P(imd \mid H_i) = 1, i \in \Psi, \Psi \equiv \{-,0,+\}$$

where imd is the abbreviation of the impossibility of making a decision

Taking into account (17), condition (7) can be rewritten as follows

$$p(H_-)P(x \in \Gamma_- \mid H_-) + p(H_0)P(x \in \Gamma_0 \mid H_0) + p(H_+)P(x \in \Gamma_+ \mid H_+) =$$
$$= p(H_-)[1 - P(x \in \Gamma_+ \mid H_-) - P(x \in \Gamma_0 \mid H_-) - P(imd \mid H_-)] +$$
$$+ p(H_0)[1 - P(x \in \Gamma_- \mid H_0) - P(x \in \Gamma_+ \mid H_0) - P(imd \mid H_0)] +$$
$$+ p(H_+)[1 - P(x \in \Gamma_- \mid H_+) - P(x \in \Gamma_0 \mid H_+) - P(imd \mid H_+)] \ge 1 - \frac{r_1}{K_0}.$$

From here follows that

$$p(H_-)[P(x \in \Gamma_+ \mid H_-) + P(x \in \Gamma_0 \mid H_-)] + p(H_0)[P(x \in \Gamma_- \mid H_0) + P(x \in \Gamma_+ \mid H_0)] +$$
$$+ p(H_+)[P(x \in \Gamma_- \mid H_+) + P(x \in \Gamma_0 \mid H_+)] \le \frac{r_1}{K_0}. \quad (18)$$

Let's denote $P_{\min} = \min\{p(H_-), p(H_0), p(H_+)\}$. Then from (18) we have

$$P(x \in \Gamma_+ \mid H_-) + P(x \in \Gamma_0 \mid H_-) +$$
$$+ P(x \in \Gamma_- \mid H_0) + P(x \in \Gamma_+ \mid H_0) +$$
$$+ P(x \in \Gamma_- \mid H_+) + P(x \in \Gamma_0 \mid H_+) \le \frac{1}{P_{\min}} \cdot \frac{r_1}{K_0}.$$

Taking into account (12), we write

$$mdFDR + P(x \in \Gamma_0 \mid H_-) + P(x \in \Gamma_0 \mid H_+) \le \frac{1}{P_{\min}} \cdot \frac{r_1}{K_0}. \quad (19)$$

This proves the theorem

Let's call false acceptance rate (FAR) the following

$$FAR = P(x \in \Gamma_0 \mid H_-) + P(x \in \Gamma_0 \mid H_+) \quad (20)$$

then from (19), we have

$$mdFDR + FAR \le \frac{1}{P_{\min}} \cdot \frac{r_1}{K_0}. \quad (21)$$

Restrictions on the conditional probabilities of acceptance of each true hypothesis (Task 2)

To minimize (6) subject to

$$P(x \in \Gamma_i \mid H_i) \ge 1 - \frac{r_2^i}{K_0 \cdot p(H_i)}, \; i \in \Psi, \; \Psi \equiv \{-,0,+\}. \quad (22)$$

The solution of the problem (6), (22) is the following

$$\Gamma_- = \{x : K_1 \cdot (p(H_0 \mid x) + p(H_+ \mid x)) < \lambda_2^- \cdot K_0 \cdot p(H_- \mid x)\},$$
$$\Gamma_0 = \{x : K_1 \cdot (p(H_- \mid x) + p(H_+ \mid x)) < \lambda_2^0 \cdot K_0 \cdot p(H_0 \mid x)\},$$
$$\Gamma_+ = \{x : K_1 \cdot (p(H_- \mid x) + p(H_0 \mid x)) < \lambda_2^+ \cdot K_0 \cdot p(H_+ \mid x)\}, \quad (23)$$

where Lagrange multipliers λ_2^-, λ_2^0 and λ_2^+ are determined so that in conditions (22) equalities takes place.

Theorem 2. CBM 2 with restriction level of (22), at satisfying a condition $\frac{r_2^-}{K_0 \cdot p(H_-)} + \frac{r_2^0}{K_0 \cdot p(H_0)} + \frac{r_2^+}{K_0 \cdot p(H_+)} = q$, where $0 < q < 1$, ensures a decision rule with $mdFDR$ (i.e. with $SERR_{III}$) less or equal to q, i.e. with the condition $mdFDR = SERR_{III} \le q$.

Proof. Taking into account (12), (17), condition

$$mdFDR = SERR_{III} \le q \quad (24)$$

can be rewritten as follows

$$mdFDR = \{1 - P(x \in \Gamma_0 \mid H_0) - P(imd \mid H_0)\} +$$
$$+ \{1 - P(x \in \Gamma_+ \mid H_+) - P(x \in \Gamma_0 \mid H_+) - P(imd \mid H_+)\} +$$

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$$+\{1-P(x\in\Gamma_-\,|\,H_-)-P(x\in\Gamma_0\,|\,H_-)-P(imd\,|\,H_-)\}\le q\,. \quad (25)$$

On the basis of (22), condition (25) takes the form

$$mdFDR\le\frac{r_2^0}{K_0\cdot p(H_0)}-P(imd\,|\,H_0)+\frac{r_2^+}{K_0\cdot p(H_+)}-P(x\in\Gamma_0\,|\,H_+)-P(imd\,|\,H_+)+$$

$$+\frac{r_2^-}{K_0\cdot p(H_-)}-P(x\in\Gamma_0\,|\,H_-)-P(imd\,|\,H_-)\le q\,. \quad (26)$$

It is obvious that if the following condition is fulfilled

$$\frac{r_2^-}{K_0\cdot p(H_-)}+\frac{r_2^0}{K_0\cdot p(H_0)}+\frac{r_2^+}{K_0\cdot p(H_+)}=q\,, \quad (27)$$

then condition (26), and accordingly the condition of the theorem, is fulfilled.

