



Catalogue of Balanced Repeated Measurement Designs in Linear Periods of Two Unequal Sizes



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Abstract

Balanced repeated measurement designs have their own importance to control the carry over or residual effects. There are many situations when experiments become infeasible to perform in periods of equal sizes. Then these designs should be used in periods of unequal sizes. In this study, some useful balanced and strongly balanced repeated measurement designs are presented in periods of two unequal sizes.

Keywords: Repeated measurement designs; Circular balanced repeated measurement designs; Circular strongly balanced repeated measurement designs; Method of cyclic shifts; Repeated measurement; Residual effect; Subclass of designs; Hamiltonian decomposition; Lexicographic; Two graphs; Design and analysis; Cross-over trials; Balanced RMDs; Linear periods

Abbreviations: RMD: Repeated Measurement Design; CSBRMD: Circular Strongly Balanced Repeated Measurements Designs; CBRMD: Circular Balanced Repeated Measurements Designs

Introduction

A repeated measurement design (RMD) is one in which multiple, or repeated measurements are made on each experimental unit. The experimental unit could be a person or an animal. In repeated measurement design each subject is measured before and one or several times after an intervention. Areas where RMDs are widely used include medicine, pharmacology, animal sciences and psychology. A characteristic feature of RMD is that the effect which a treatment has during its period of application (its direct effect) may persist into the following period(s). If the effect persists only into the immediately following period, the effect is called the first-order residual effect or residual effect. The choice of RMD must be made in a way that the treatments can be efficiently compared after allowing for the residual effects. In this paper circular balanced and strongly balanced repeated measurement designs are constructed to extend the work of Iqbal and Tahir [1] for $v \leq 100$ in 3 periods.

Williams [2,3] first initiated repeated measurements designs. Magda [4] introduced the idea of a circular balance repeated measurement design when proper balance for different effects is considered, also proved its universal optimality over the subclass of designs with the same set of parameters. Roy [5] constructed strongly balanced uniform repeated measurements designs for $v=0,1$ or 3 modulo 4, by using the methods of differences and Hamiltonian decomposition of the lexicographic product of two graphs. Jones & Kenward [6] gave a thorough review of the design and analysis of cross-over trials. They also discussed its importance. Afsarinejed [7] presented some construction methods

for repeated measurement design. Cheng & Wu [8] explained two different types of repeated measurements designs (RMD), the balanced uniform RMD and the strongly balanced uniform RMD. Iqbal & Jones [9] constructed

- i. Efficient RMDs with equal and unequal period sizes using method of cyclic shifts for $3 \leq v \leq 10$, $3 \leq p \leq 10$,
- ii. Strongly balanced RMDs for $3 \leq v \leq 10$, $3 \leq p \leq 10$,
- iii. RMDs that are balanced for first and second order residual effect for $6 \leq v \leq 9$, $4 \leq p \leq 6$ and
- iv. combinatorial balanced designs for two unequal number of periods for $5 \leq v \leq 10$, $3 \leq p_1 \leq 6$ and $3 \leq p_2 \leq 10$.

Sharma et al. [10] introduced a general strategy of construction of balanced RMDs for odd number of treatments and their analysis. Iqbal & Tahir [1] constructed CSBRMD (circular strongly balanced repeated measurements designs) for some classes. Iqbal et al. [11] constructed some first- and second-order CBRMD (circular balanced repeated measurements designs). They also constructed some CSBRMDs. In this article, BRMDs and SBRMDs are constructed for linear periods in two different period sizes.

Definition 1.1: The set of all RMD with p periods, n experimental units and v treatments is denoted by $RM(v, n, p)$. A repeated measurements design is balanced with respect to the first-order residual effects if each treatment is immediately preceded λ' times by each other treatment (excluding itself).

Definition 1.2: A repeated measurements design is strongly balanced with respect to the first-order residual effects if each treatment is immediately preceded λ' times by each other treatment (including itself).

Definition 1.3: For given v and p , a balanced or strongly balanced repeated measurements design is minimal if $\lambda' = 1$.

In this paper, method of cyclic shifts is explained briefly in Section 2. BRMDs and SBRMDs are constructed in Section 3 and 4 respectively.

Method of Cyclic Shifts

Method of cyclic shifts is explained here briefly. For detail, see Iqbal and Tahir [1] and Iqbal et al. [11].

BRM Designs in Linear Blocks of two unequal Sizes

In this Section BRMDs are constructed for linear periods two different sizes (Table 1,2).

