Biostatistics and Biometrics
Open Access Journal
ISSN: 2573-2633

Review Article
Volume 8 Issue 4 - November 2018
DOI: 10.19080/BBOAJ.2018.08.555742

Biostat Biometrics Open Acc J

The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data

Jehan A. Almamy[1], Saeed Al-mualim[1,2], M. S. Hamed[1,3] and Haitham M. Yousof[3]

[1]*Management Information System Department, Taibah University, Saudi Arabia*

[2]*Department of Statistics, Sana'a University, Yemen*

[3]*Department of Statistics, Mathematics and Insurance, Benha University, Egypt*

Submission: September 27, 2018; **Published:** November 14, 2018

***Corresponding author:** Haitham M. Yousof, Department of Statistics, Mathematics and Insurance, Benha University, Egypt

Abstract

In this work, we derive a new inverse exponential continuous model via the zero truncated Poisson distribution with a strong physical interpretation. A round of mathematical properties are derived. The maximum likelihood method is considered for estimating the unknown parameter of our model. The new model is much better other important competitive models.

Keywords: Zero Truncated Poisson; Inverse Exponential; Maximum Likelihood; Modeling Real Data

Abbreviations: IE: Inverse Exponential; PDF: Probability Density Function; CDF: Cumulative Distribution Function; BXII: Burr XII; LxIE: Lomax IE; LLIE: Log-logistic IE; ZTP: Zero Truncated Poisson; PMF: Probability Mass Function; HRF: Hazard Rate Function; AIC: Akaike Information Criterion; HQIC: Hannan-Quinn Information Criterion; CAIC: Consistent Akaike Information Criterion; BIC: Bayesian Information Criterion; QF: Quantile Function

Introduction and physical motivation

The Inverse Exponential (IE) distribution was originally introduced by Keller & Kamath [1] and it has been derived and considered as a lifetime model. If a random variable (r.v.) X has an E distribution, the variable $Y = X^{-1}$ will have an IE distribution. The probability density function (PDF) and cumulative distribution function (CDF) of the inverse exponential (IE) distribution are given by (for $x \geq 0$)

$$g_{IE}^{(a)}(x) = ax^{-2}\exp\left[-\left(ax^{-1}\right)\right] \quad G_{IE}^{(a)}(x) = \exp\left[-\left(ax^{-1}\right)\right],$$

respectively, where $a > 0$ is the scale parameter. In this paper we propose and study a new extension of the IE distribution using the zero truncated Poisson (ZTP) distribution. Suppose that a system has N subsystems which functioning independently at a given time where N has ZTP distribution with parameter α. It is the conditional probability distribution of a Poisson-distributed r.v., given that the value of the r.v. is not zero $(\neq 0)$. The probability mass function (PMF) of N is given by

$$PMF_{ZTP}^{(\alpha)}(N=n) = \left[\exp(-\alpha)\alpha^{n}\right] \Big/ \left\{n!\left[-\exp(-\alpha)+1\right]\right\}\Big|_{(n=1,2,\ldots)}. \quad (1)$$

Note that for ZTP r.v., the expected value $E(N|\alpha)$ and variance $Var(N|\alpha)$ are, respectively, given by

$$E(N|\alpha) = \alpha / \left[-\exp(-\alpha)+1\right],$$

and

$$Var(N|\alpha) = \frac{\alpha^2}{\left[-\exp(-\alpha)+1\right]^2} + \frac{\alpha+\alpha^2}{\left[-\exp(-\alpha)+1\right]}.$$

Suppose that the failure time of each subsystem has the Burr X IE "$\mathrm{BXIE}(\theta,a)$" for short) defined

by the CDF and PDF given by

$$H_{\mathrm{BXIE}}^{(\theta,a)}(x) = \left[1-\exp\left(-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)\right]^{\theta}. \quad (2)$$

$$H_{\mathrm{BXIE}}^{(\theta,a)}(x) = 2\theta_{ax^{-2}} \frac{\exp\left[-2\left(ax^{-1}\right)\right]}{\left\{1-\exp\left[-\left(ax^{-1}\right)\right]\right\}^{3}}$$

$$\times\exp\left(-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)$$

$$\times\left[1-\exp\left(-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)\right]^{\theta-1}. \quad (3)$$

respectively, where $a > 0$ is a scale parameter and $\theta > 0$ is the shape parameters. Let Y_i denote the failure time of the ith subsystem and let $X = \min\{Y_1, Y_2, \ldots, Y_N\}$. Then the conditional CDF of X given N is

$$F\left(x|N\right)=1-\Pr\left(X>x|N\right)=1-\left[1-H_{BXIE}^{(\theta,\alpha)}\left(x\right)\right]^{N}. \quad (4)$$