Using denotation (20), at fulfillment of (27) the following inequality can be written

$$mdFDR+FAR\le q-\left(P(imd\,|\,H_0)+P(imd\,|\,H_+)+P(imd\,|\,H_-)\right). \quad (28)$$

Restrictions on the averaged probability of rejection of true hypotheses (Task 4)

In the statement of Tasks 1 and 2, Type I error rates are minimized and Type II error rates are restricted. Now let's make inversion in the statement, i.e. let's introduce the statements when Type II error rates are minimized (that means the power is maximized) and Type I error rates are restricted.

Let's consider the following problem

$$G_\delta=\max_{\{\Gamma_-,\Gamma_0,\Gamma_+\}}\{K_0\cdot[p(H_-)\cdot P(x\in\Gamma_-\,|\,H_-)+p(H_0)\cdot P(x\in\Gamma_0\,|\,H_0)+p(H_+)\cdot P(x\in\Gamma_+\,|\,H_+)]\} \quad (29)$$

subject to

$$p(H_-)\cdot K_1\cdot[P(x\in\Gamma_0\,|\,H_-)+P(x\in\Gamma_+\,|\,H_-)]+$$

$$+p(H_0)\cdot K_1\cdot[P(x\in\Gamma_-\,|\,H_0)+P(x\in\Gamma_+\,|\,H_0)]+$$

$$+p(H_+)\cdot K_1\cdot[P(x\in\Gamma_-\,|\,H_+)+P(x\in\Gamma_0\,|\,H_+)]\le r_4\,. \quad (30)$$

Solution of the problem (29), (30) by the method of Lagrange undetermined multipliers gives

$$\Gamma_-=\left\{x: K_1\cdot(p(H_0\,|\,x)+p(H_+\,|\,x))<\frac{1}{\lambda_4}\cdot K_0\cdot p(H_-\,|\,x)\right\},$$

$$\Gamma_0=\left\{x: K_1\cdot(p(H_-\,|\,x)+p(H_+\,|\,x))<\frac{1}{\lambda_4}\cdot K_0\cdot p(H_0\,|\,x)\right\},$$

$$\Gamma_+=\left\{x: K_1\cdot(p(H_-\,|\,x)+p(H_0\,|\,x))<\frac{1}{\lambda_4}\cdot K_0\cdot p(H_+\,|\,x)\right\}, \quad (31)$$

where Lagrange multiplier λ_4 is determined so that in condition (31) equality takes place.

Theorem 3. CBM 4 with restriction level of (30), at satisfying condition $\frac{1}{P_{\min}}\cdot\frac{r_4}{K_1}=q$, where $0<q<1$, $P_{\min}=\min\{p(H_-)\ p(H_0)\ p(H_+)\}$, ensures a decision rule with $mdFDR$ (i.e. with $SERR_{III}$) less or equal to q, i.e. with the condition $mdFDR=SERR_{III}\le q$.

Proof. Let's rewrite (30) as follows

$$P(x\in\Gamma_0\,|\,H_-)+P(x\in\Gamma_+\,|\,H_-)+P(x\in\Gamma_-\,|\,H_0)+P(x\in\Gamma_+\,|\,H_0)+$$

$$+P(x\in\Gamma_-\,|\,H_+)+P(x\in\Gamma_0\,|\,H_+)\le\frac{1}{P_{\min}}\cdot\frac{r_4}{K_1}\,. \quad (32)$$

Taking into account (12), condition (32) can be rewritten as follows

$$mdFDR+P(x\in\Gamma_0\,|\,H_-)+P(x\in\Gamma_0\,|\,H_+)\le\frac{1}{P_{\min}}\cdot\frac{r_4}{K_1}\,. \quad (33)$$

Because of $P(x\in\Gamma_0\,|\,H_-)+P(x\in\Gamma_0\,|\,H_+)\ge 0$, the correctness of the stated theorem is obvious.

