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Exact Degree Distributions of a Simple Preferential Attachment Model

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Abstract

We study the exact degree distribution for a simple preferential attachment model. Suppose that there is only one initial node and each new node is allowed to connect to one existing node, the exact degree distributions are obtained using the finite Markov chain imbedding technique. Numerical results are given to demonstrate our result.

Keywords: Preferential attachment models; Degree distributions; Finite Markov chain imbedding

Introduction

Preferential attachment models have been successfully used to model human networks. Examples of this line of research include some empirical studies such as biological network and human behavior network. Many graph models including preferential attachment models are shown to have power laws. Approximations and bounds have also been developed for degree distributions [1-5]. In this paper, we will show how to compute the exact degree distribution for a simple preferential attachment model based on the finite Markov chain imbedding (FMCI) technique [3].

We consider the simple model where there is only $l=1$ initial node (label 0) and each new node (child) is only allowed to connect to $m=1$ existing node (parent) in the network. The k-th node joining the network is given a label k. For each node, the in-degree is the number of children and the outdegree is always 1 for new coming node. The only exception is the initial node whose out-degree is 0. The degree of a node is defined to be the sum of indegree and out-degree. The probability of a new node connecting to any node is proportional to its degree.

Let $D_k(n)$ be the degree of label k at time n. It is not difficult to see that $D_k(n)$ forms a discrete Markov chain. We can define a Markov chain $\{Y_t\}_{t=0}^n$ on state space $\Omega_t=\{0,1,2,\dots,t-k+1\}$ with initial probability $P(Y_0=0)=1$. As the model grows over time with new nodes, the probability of a new node connecting to label i is also changing over time. Thus, $\{Y_t\}_{t=0}^n$ is a non-homogeneous Markov chain. For a node labeled $k\ge 1$, the maximal degree at time t is $t-k+1$. At time $t=1$,

$$P(Y_t=v|Y_0=0)=\begin{cases}1 & \text{if } v=1,k=0,1,\\ 1 & \text{if } v=0,k\ge 2,\\ 0 & \text{otherwise.}\end{cases} \quad (1)$$

Also, $P(Y_t=0|Y_{t-1}=0)=1$ for $2\le t\le k-1$. The transition probabilities of the imbedded Markov chain $\{Y_t\}$ are given as follows: at time $t\ge k$,

$$P(D_k(t)=i|D_k(t-1)=i)=\frac{i}{2(t-1)}.$$

$$P(D_k(t)=i+1|D_k(t-1)=i)=1-\frac{i}{2(t-1)}.$$

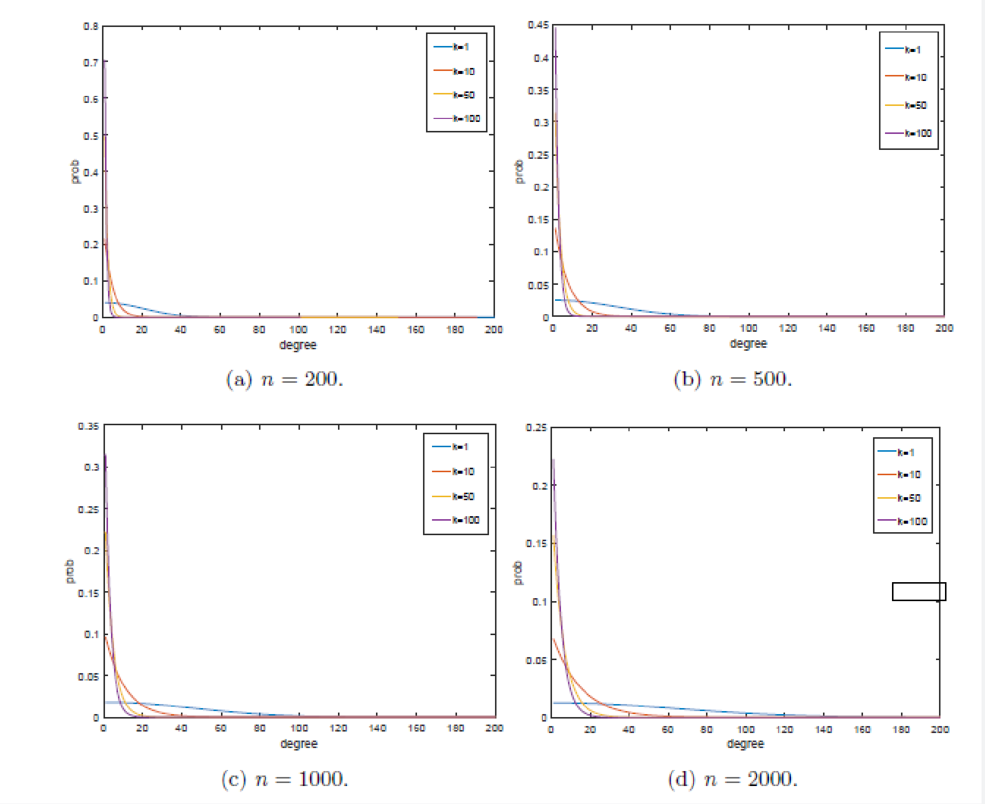
The transition matrices of $\{Y_t\}_{t=0}^n$ are of the following form for $t\ge k$,

$$M_t=\begin{array}{c|cccccccc} & 0 & 1 & \cdots & i & \cdots & t-k & t-k+1 \\ \hline 0 & 0 & 1 & & & & & \\ 1 & & 1-\frac{1}{2(t-1)} & \frac{1}{2(t-1)} & & & 0 & \\ \vdots & & & \ddots & & & & \\ i & & & & 1-\frac{i}{2(t-1)} & \frac{i}{2(t-1)} & & \\ \vdots & & & & & \ddots & & \\ t-k & & 0 & & & & 1-\frac{t-k}{2(t-1)} & \frac{t-k}{2(t-1)} \end{array}.$$

Hence, the degree distribution of label k is given by

$$P(D_k(n)=i)=\xi_0\prod_{t=k}^{n}M_t e_i, \quad (2)$$

Where $\xi_0=1$ and $e_i'=(0,\dots,0,1,0,\dots,0)$ with one corresponding to state i. The exact degree distributions of labels $k=1,10,50,100$ for $n=200,500,1000,2000$ are given in Figure 1.



(a) $n = 200$. (b) $n = 500$. (c) $n = 1000$. (d) $n = 2000$.

Figure 1: The exact degree distributions with different n.

Conclusion

Preferential attachment models have recently gained popularity in modeling human networks. In this short paper, we illustrate how to obtain the exact degree distributions based on the FMCI technique for a simple model and hope that this will attract more interest in this line of work.

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