Therefore, the unconditional CDF of the PBXIE density function, as described in Ramos et al. (2015), can be expressed as

$$F_{PBXIE}^{(x,\theta,a)}\left(x\right)=\frac{1-\exp\left\{-\alpha\left[1-\exp\left(-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)\right]^{\theta}\right\}}{1-\exp\left(-\alpha\right)}, \quad (5)$$

with the corresponding PDF as

$$F_{PBrXIE}^{(x,\theta,a)}\left(x\right)=\frac{2\theta\alpha a}{1-\exp\left(-\alpha\right)}x^{-2}\left\{1-\exp\left[-\left(ax^{-1}\right)\right]\right\}^{-3}$$

$$\times\exp\left(-2\left(ax^{-1}\right)-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)$$

$$-\left[1-\exp\left(-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)\right]^{\theta-1}$$

$$\times\exp\left\{-\alpha\left[1-\exp\left(-\left\{\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right\}^{2}\right)\right]^{\theta}\right\}. \quad (6)$$

The hazard rate function (HRF) can be easily obtained via $F_{PBrXIE}^{(x,\theta,a)}(x)/\left[1-F_{PBrXIE}^{(x,\theta,a)}(x)\right]$. Figure 1 shows that the PDF of the PBXIE model exhibits various shapes like right skewed symmetric and from Figure 2 we see that the HRF of the PBXIE model exhibits increasing, unimodal then bathtub and bathtub and unimodal then increasing hazard rates.

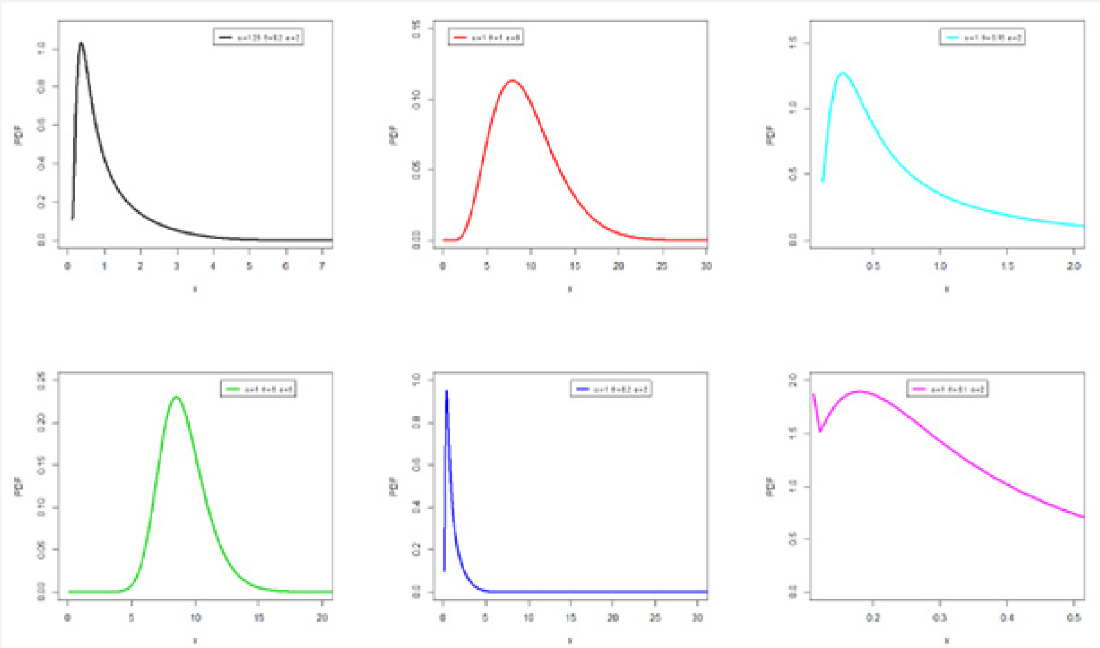

Figure 1: Plots of the PBXIE density.

This paper is organized as follows

In Section 2, we derived some properties of the PBXIE model. Maximum likelihood (ML) method for the PBXIE model parameters under the uncensored case is addressed in Section 3. In Sections 4 potentiality of the PBXIE model is illustrated by means of two real data sets. Finally, Section 5 provides some concluding remarks.

Mathematical Properties

Useful expansions

Using the power series

$$\exp\left(\phi\right)=\sum_{m=0}^{\infty}\frac{\phi^{m}}{m!},$$

the PDF in (6) can be written as

$$F_{PBXIE}^{(\alpha,\theta,a)}\left(x\right)=\sum_{h=0}^{\infty}\frac{2\theta\alpha^{1+h}a\left(-1\right)^{h}}{h!\left[-\exp\left(-\alpha\right)+1\right]}$$

$$\times x^{-2}\frac{\exp\left[-2\left(ax^{-1}\right)\right]}{\left[1-\exp\left[-\left(ax^{-1}\right)\right]\right]^{3}}$$

$$\times\exp\left[-\left(\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right)^{2}\right]$$

$$\times\left\{1-\exp\left[-\left(\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right)^{2}\right]\right\}^{\theta(h+1)-1}. \quad (7)$$