Recalling (20), condition (33) can be rewritten as follows

$$mdFDR+FAR\le\frac{1}{P_{\min}}\cdot\frac{r_4}{K_1}\,. \quad (34)$$

Restrictions on the conditional probabilities of rejection of each true hypothesis (Task 5)

In this case, it is necessary to maximize (29) subject to

$$P(x\in\Gamma_0\,|\,H_-)\le\frac{r_5^{0,-}}{K_1},\quad P(x\in\Gamma_+\,|\,H_-)\le\frac{r_5^{+,-}}{K_1},$$

$$P(x\in\Gamma_-\,|\,H_0)\le\frac{r_5^{-,0}}{K_1},\quad P(x\in\Gamma_+\,|\,H_0)\le\frac{r_5^{+,0}}{K_1}, \quad (35)$$

$$P(x\in\Gamma_0\,|\,H_+)\le\frac{r_5^{0,+}}{K_1},\quad P(x\in\Gamma_-\,|\,H_+)\le\frac{r_5^{-,+}}{K_1}.$$

Application of the Lagrange method gives

$$\Gamma_-=\left\{x: K_1\cdot\left(\lambda_5^{-,0}\cdot p(x\,|\,H_0)+\lambda_5^{-,+}\cdot p(x\,|\,H_+)\right)<K_0\cdot p(H_-)p(x\,|\,H_-)\right\},$$

$$\Gamma_0=\left\{x: K_1\cdot\left(\lambda_5^{0,-}\cdot p(x\,|\,H_-)+\lambda_5^{0,+}\cdot p(x\,|\,H_+)\right)<K_0\cdot p(H_0)p(x\,|\,H_0)\right\}, \quad (36)$$

$$\Gamma_+=\left\{x: K_1\cdot\left(\lambda_5^{+,-}\cdot p(x\,|\,H_-)+\lambda_5^{+,0}\cdot p(x\,|\,H_0)\right)<K_0\cdot p(H_+)p(x\,|\,H_+)\right\},$$

where Lagrange multiplier vectors $\lambda_5^-=(\lambda_5^{-,0},\lambda_5^{-,+})$, $\lambda_5^0=(\lambda_5^{0,-},\lambda_5^{0,+})$ and $\lambda_5^+=(\lambda_5^{+,-},\lambda_5^{+,0})$ are determined so that in (35) the equality takes place.

Theorem 4. CBM 5 with restriction level of (35), at satisfying condition $\frac{r_5^{-,+}+r_5^{-,0}+r_5^{+,-}+r_5^{+,0}}{K_1}=q$ (i.e. $r_5^{-,+}+r_5^{-,0}+r_5^{+,-}+r_5^{+,0}=q\cdot K_1$), where $0<q<1$, ensures a decision rule with $mdFDR$ (i.e. with $SERR_{III}$) less or equal to q, i.e. with the condition $mdFDR=SERR_{III}\le q$.

Proof. It is not difficult to be convinced in the correctness of the theorem, putting restrictions from (35) into expression (12) that gives

$$mdFDR\le\frac{r_5^{-,+}+r_5^{-,0}+r_5^{+,-}+r_5^{+,0}}{K_1}=q\,. \quad (37)$$

On the other hand, taking into account equations (11) and (20) and restrictions (35), for pure directional false discovery rate (pdFDR) and for false acceptance rate (FAR) we have

$$pdFDR\le\frac{r_5^{-,+}+r_5^{+,-}}{K_1}, \quad (38)$$

and

$$FAR\le\frac{r_5^{0,-}+r_5^{0,+}}{K_1}, \quad (39)$$

respectively.

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In the condition of Theorem 4, there takes place $pdFDR < mdFDR \le q$ and, if restriction levels in (35) satisfy condition $r_5^{-,+} + r_5^{-,0} + r_5^{+,-} + r_5^{+,0} + r_5^{0,-} + r_5^{0,+} = q \cdot K_1$, then the following inequality $mdFDR + FAR \le q$ is correct.

To compare conditions (21), (28), (34) and (37), we conclude that in the conditions of the stated theorems the less strict is Task 5 by restriction level of the $mdFDR$ as in this case we have $mdFDR \le q$, then the less strict are Task 1 and Task 4 as in these cases we have $mdFDR \le q - FAR$. Task 2 is the most strict as in this case we have $mdFDR \le q - FAR - \left(P(imd \mid H_-) + P(imd \mid H_0) + P(imd \mid H_+)\right)$.

The consideration of the statements of different Tasks and hypotheses acceptance regions allow us to see that the directionality can be foreseen by a priory probabilities in Tasks 1 and 4 while it can be foreseen not only by a priory probabilities but by restriction levels too in Tasks 2 and 5. Therefore consideration of Tasks 2 and 5 for testing directional hypotheses is preferable than of Tasks 1 and 4.

Unfortunately, not for every observation result x can be made decision at given restriction level of $mdFDR$, i.e. for given q there are observation results for which simple decision concerning validity of tested hypotheses can't be made. In such cases there are two possible ways: to change q or collect extra observations and decision make on the basis of arithmetic mean $\bar{x}$ of these observations. Decision making rules with necessary restriction levels in the abovementioned tasks that guaranty the fulfillment of the condition $mdFDR \le q$ can be described by the following procedure, sequential in principle by its nature [31,26].

