



The Non-commutative Geometry and Matrix Model



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Abstract

From the analysis of the harmonic algebra of the three and two spheres, we study the compactification of the IKKT model on this spheres surface. Like the tori and orbifolds, we show that there exists here also a possibility to see clearly the compactifications of matrix models of M-theory on non-commutative geometry.

Keywords: Matrix Model; Non-commutative geometry; Compactification; M-theory; Super-symmetric; Dimensional reduction; SYM theory; Toroidal compactification; Standard solutions; Higher dimensional; Compact manifolds; Compact cycles; Geometry ideas; Commutative tori; Cartan subsymmetry; BFSS matrix theory; Compactification constraints; Hirzebruch complex surface

Abbreviations: SYM: Supersymmetric Yang Mills

Introduction

The matrix model formulation of M-theory is described by maximally super-symmetric $U(N)$ gauge quantum mechanics in the large N limit [1]. This model to which we shall refer hereafter to as the BFSS model may be obtained from 10d supersymmetric Yang Mills (SYM) theory by means of dimensional reduction. The BFSS model is closely related to another basic model often known as the IKKT matrix theory [2] and obtained by reduction of 10d SYM theory to a point [3]. Recently an important development has been made in the study of toroidal compactification of M-theory using the matrix model approach where it has been shown that non commutative geometry ideas emerge in a natural way. Indeed it has been shown in that the general solutions of the constraint Eqs defining the toroidal compactification of the IKKT model are related to the value of the flux of the three form potential of 11d supergravity [3-6].

The general solutions go beyond the standard solutions involving commutative tori which are recovered as a special case. Using the solution of the periodicity constraints of the variables of the IKKT model compactified on, one gets a 2d SYM theory on non commutative torus [3,4,6]. Recent analysis concerning AdS/CFT correspondence in particular type IIB on $AdS_5 \times S^5 / Z_3$ with 4d $N=1$ supersymmetric $su(N)^3$ gauge theory with fundamental matter involve also results which seem to have some thing to do with the non commutative torus [7,8]. Since the Connes et al. [3] development, several types of toroidal and orbifold compactifications of matrix models using non commutative geometry ideas have been studied and general results have been obtained [8-10].

The aim of this paper is to contribute to these efforts by extending the results of to the compactification of the IKKT model on higher dimensional compact manifolds which have no one cycles only. In other words we want to extend the analysis made where the role of the one cycles S^1 of the two torus $T^2 = S^1 \times S^1$ in the Connes et al. [3] analysis are played by the irreducible two cycles S^2 in the present study. Generalizations of this construction to higher $2k$ compact cycles involving k S^2 spheres is also possible. To address this question we have first to identify the defining constraint Eqs of the compactification of the IKKT model on F_0 . To that purpose we shall use the harmonic analysis of the three sphere $S^3 = SU(2)$ and its isometries to write down the compactification on S^2 .

The latter is obtained from S^3 by gauging out the Cartan subsymmetry of $SU(2)$. The compactification on $S^2 \times S^2$ is then obtained by using $S^3 \times S^3 = SU(2) \times SU(2)$ and gauging out the subgroup $U(1) \times U(1)$. The presentation of this paper is as follows: In section 2 we review briefly the main lines of the toroidal compactification of the IKKT model. This study, which is also valid for the BFSS matrix theory, allows us to give a reformulation of the defining constraints Eq of Banks et al. [11], useful when discussing the compactification on S^2 , after we give the solution of the constraints of periodicity in presence of winding numbers and we consider the compactification on S^2 by using the above mentioned harmonic analysis. As a result, first derive the compactification constraints Eqs on S^3 ; then we give their solution for both S^3 and S^2 .

Conclusion

In this paper we have studied the compactification of the IKKT model on the three and two spheres. This method can be done by other way such that the Hirzebruch complex surface F_0 geometry for the next works.

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