If $|s|<1$ and $c>0$ is a real non-integer, the following power series holds

$$\left(1-s\right)^{c-1}=\sum_{i=0}^{\infty}\frac{\left(-1\right)^{i}\Gamma\left(c\right)}{i!\Gamma\left(c-i\right)}s^{i}. \quad (8)$$

Applying (8) to (7) we have

$$F_{PBXIE}^{(\alpha,\theta,a)}\left(x\right)=\sum_{h,i=0}^{\infty}\frac{2\theta\alpha^{1+h}ax^{-2}\exp\left[-2\left(ax^{-1}\right)\right]}{\left[-\exp\left(-\alpha\right)+1\right]}\left[1-\exp\left[ax^{-1}\right]\right]^{-3}$$

$$\times\frac{\left(-1\right)^{h+1}\Gamma\left(\theta\left(h+1\right)\right)}{i!\Gamma\left(\theta\left(h+1\right)-i\right)}\exp\left[-\left(1+i\right)\left(\frac{\exp\left[-\left(ax^{-1}\right)\right]}{1-\exp\left[-\left(ax^{-1}\right)\right]}\right)^{2}\right]. \quad (9)$$

How to cite this article: Jehan A. A, Saeed Al-mualim, M S Hamed, Haitham M Y. The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data. Biostat Biometrics Open Acc J. 2018; 8(4): 555742. DOI: 10.19080/BBOAJ.2018.08.555742.

Applying the power series to the term $\exp\left[-(1+i)\left(\frac{\exp\left[-(ax^{-1})\right]}{1-\exp\left[-(ax^{-1})\right]}\right)^2\right]$, Equation (9) becomes

$$F_{PBXIE}^{(\alpha,\theta,a)}(x)=\sum_{h,i,j=0}^{\infty}\frac{2\theta\alpha^{1+h}(-1)^{h+i+j}(1+j)^j\,\Gamma(\theta(h+1))}{i!\,j!\,\Gamma(1-e^{-\alpha})\Gamma(\theta(h+1)-i)}$$

$$\times\frac{ax^{-2}\exp\left[-(ax^{-1})\right]\left\{\exp\left[-(ax^{-1})\right]\right\}^{2j+1}}{\left\{1-\exp\left[-(ax^{-1})\right]\right\}^{2j+3}}.\ (10)$$

Consider the series expansion

$$(1-\varsigma)^{-c}\Big|_{(|\varsigma|<1,c>0)}=\sum_{k=0}^{\infty}\frac{\Gamma(c+k)}{k!\,\Gamma(c)}\varsigma^k.\ (11)$$

Applying the expansion in (11) to (10) for the term $\left\{1-\exp\left[-(ax^{-1})\right]\right\}^{2j+3}$, Equation (10) becomes

$$F_{PBXIE}^{(\alpha,\theta,a)}(x)=\sum_{h,i,j=0}^{\infty}\frac{2\theta\alpha^{1+h}(-1)^{h+i+j}(1+j)^j\left[2(1+j)+k\right]}{i!\,j!\,k!\left[-\exp(-\alpha)+1\right]\left[2(1+j)+k\right]}$$

$$\times\frac{\Gamma(\theta(h+1))\Gamma(3+2j+k)}{\Gamma(\theta(h+1)-i)\Gamma(2j+3)}\left[2(1+j)+k\right]$$

$$\times ax^{-2}\exp\left\{-\left[2(1+j)+k\right](ax^{-1})\right\}.$$

This can be written as

$$F_{PBXIE}^{(\alpha,\theta,a)}(x)=\sum_{h,i,j=0}^{\infty}v_{j,k}h_{[2(1+j)+k]}(x;a),\ (12)$$

where

$$v_{j,k}=\frac{2\theta\alpha^{1+h}(-1)^j\,\Gamma(+32j+k)}{j!\,k!\left[-\exp(-\alpha)+1\right]\Gamma(2j+3)\left[2(1+j)+k\right]}$$

$$\times\sum_{h,i=0}^{\infty}\frac{(-1)^{h+1}\Gamma(\theta(h+1))(1+i)^j}{i!\,\Gamma(\theta(h+1)-i)}$$

and $h_{[2(1+j)+k]}(x;a)$ is the IE density with scale parameter $a[2(1+j)+k]$. Similarly, the CDF

of the PBXIE model can also be expressed as

$$f_{PBXIE}^{(\alpha,\theta,a)}(x)=\sum_{j,k=0}^{\infty}v_{j,k}H_{[2(1+j)+k]}(x;a),\ (13)$$

Where $H_{[2(1+j)+k]}(x;a)$ is the IE density with scale parameter $a[2(1+j)+k]$.