I. Procedure A

To accept hypothesis H_i, $i \in \Psi$, where $\Psi \equiv \{-,0,+\}$ is a set of indices, if test statistics $x \in \Gamma_i$ and $x \notin \Gamma_j$, where $j \in \Psi / i$. Not to make decision, if $x \notin \Gamma_i$, $i \in \Psi$, or $(x \in \Gamma_i) \bigcap (x \in \Gamma_j)$, $i \neq j$, $i, j \in \Psi$ or $(x \in \Gamma_-) \bigcap (x \in \Gamma_0) \bigcap (x \in \Gamma_+)$. In the cases, when test statistics does not belong to any region of acceptance of hypotheses or belongs to more than one of them, for making decision with given reliability, it is necessary to increase the amount of existed information, i.e. to get one more observation result and to make decision on the basis of total information existing in this moment (Lagrange multipliers that are determined for $n = 1$ are used in hypotheses acceptance regions for making decision at increasing sample size).

The following assertion is correct for above given and, in general, for all possible statements of CBM.

Theorem 5. Let us assume that the probability distributions $p(x \mid H_i)$, $i \in \Psi$, where $\Psi \equiv \{-,0,+\}$, are such that, at increasing number of observations n in the sample, the entropy concerning distribution parameter θ, in relation to which the hypotheses are formulated, decreases*). In such case, for given set of hypotheses H_0, H_- and H_+, there always exists such a sample of size n on the basis of which decision concerning tested hypotheses can be made with given reliability when in decision making regions Lagrange multipliers are determined for $n = 1$ and the condition $mdFDR \le q$ is satisfied.

Note: *) hereinafter it is assumed that this supposition is fulfilled.

Proof. Let us denote the necessary reliability of making a decision by α $(0 < \alpha < 1)$. Under making decision with given reliability, we mean the fulfillment of the following inequality $P(x \in \Gamma_i \mid H_i) \ge \alpha$, when H_i, $i \in \Psi$, $\Psi \equiv \{-,0,+\}$, is true and x is a statistics on the basis of which decision is made. When sample size increases and we make decision on the basis of $\bar{x}_n$ (index n indicates that arithmetic mean $\bar{x}_n$ is computed on the basis of the sample with n observations), a posteriori distribution $p(H_i \mid \bar{x}_n)$, when H_i, $i \in \Psi$, is true, increases and other two probabilities $p(H_j \mid \bar{x}_n)$ and $p(H_k \mid \bar{x}_n)$, $j \neq k$, $j, k \in \Psi / i$, decrease. In decision making regions (8), (23), (31) and (36), fraction, in the denominator of which $p(H_i \mid \bar{x}_n)$ attends, when H_i is true, must be less than some constant that does not depend on n. Therefore, the bigger is $p(H_i \mid \bar{x}_n)$, the bigger is hypothesis H_i acceptance region and, accordingly, the bigger is the probability $P(\bar{x}_n \in \Gamma_i \mid H_i)$ that proves the first statement of the theorem. For the reason, mentioned above, in the considered case, probabilities $P(\bar{x}_n \in \Gamma_j \mid H_i)$, $j \neq i$, $i, j \in \Psi$, $\Psi \equiv \{-,0,+\}$, are decreased, consequently $mdFDR$ does not increase (see (12)) and inequality $mdFDR \le q$ is fulfilled.

Consideration of the normally distributed directional hypotheses

With the purpose to check the rightness of theoretical results given above, let us consider the following example. Let us suppose that an observation result x is distributed by the normal distribution $N(\theta, \sigma^2)$ with known σ^2 at H_0, and it is distributed by the truncated $N(0, \omega_0^{-1}\sigma^2)$ (ω_0 known) densities over $(-\infty, 0)$ and $(0, \infty)$, at H_- and at H_+, respectively.

The arithmetic mean is sufficient statistics for this example. For determination of hypotheses acceptance regions (8), (23) and (31), the following ratios must be determined:

$$\frac{p(H_0 \mid \bar{x}) + p(H_+ \mid \bar{x})}{p(H_- \mid \bar{x})}, \quad \frac{p(H_- \mid \bar{x}) + p(H_+ \mid \bar{x})}{p(H_0 \mid \bar{x})} \text{ and } \frac{p(H_- \mid \bar{x}) + p(H_0 \mid \bar{x})}{p(H_+ \mid \bar{x})}.$$

(Notation 3. Hypotheses acceptance regions (36) a little differ from these ones. Therefore, the computational formulae for Task 5 are slightly different from the formulae given below. Because this is not related with any principal difficulties, we will not concentrate our attention on it.)

Taking into account the conditions of the stated problem, after routine computation, we have

$$\frac{p(H_0 \mid \bar{x}) + p(H_+ \mid \bar{x})}{p(H_- \mid \bar{x})} = \frac{p(H_0) \cdot \sqrt{n + \omega_0} \cdot \exp\{-u^2/2\} + p(H_+) \cdot 2 \cdot \sqrt{\omega_0} \cdot \Phi(u)}{p(H_-) \cdot 2 \cdot \sqrt{\omega_0} \cdot (1 - \Phi(u))}, \quad (40)$$

$$\frac{p(H_- \mid \bar{x}) + p(H_+ \mid \bar{x})}{p(H_0 \mid \bar{x})} = \frac{p(H_-) \cdot 2 \cdot \sqrt{\omega_0} \cdot (1 - \Phi(u)) + p(H_+) \cdot 2 \cdot \sqrt{\omega_0} \cdot \Phi(u)}{p(H_0) \cdot \sqrt{n + \omega_0} \cdot \exp\{-u^2/2\}}, \quad (41)$$

$$\frac{p(H_- \mid \bar{x}) + p(H_0 \mid \bar{x})}{p(H_+ \mid \bar{x})} = \frac{p(H_-) \cdot 2 \cdot \sqrt{\omega_0} \cdot (1 - \Phi(u)) + p(H_0) \cdot \sqrt{n + \omega_0} \cdot \exp\{-u^2/2\}}{p(H_+) \cdot 2 \cdot \sqrt{\omega_0} \cdot \Phi(u)}, \quad (42)$$

where $\Phi(\cdot)$ is the standard normal distribution function and $u = n\bar{x} / \left(\sigma\sqrt{n + \omega_0}\right)$.

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Application of these ratios to hypotheses acceptance regions of the considered Tasks 1, 2 and 4, i.e. to formulae (8), (23) and (31), respectively, gives

$$\Gamma_- = \left\{ x : \frac{p(H_0)\cdot\sqrt{n+\omega_0}\cdot\exp\{-u^2/2\} + p(H_+)\cdot 2\cdot\sqrt{\omega_0}\cdot\Phi(u)}{p(H_-)\cdot 2\cdot\sqrt{\omega_0}\cdot(1-\Phi(u))} < \lambda'\cdot\frac{K_0}{K_1} \right\},$$

$$\Gamma_0 = \left\{ x : \frac{p(H_-)\cdot 2\cdot\sqrt{\omega_0}\cdot(1-\Phi(u)) + p(H_+)\cdot 2\cdot\sqrt{\omega_0}\cdot\Phi(u)}{p(H_0)\cdot\sqrt{n+\omega_0}\cdot\exp\{-u^2/2\}} < \lambda''\cdot\frac{K_0}{K_1} \right\},$$

$$\Gamma_+ = \left\{ x : \frac{p(H_-)\cdot 2\cdot\sqrt{\omega_0}\cdot(1-\Phi(u)) + p(H_0)\cdot\sqrt{n+\omega_0}\cdot\exp\{-u^2/2\}}{p(H_+)\cdot 2\cdot\sqrt{\omega_0}\cdot\Phi(u)} < \lambda'''\cdot\frac{K_0}{K_1} \right\}, \quad (43)$$

where $\lambda' = \lambda'' = \lambda''' = \lambda_1$ for Task 1, $\lambda' = \lambda_2^-$, $\lambda'' = \lambda_2^0$ and $\lambda''' = \lambda_2^+$ for Task 2 and $\lambda' = \lambda'' = \lambda''' = 1/\lambda_4$ for Task 4.

The Lagrange multipliers are determined so that, in the appropriate conditions (7), (22) and (30), the equalities were provided. For the solution of these equations, the computation of the following integrals is necessary

$\int_{\Gamma_j} p(\bar{x}\,|\,H_i)d\bar{x}$, $i,j\in\Psi$, $\Psi\equiv\{-,0,+\}$, (44)

that can be easily made by the Monte-Carlo method described in Kachiashvili et al. [27]. In particular, it is necessary to generate the random variables $\bar{x}$ with distribution law $p(\bar{x}\,|\,H_i)$ N times and to check the condition $\bar{x}\in\Gamma_j$. Let the condition $\bar{x}\in\Gamma_j$ be fulfilled $N_1 \le N$ times.

Then

$$\int_{\Gamma_j} p(\bar{x}\,|\,H_i)d\bar{x} \approx N_1/N$$

Computation results

Let's compute a concrete example with the initial data from Bansal & Sheng [15] and Kachiashvili et al. [27] a priori probabilities $p=\{p_-,p_0,p_+\}=\{0.3975,0.3975,0.205\}$; the values of the loss functions $K_0=K_1=1$; coefficient $\omega_0=1$; variance $\sigma^2=1$; the levels of $mdFDR$ in all the considered cases $q=0.05$.

For these data, for satisfaction of the conditions of Theorems 1, 2, 3 and 4, probabilities in restrictions (7), (22), (30) and (35), must be taken: $r_1=0.01025$; $r_2^-=r_2^0=r_2^+=0.0050456$; $r_4=0.01025$ and $r_5^{-,+}=r_5^{-,0}=r_5^{+,-}=r_5^{+,0}=0.0125$. Otherwise, we can take different restriction levels for different restriction conditions of the considered Tasks, for example, proportionally to the a priori probabilities.

Computation results for only Task 1 for saving room and simplifying the reading of the work are given below. Computation results for Task 1 when decisions are made on the basis of $\bar{x}_n$, computed by n observation results and the appropriate λ_n, obtained by solving equation (7) for $p(\bar{x}_n\,|\,H_i)$, $i\in\Psi$, $\Psi\equiv\{-,0,+\}$, are given in Table 1. Computation results for Task 1 when decisions are made on the basis of $\bar{x}_n$, computed by n observation results and λ, obtained by solving equation (7) for $n=1$, are given in Table 2. We used 3,000 simulated results for computation probabilities integrals (44) at determination Lagrange multipliers and we used simulated samples with 10,000 observations for making decisions at different values of n.