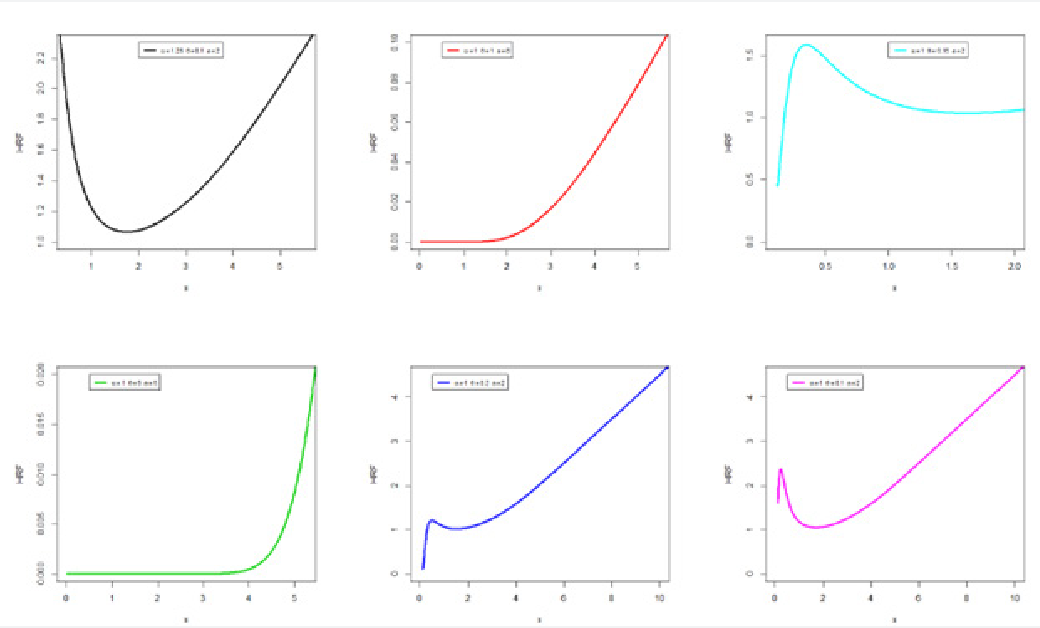

Figure 2: Plots of the PBXIE HRF.

Quantile and random number generation

The quantile function (QF) of X, where $X\sim PBXIE(\alpha,\theta,a)$, is obtained by inverting (5) as

$$Q(U)=a\left\{-\text{In}\left[\left(1+\left\{-\text{In}\left[1-\left(\frac{-\text{In}\left\{1-u\left[-\exp(-\alpha)+1\right]\right\}}{\alpha}\right)^{\frac{1}{\theta}}\right]\right\}^{\frac{1}{2}}\right)\right]\right\}^{-1},$$

Simulating the PBXIE r.v. is straightforward. If U is a uniform variate on the unit interval (0, 1); then the r.v. $X=Q(U)$ follows (6).

Moments

The $r^{(th)}$ ordinary moment of X, say μ'_r, follows from (12) as

$$\mu'_r\Big|_{(r<1)}=E(X^r)=\Gamma(1-r)\sum_{j,k=0}^{\infty}v_{j,k}a^r\left[2(1+j)+k\right]^r,\ (14)$$

Where

$$\Gamma(1+\tau)\Big|_{(\tau\in\mathbb{R}^+)}=\tau!$$

$$=\tau\times(\tau-1)\times(\tau-2)\times\ldots\times1$$

$$=\prod_{w=0}^{\tau-1}(\tau-w),$$

and

$$\int_0^{\infty}x^{\tau-1}e^{-t}dx=\Gamma(\tau)$$

is the complete gamma function.

Incomplete moments

The $r^{(th)}$ incomplete moment of X is defined by $m_r(y)\int_{-\infty}^{y}x^r f(x)dx$. We can write from (12)

How to cite this article: Jehan A. A, Saeed Al-mualim, M S Hamed, Haitham M Y. The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data. Biostat Biometrics Open Acc J. 2018; 8(4): 555742. DOI: 10.19080/BBOAJ.2018.08.555742.

$$m_r(y)\Big|_{(r<1)} = \gamma\left(1-r,\frac{a}{t}\right)\sum_{j,k=0}^{\infty} v_{j,k}a^r\left[2(1+j)+k\right]^r. \ (15)$$

Where $\gamma(\zeta,q)$ is the incomplete gamma function.

$$\gamma(\tau,q)\Big|_{(\tau\neq 0,-1,-2,\ldots)} = \int_0^q t^{\tau-1}\exp(-t)\,dt$$

$$=\frac{q^\tau}{\tau}\left\{{}_1F_1\left[\tau;\tau+1;-q\right]\right\}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(\tau+k)}q^{\tau+k},$$

and ${}_1F_1[\cdot,\cdot,\cdot]$ is a confluent hypergeometric function.