The correctness of the theoretical results of Item 2.1., in particular, of Theorem 1, is obvious from these results. For changing the informational distance between tested hypotheses, we were changing the value of n (Table 1). It is evident that to the greater n corresponds the great informational distance between hypotheses. For easy perception, the results of Table 1 are presented graphically on Figures 1-4. From here, it is seen that when information distance among hypotheses increase, the probability of acceptance of hypotheses increase, the probability of impossibility of making decision and mixed directional false discovery rate decrease that are logically correct.

Discussion

The results of Table 1 allow us to make the following reasoning.

Table 1: Computation results for Task 1 obtained on the basis of $\bar{x}_n$ for the appropriate Lagrange multipliers λ_n.

Sample size	Lagrange multiplier	Probabilities of acceptance of Hypotheses			Probabilities of impossibility of making a decision			Pure directional false discovery rate	Mixed directional false discovery rate	Left side of equation (18)	Left side of equation (19)
n	λ	at H_0	at H_-	at H_+	at H_0	at H_-	at H_+	$pdFDR$	$mdFDR$ <0.05	It must be <0.01025	It must be <0.05
1	4.681218057687266	0/0.0128/0.0038	0/0.12/0	0/0/0.0535	0.9834	0.88	0.9465	0	0.0166	0.0065985	0.0166
2	5.297517776489258	0/0.0156/0.005	0/0.2122/0	0/0/0.1418	0.9794	0.7878	0.8582	0	0.0206	0.0081885	0.0206
3	5.817072043658022	0/0.0138/0.0047	0/0.2748/0	0/0/0.2065	0.9815	0.7252	0.7935	0	0.0185	0.00735375	0.0185
4	6.27502441406250	0/0.0137/0.0052	0/0.3195/0	0/0/0.2534	0.9811	0.6805	0.7466	0	0.0189	0.00751275	0.0189
5	6.68864345672742	0/0.0111/0.0053	0/0.3621/0	0/0/0.2865	0.9836	0.6379	0.7135	0	0.0164	0.006519	0.0164
6	7.069199352497151	0/0.0111/0.0044	0/0.3847/0	0/0/0.3187	0.9845	0.6153	0.6813	0	0.0155	0.00616125	0.0155

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7	7.423412142832838	0/0.0103/0.0037	0/0.4114/0	0/0/0.3599	0.986	0.5886	0.6401	0	0.014	0.005565	0.014
8	7.8125	0/0.0085/0.0045	0/0.4371/0	0/0/0.3684	0.987	0.5629	0.6316	0	0.013	0.0051675	0.013
9	8.070774078369141	0/0.0086/0.0037	0/0.4451/0	0/0/0.3948	0.9877	0.5549	0.6052	0	0.0123	0.00488925	0.0123
10	8.370046615600586	0/0.0076/0.0031	0/0.4678/0	0/0/0.4196	0.9893	0.5322	0.5804	0	0.0107	0.00425325	0.0107
11	8.65600024076107	0/0.0071/0.004	0/0.4867/0	0/0/0.4292	0.9889	0.5133	0.5708	0	0.0111	0.00441225	0.0111
12	8.930274526515243	0/0.0065/0.0032	0/0.492/0	0/0/0.446	0.9903	0.508	0.554	0	0.0097	0.00385575	0.0097
13	9.194187541976135	0/0.0059/0.0024	0/0.506/0	0/0/0.4647	0.9917	0.494	0.5353	0	0.0083	0.00329925	0.0083
14	9.448831728659570	0/0.0049/0.0027	0/0.5181/0	0/0/0.4773	0.9924	0.4819	0.5227	0	0.0076	0.003021	0.0076
15	9.695120066094205	0/0.0052/0.0032	0/0.5266/0	0/0/0.4798	0.9916	0.4734	0.5202	0	0.0084	0.003339	0.0084
16	9.933824826543038	0/0.0047/0.0021	0/0.5308/0	0/0/0.4962	0.9932	0.4692	0.5038	0	0.0068	0.002703	0.0068
17	10.165596008300781	0/0.0055/0.0031	0/0.5429/0	0/0/0.5032	0.9914	0.4571	0.4968	0	0.0086	0.0034185	0.0086
18	10.391025543212891	0/0.0044/0.0025	0/0.5459/0	0/0/0.5165	0.9931	0.4541	0.4835	0	0.0069	0.00274275	0.0069
19	10.610603003517735	0/0.004/0.0029	0/0.5578/0	0/0/0.5204	0.9931	0.4422	0.4796	0	0.0069	0.0027427	0.0069
20	10.82275390625	0/0.0042/0.0028	0/0.5743/0	0/0/0.5268	0.993	0.4257	0.4732	0	0.007	0.0027825	0.007
100	12.5	0.1361/0.0014/ 0.0007	0/0.7773/0	0.0155/0/ 0.7528	0.8618	0.2227	0.2317	0	0.0021	0.00401225	0.0176

Remark: the results corresponding to hypotheses H_0, H_- and H_+ are given in the third, fourth and fifth columns divided by oblique lines.