Generating functions

The moment generating function (mgf) of X, say $M(t)=E(\exp(tX))$, is obtained from (12) as

$$M(t)\Big|_{(r<1)} = \Gamma(1-r)\sum_{j,k,r=0}^{\infty} v_{j,k}\left(t^r/r!\right)a^r\left[2(1+j)+k\right]^r,$$

Moment of residual life

The $n^{(th)}$ moment of the residual life, say

$$\tau_n(t)=E\left((X-t)^n\left|\left[\begin{matrix}(n=1,2,\ldots,)\\(X>t)\end{matrix}\right]\right.\right)$$

uniquely determines $F(x)$. The $n^{(th)}$ moment of the residual life of X is given by

$$\tau_n(t)=\frac{\int_t^\infty (x-t)^n dF_{PBXIE}^{(\alpha,\theta,a)}(x)}{1-F_{PBXIE}^{(\alpha,\theta,a)}(x)},$$

then

$$\tau_n(t)=\frac{\Gamma\left(1-n,\frac{a}{t}\right)}{1-F_{PBXIE}^{(\alpha,\theta,a)}(x)}\sum_{j,k=0}^{\infty} v_{j,k}^{*}a^n\left[2(1+j)+k\right]^n,$$

where

$$v_{j,k}^{*}=v_{j,k}(1-t)^n,$$

$$\Gamma(\tau,q)\Big|_{(x>0)}=\int_q^\infty t^{\tau-1}\exp(-t)\,dt,$$

and

$$\Gamma(\tau,q)+\gamma(\tau,q)=\Gamma(\tau)$$

The mean residual life function or the life expectation at age t defined by

$$\tau_1(t)=E\left((X-t)\left|\begin{matrix}(n=1)\\(X>t)\end{matrix}\right.\right)$$

The MRL of X can be derived by setting $n=1$ in $\tau_n(t)$.

Moment of reversed residual life

The $n^{(th)}$ moment of the reversed residual life, say

$$T_n(t)=E\left((t-X)^n\left|\left[\begin{matrix}(n=1,2,\ldots)\\(X\le t,t>0)\end{matrix}\right]\right.\right)^n,$$

uniquely determines $F(x)$. We obtain

$$T_n(t)=\frac{\int_0^t (t-x)^n\,dF_{PBXIE}^{(\alpha,\theta,a)}(x)}{F_{PBXIE}^{(\alpha,\theta,a)}(x)}.$$

Then, the $n^{(th)}$ moment of the reversed residual life of X comes from

$$T_n(t)=\frac{\gamma\left(1-n,\frac{a}{t}\right)}{F(t)}\sum_{j,k=0}^{\infty} v_{j,k}^{*}a^n\left[2(1+j)+k\right]^n,$$

where

$$v_{j,k}^{**}=v_{j,k}\sum_{r=0}^{n}(-1)^r\binom{n}{r}t^{n-r}.$$

The mean inactivity time (the mean reversed residual life function) is given by

$$T_1(t)=E\left((t-X)\left|\left[\begin{matrix}(n=1)\\(X\le t,t>0)\end{matrix}\right]\right.\right),$$

The mean inactivity time of the new is obtained easily by setting $n=1$ in $T_n(t)$.

Estimation

In this work we will consider the ML Method for estimating the unknown parameter of our model. The log likelihood $(\log \mathrm{L})$ can be expressed as

$$\log \mathrm{L} = n\log 2 + n\log\theta + n\log\alpha + n\log a$$

$$-n\log\left[-\exp(-\alpha)+1\right]-2\sum_{i=1}^{n}\log x_i - 3\log(1-\xi_i)$$

$$+2\sum_{i=1}^{n}\log\xi_i-\alpha\sum_{i=1}^{n}\left[1-\exp(-\tau_i)\right]^\theta-\sum_{i=1}^{n}\tau_i+(\theta-1)\sum_{i=1}^{n}\log\left[1-\exp(-\tau_i)\right],$$

where

$$\xi_i=\exp\left[-\left(ax^{-1}\right)\right]$$

and

$$\tau_i=\left(\frac{\xi_i}{1-\xi_i}\right)^2.$$

The score vector can be easily derived. Procedures of the method are available in the literature. Other estimation methods can be considered in other future works.

Modeling real data

We provide two applications to show empirically the potentiality of the PBXIE. In order to compare the fits of the PBXIE with other related distributions, we will consider the goodness-of-fit measures including the Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC), consistent Akaike information criterion (CAIC) and Bayesian information criterion (BIC), $-l$, Cramér-von Mises (W^*), the Anderson-Darling (A^*) and KS(p-value) statistics. These statistics are widely used to determine how closely a specific CDF fits the empirical distribution of a given data set. These statistics are given by

$$AIC=2\left(p-\hat{l}\right),$$

$$BIC=2\left[-\hat{l}+\frac{1}{2}p\log(n)\right],$$

$$CAIC=2\left[-\hat{l}+pn/(n-p-1)\right],$$

and

$$HQIC=2\left\{-\hat{l}++p\log\left[\log(n)\right]\right\},$$

where p is the number of parameters, n is the sample size and $\hat{l}$ is the maximized log-likelihood.

How to cite this article: Jehan A. A, Saeed Al-mualim, M S Hamed, Haitham M Y. The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data. Biostat Biometrics Open Acc J. 2018; 8(4): 555742. DOI: 10.19080/BBOAJ.2018.08.555742.