I. At validity of H_0, for the reason of informational closeness of hypotheses, hypothesis H_0 practically never is accepted on the basis of an observation, except of one case when n=100. In this case, informational distance between tested hypotheses allows us to distinguish the simple H_0 from complex H_- and H_+ with low reliability (Figure 1). In other cases incorrect decisions (H_- or H_+ hypotheses) are made.

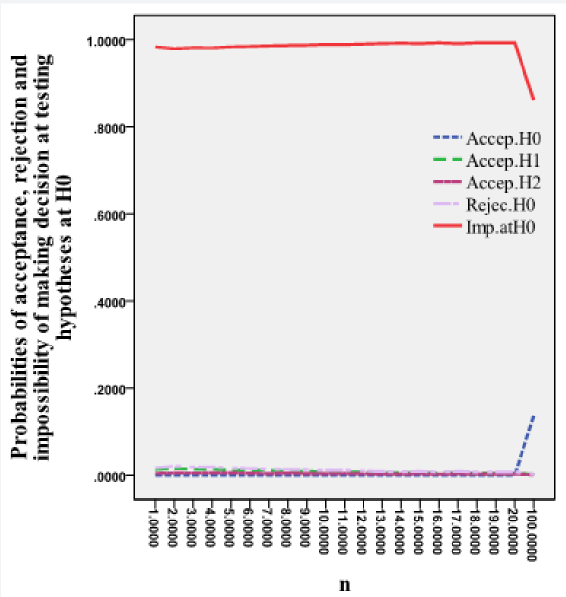

Figure 1: Probabilities of acceptance, rejection and impossibility of making decision at validity of H_0 obtained on the basis of $\bar{x}_n$ for the appropriate Lagrange multipliers λ_n .

II. At validity of H_- or H_+, only true hypothesis is accepted when decision is made. Otherwise decision is not made. Exclusion is the case when hypothesis H_+ is true and $n=100$. In this case, very seldom (with probability equal to 0.0155) but still hypothesis H_0 is accepted.

III. The increase of the informational distance between tested hypotheses (that is determined by increasing of n in normal and truncated normal distributions) entails the increase of probabilities of acceptance of hypotheses H_0, H_- and H_+ when they are true and the decrease of the probabilities of impossibilities of making a decision, accordingly (Figures 1-3).

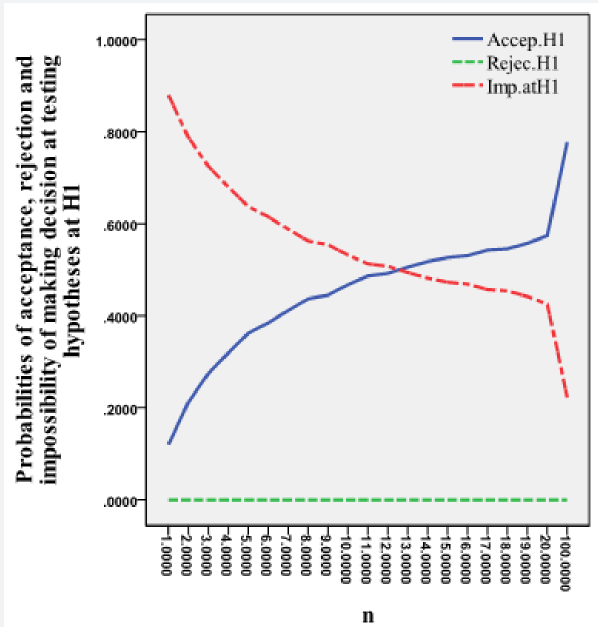

Figure 2: Probabilities of acceptance, rejection and impossibility of making decision at validity of H_- obtained on the basis of $\bar{x}_n$ for the appropriate Lagrange multipliers λ_n .

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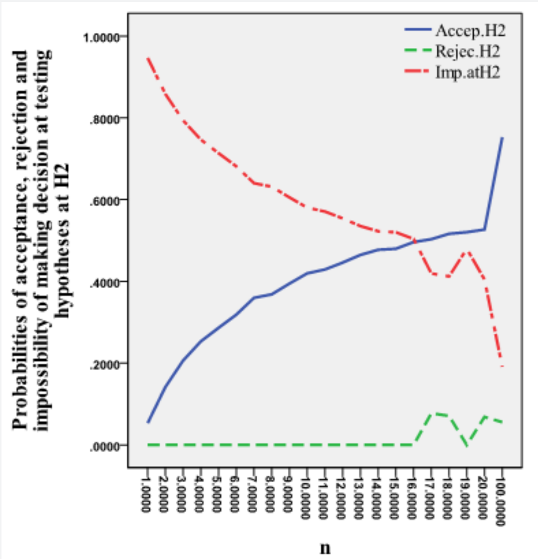

Figure 3: Probabilities of acceptance, rejection and impossibility of making decision at validity of H_+ obtained on the basis of $\bar{x}_n$ for the appropriate Lagrange multipliers λ_n .

IV. The increase of the informational distance between tested hypotheses entails the decrease of mixed directional false discovery rate (mdFDR) (Figure 4).