$W^* = (1+1/2n)\left[(1/12n) + \sum_{j=1}^{n} w_j\right],$

and

$A^* = \left\{n + \frac{1}{n}\sum_{j=1}^{n} a_j\right\}\left(1 + \frac{9}{4n^2} + \frac{3}{4n}\right),$

respectively, where

$w_j = \left[z_i - (2j-1)/2n\right]^2,$

$a_j = (2j-1)\log\left[z_i\left(1 - z_{n-j+1}\right)\right],$

and $z_i = F(y_i)$ and the y_i's values are the ordered observations. The smaller these statistics are, the better the fit. The required computations are carried out using the R software. The MLEs and the corresponding standard errors (in parentheses) of the model parameters are given in Tables 1-4. The numerical values of the statistics W^* and A^* are listed in Tables 2 and 4. The histograms of the two data sets and the estimated PDF of the proposed model are displayed in Figures 2 and 3.

Table 1: MLEs (standard errors in parentheses) for data set I.

Distribution	Estimates		
$PBXIE(\alpha,\theta,a)$	-3.114 (1.0311)	1.647 (0.538)	2.310 (0.088)
$BXIIIE(\alpha,\beta,a)$	10.836 (8.861)	2.939 (0.527)	5.420 (1.339)
$LxIE(\alpha,a)$	88.155 (30.159)	20.654 (1.628)	
$LLIE(\beta,a)$	4.867 (0.382)	2.976 (0.068)	

Table 2: AIC , CAIC, BIC, HQIC, $-l$, W^* , A^* and KS(p-value) for data set I.

Distribution	AIC	CAIC	BIC	HQIC	$-l$	W^*	A^*	KS(p-value)
PBXIE	346.899	347.109	355.237	350.285	170.45	0.147	0.942	0.0789 (0.448)
BXIIIE	351.079	351.287	359.416	354.464	172.539	0.210	1.296	0.119 (0.070)
LxIE	356.429	356.533	361.988	358.686	176.215	0.334	2.026	0.115 (0.087)
LLIE	370.133	370.237	375.691	372.390	183.067	0.406	2.523	0.332 ($8.12e^{-12}$)

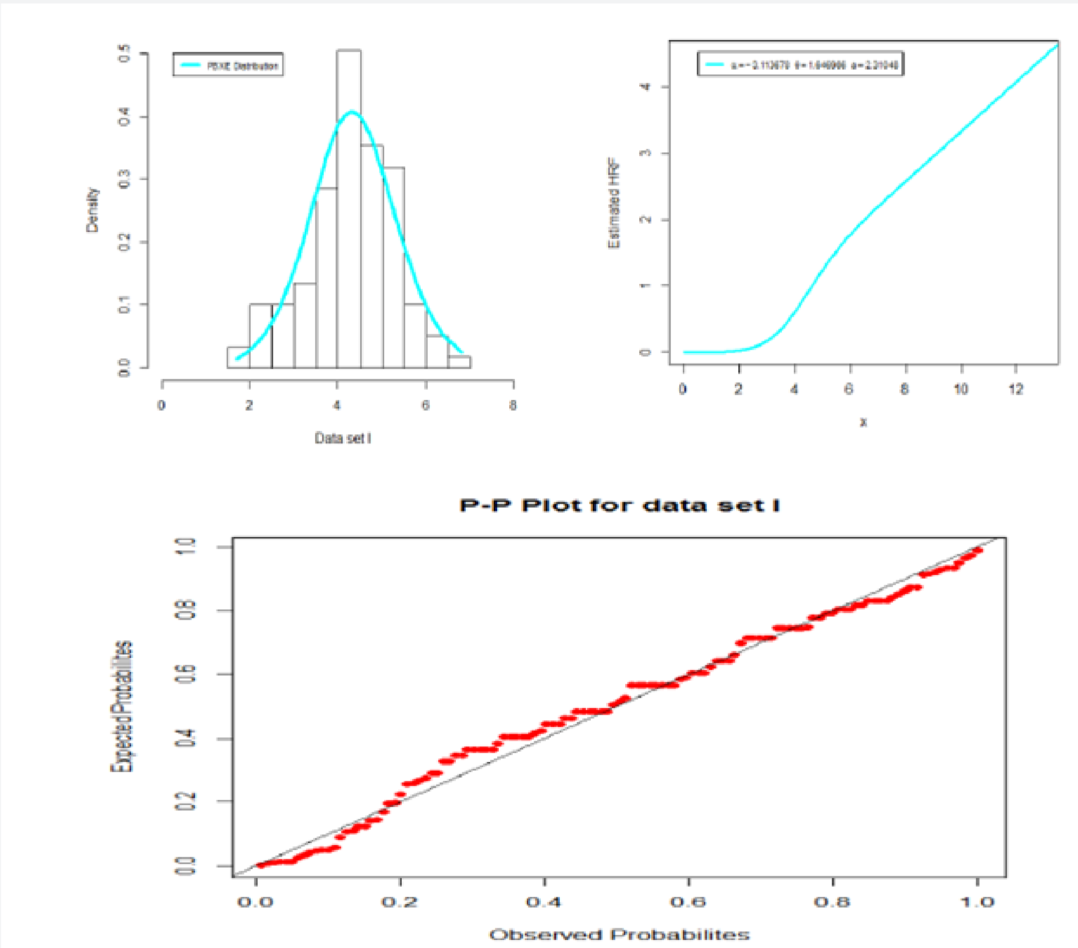

Figure 3: Estimated PDF, Estimated HRF and PP plot for data set I.