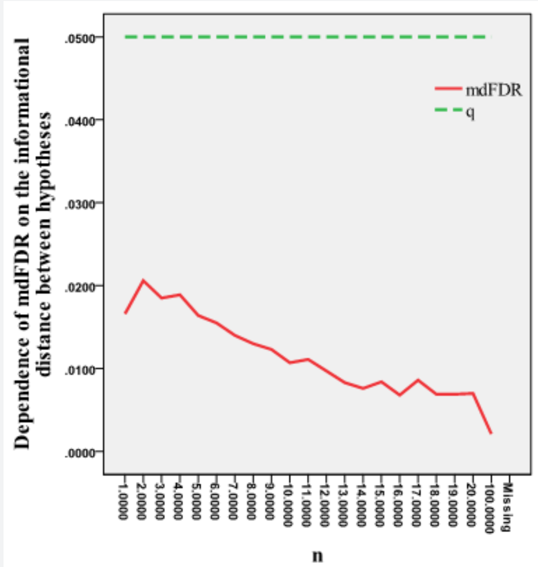

Figure 4: Dependence of Mixed directional false discovery rate (mdFDR) on the informational distance between hypotheses (q restriction level) obtained on the basis of $\bar{x}_n$ for the appropriate Lagrange multipliers λ_n .

The results of Table 2 allow us to make the following reasoning.

Table 2: Computation results for Task 1 obtained on the basis of $\bar{x}_n$ for Lagrange multiplier λ computed for $n=1$.

Sample size	Lagrange multiplier	Probabilities of acceptance of Hypotheses			Probabilities of impossibility of making a decision			Pure directional false discovery rate	Mixed directional false discovery rate	Left side of equation (18)	Left side of equation (19)
n	λ	at H_0	at H_-	at H_+	at H_0	at H_-	at H_+	$pdFDR$	$mdFDR$ <0.05	It must be <0.01025	It must be <0.05
1	4.681218057687266	0/0.0128/0.0038	0/0.12/0	0/0/0.0535	0.9834	0.88	0.9465	0	0.0166	0.0065985	0.0166
2		0/0.0198/0.0065	0/0.2228/0	0/0/0.1474	0.9737	0.7772	0.8526	0	0.0263	0.01045425	0.0263
3		0/0.0208/0.0082	0/0.3108/0	0/0/0.2311	0.971	0.6892	0.7689	0	0.029	0.0115275	0.029
4		0/0.0202/0.0079	0/0.352/0	0/0/0.2885	0.9719	0.648	0.7115	0	0.0281	0.01116975	0.0281
5		0/0.0189/0.0078	0/0.3932/0	0/0/0.3278	0.9733	0.6068	0.6722	0	0.0267	0.01061325	0.0267
6		0/0.0175/0.0077	0/0.4203/0	0/0/0.3544	0.9748	0.5797	0.6456	0	0.0252	0.010017	0.0252
7		0.0349/0/0	0/0.9975/0	0/0/0.9579	0.9651	0.0025	0.0421	0	0	0	0
8		0.0873/0/0	0/0.9988/0	0/0/0.9857	0.9127	0.0012	0.0143	0	0	0	0
9		0.1442/0/0	0/0.9999/0	0/0/0.9949	0.8558	0.0001	0.0051	0	0	0	0
10		0.1938/0/0	0/1/0	0/0/0.999	0.8062	0	0.001	0	0	0	0
15		0.4388/0/0	0/1/0	0/0/1	0.5612	0	0	0	0	0	0
20		0.6671/0/0	0/1/0	0/0/1	0.3329	0	0	0	0	0	0

Remark: The results corresponding to hypotheses H_0, H_- and H_+ are given in the third, fourth and fifth columns divided by oblique lines.

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a) Hypothesis H_0 is informational so closed to other two hypotheses that it can be recognized on the basis of arithmetic mean computed by seven or more observation results.

b) For reliable recognition of tested hypotheses the number of averaged observations in parallel experiments must be no less than 7 at validity of hypotheses H_- and H_+ and, no less that 15, at validity of hypothesis H_0.

c) The correctness of Theorem 5 is evident.

From the results of Table 1 and Table 2 the following is obvious.

i. The correctness of (17) is evident.

ii. The correctness of Theorem 1 and, accordingly, of the conditions (18) and (19) are evident from the results given in the three last right columns.

Remark: insignificant violation of the condition (18) (see Table 2) for some n is related to the accuracy of computation of small values of probability.

iii. To the big informational distance between tested hypotheses or to the big information concerning tested hypotheses corresponds the big opportunity to make a simple decision in CBM.

iv. probability of acceptance of H_i at H_i + probability of rejection of H_i at H_i + probability of impossibility of making decision at H_i = 1, $i \in \Psi$, $\Psi \equiv \{-,0,+\}$.

v. probability of acceptance of H_- at H_i + probability of acceptance of H_0 at H_i + probability of acceptance of H_+ at H_i + probability of impossibility of making decision at H_i = 1, $i \in \Psi$, $\Psi \equiv \{-,0,+\}$.

Conclusion

CBM for testing directional hypotheses is considered. Theoretically is proved that at the suitable choice of restriction levels, we can control mixed directional false discovery rates on the desired levels in different statements of CBM. This fact is demonstrated by computation of concrete examples for one of possible statement of CBM, in particular, for Task 1.

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