How to cite this article: Jehan A. A, Saeed Al-mualim, M S Hamed, Haitham M Y. The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data. Biostat Biometrics Open Acc J. 2018; 8(4): 555742. DOI: 10.19080/BBOAJ.2018.08.555742.

Table 3: MLEs (standard errors in parentheses) for data set II.

Distribution	Estimates		
$PBXIE(\alpha,\theta,a)$	-6.3262 (5.228)	0.189 (0.197)	1.937 (0.133)
$BXIIIE(\alpha,\beta,a)$	3.406 (1.568)	1.865 (0.340)	2.832 (0.671)
$LxIE(\alpha,a)$	9.197 (2.111)	6.214 (0.599)	
$LLIE(\beta,a)$	2.770 (0.233)	1.676 (0.073)	

Application 1

The 1st real data set represents fracture toughness data given in Nadarajah & Kotz [2]. The data are: 5.5, 5, 4.9, 6.4, 5.1, 5.2, 5.2, 5, 4.7, 4, 4.5, 4.2, 4.1, 4.56, 5.01, 4.7, 3.13, 3.12, 2.68, 2.77,

2.7, 2.36, 4.38, 5.73, 4.35, 6.81, 1.91, 2.66, 2.61, 1.68, 2.04, 2.08, 2.13, 3.8, 3.73, 3.71, 3.28, 3.9,

4, 3.8, 4.1, 3.9, 4.05, 4, 3.95, 4, 4.5, 4.5, 4.2, 4.55, 4.65, 4.1, 4.25, 4.3, 4.5, 4.7, 5.15, 4.3, 4.5, 4.9,

5, 5.35,5.15, 5.25, 5.8, 5.85, 5.9, 5.75, 6.25, 6.05, 5.9, 3.6, 4.1, 4.5, 5.3, 4.85, 5.3, 5.45, 5.1, 5.3,

5.2, 5.3, 5.25, 4.75, 4.5, 4.2, 4, 4.15, 4.25, 4.3, 3.75, 3.95, 3.51, 4.13, 5.4, 5, 2.1, 4.6, 3.2, 2.5, 4.1,

3.5, 3.2, 3.3, 4.6, 4.3, 4.3, 4.5, 5.5,4.6, 4.9, 4.3, 3, 3.4, 3.7, 4.4, 4.9, 4.9, 5. Here, we shall compare the fits of the PBXIE distribution with those of other competitive models, namely the Burr IE (BXIIIE), Lomax IE (LxIE) and Log-logistic IE (LLIE) distributions (Tables 1&2).

Table 4: AIC , CAIC, BIC, HQIC, $-l$, W^* , A^* and KS(p-value) for data set II.

Distribution	AIC	CAIC	BIC	HQIC	$-l$	W^*	A^*	KS(p-value)
PBXIE	287.295	287.545	295.11	290.458	140.647	0.069	0.418	0.0584 (0.885)
BXIIIE	300.717	300.967	308.533	303.881	147.359	0.225	1.1826	0.207 (0.0003)
LxIE	304.832	304.956	310.043	306.941	150.4161	0.315	1.694	0.1703 (0.006)
LLIE	307.883	308.007	313.093	309.992	151.941	0.3281	1.7340	0.3634 ($6.7e^{-12}$)

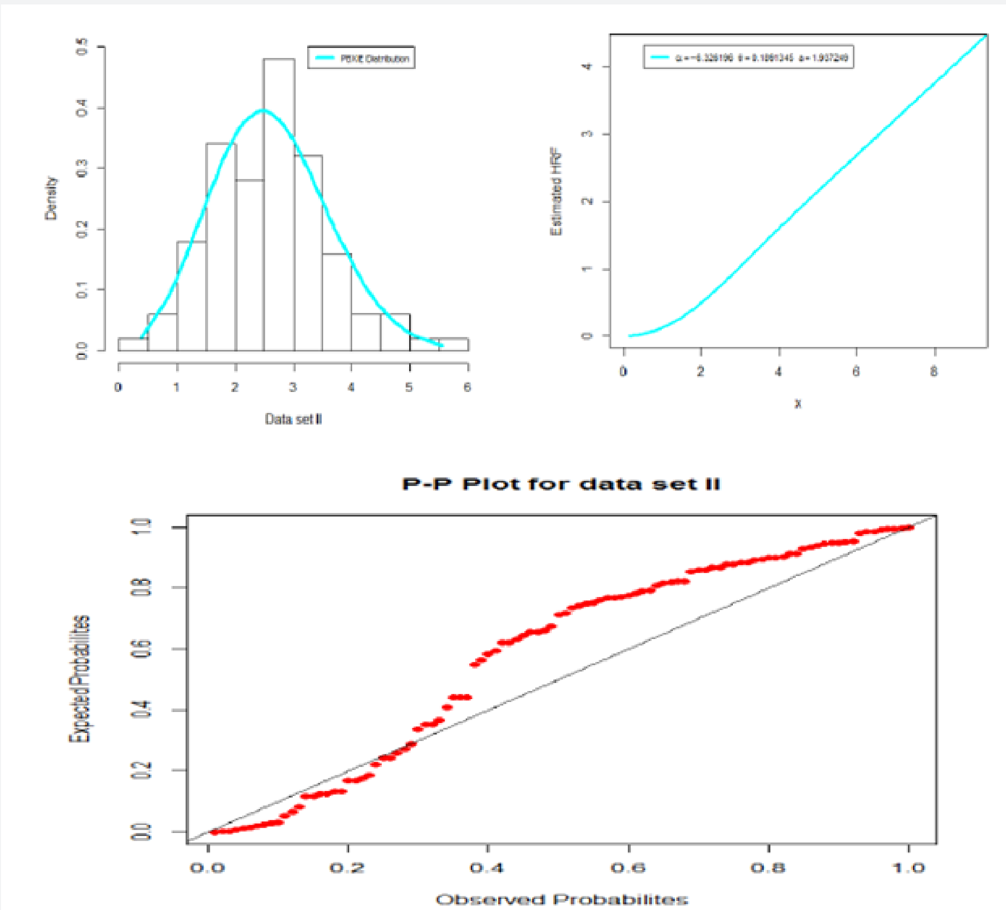

Figure 4: Estimated PDF, Estimated HRF and PP plot for data set II.

How to cite this article: Jehan A. A, Saeed Al-mualim, M S Hamed, Haitham M Y. The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data. Biostat Biometrics Open Acc J. 2018; 8(4): 555742. DOI: 10.19080/BBOAJ.2018.08.555742.

Based on Table 4 and Figure 4 we note that the new lifetime model is much better than the all other considered competitive models.

Application 2

The 2nd real data set represents tax revenue data.

This data is given by: 3.7, 3.11, 4.42, 3.28,

3.75, 2.96, 3.39, 3.31, 3.15, 2.81, 1.41, 2.76, 3.19, 1.59, 2.17, 3.51, 1.84, 1.61, 1.57, 1.89, 2.74, 3.27,

2.41, 3.09, 2.43, 2.53, 2.81, 3.31, 2.35, 2.77, 2.68, 4.91, 1.57, 2.00, 1.17, 2.17, 0.39, 2.79, 1.08, 2.88,

2.73, 2.87, 3.19, 1.87, 2.95, 2.67, 4.20, 2.85, 2.55, 2.17, 2.97, 3.68, 0.81, 1.22, 5.08, 1.69, 3.68, 4.70,

2.03, 2.82, 2.50, 1.47, 3.22, 3.15, 2.97, 2.93, 3.33, 2.56, 2.59, 2.83, 1.36, 1.84, 5.56, 1.12, 2.48, 1.25,

2.48, 2.03, 1.61, 2.05, 3.60, 3.11, 1.69, 4.90, 3.39, 3.22, 2.55, 3.56, 2.38, 1.92, 0.98, 1.59, 1.73, 1.71,

1.18, 4.38, 0.85, 1.80, 2.12, 3.65.

Based on Table 4 and Figure 4 we note that the new lifetime model is much better than the all other considered competitive models (Tables 3&4).

Concluding Remarks

In this work, we derive a new inverse exponential continuous model via the zero truncated Poisson with a strong physical interpretation. A round of mathematical properties are derived. The maximum likelihood Method is considered for estimating the unknown parameter of our model. The new model is much better other important competitive models like Burr XII inverse exponential, Lomax inverse exponential and Log-logistic inverse exponential distributions.

References

1. Keller AZ, Kamath ARR (1982) Alternative Reliability Models for Mechanical Systems. Proceeding of the 3rd International Conference on Reliability and Maintainability 411-415.
2. Nadarajah S, Kotz S (2007) On the alternative to the Weibull function. Eng Frac Mech 74(3): 577-579.

How to cite this article: Jehan A. A, Saeed Al-mualim, M S Hamed, Haitham M Y. The Zero Truncated Poisson Burr X Inverse Exponential Distribution for Modeling Fracture Toughness and Taxes Revenue Data. Biostat Biometrics Open Acc J. 2018; 8(4): 555742. DOI: 10.19080/BBOAJ.2018.08.555742.