Table 1: BRMDs for $p_1 = 5$ and $p_2 = 3$ (Linear Periods).

V	p_1	p_2	Sets of Shifts
7	5	3	[1,3,2,4]+[5,6]
9	5	3	[1,2,3,5]+[4,6]+[7,8]
11	5	3	[1,2,3,4]+[5,7,6,8]+[9,10]
13	5	3	[1,2,3,4]+[5,6,8,9]+[7,10]+[11,12]
15	5	3	[1,2,3,4]+[5,6,7,9]+[8,10,11,12]+[13,14]
17	5	3	[1,2,3,4]+[5,6,7,8]+[9,10,11,12]+[13,14]+[15,16]
19	5	3	[1,2,3,4]+[5,6,7,8]+[9,11,10,12]+[13,14,15,16]+[17,18]
21	5	3	[1,2,3,4]+[6,5,7,8]+[9,10,13,11]+[12,14,15,16]+[17,18]+[19,20]
23	5	3	[1,2,3,4]+[6,5,7,8]+[9,10,11,13]+[12,14,15,16]+[17,18,19,20]+[21]+[22]
25	5	3	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,15,16]+[17,18,19,20]+[21,22]+[23,24]
27	5	3	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,15,14,16]+[17,18,20,19]+[21,22,23,24]+[25,26]
29	5	3	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26]+ [27,28]
31	5	3	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[14,13,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30]
33	5	3	[1,2,3,4]+[6,5,7,8]+[11,9,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,24,23]+[25,26,27,28]+[29,30]+[31,32]
35	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34]
37	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34]+[35,36]
39	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,28,27]+[29,30,31,32]+[33,34,35,36]+[37]+[38]
41	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38]+[39,40]
43	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34,35,36]+[37,38,39,40]+[41,42]
45	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,17]+[16,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42]+[43,44]
47	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43,44]+[45,46]
49	5	3	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43,44]+[45,46]+[47,48]

Table 2: BRMDs for $p_1 = 5$ and $p_2 = 4$ (Linear Periods).

V	p_1	p_2	Sets of Shifts
8	5	4	[1,2,3,4]+[5,6,7]
11	5	4	[1,2,3,4]+[5,7,6]+[8,9,10]

Rule I: Let $S = [q_1, q_2, \dots, q_{p-1}]$ be a set of shifts, where $1 \leq q_i \leq v-1$. If each element $1, 2, \dots, v-1$ appears an equal number of times, say λ' in a new set of shifts $S^* = [q_1, q_2, \dots, q_{p-1}]$ then it will be BRMD. If $0 \leq q_i \leq v-1$ and each element $0, 1, \dots, v-1$ appears an equal number of times, say λ' in a new set of shifts S^* then it will be SBRMD [12].

Rule II: Let $S = [q_1, q_2, \dots, q_{(p-2)}]t$ be a set of shifts, where $1 \leq q_i \leq v-2$. If each element $1, 2, \dots, v-2$ appears an equal number of times, say λ' in a new set of shifts $S^* = [q_1, q_2, \dots, q_{(p-2)}]$ then it will be BRMD. If $0 \leq q_i \leq v-2$ and each element $0, 1, \dots, v-2$ appears an equal number of times, say λ' in a new set of shifts S^* then it will be SBRMD.

12	5	4	[1,2,3,4]+[5,6,7,8]+[9,10,11]
ν	p_1	p_2	Sets of Shifts
14	5	4	[1,2,3,4]+[5,6,7]+[8,9,10]+[11,13,12]
15	5	4	[1,2,3,4]+[5,6,7,9]+[8,10,11]+[12,13,14]
16	5	4	[1,2,3,4]+[5,6,7,8]+[9,10,11,12]+[13,14,15]
17	5	4	[1,2,3,4]+[5,6,7]+[8,9,10]+[11,12,13]+[14,15,16]
18	5	4	[1,2,3,4]+[5,6,8,7]+[9,10,11]+[12,13,14]+[15,16,17]
19	5	4	[1,2,3,4]+[5,6,7,8]+[9,11,10,12]+[13,14,15]+[16,17,18]
20	5	4	[1,2,3,4]+[5,6,7,8]+[9,10,11,12]+[13,14,15,16]+[17,18,19]
21	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,13]+[11,12,14]+[15,16,17]+[18,19,20]
22	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,13]+[12,14,15]+[16,17,18]+[19,20,21]
23	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,13]+[12,14,15,16]+[17,18,19]+[20,21,22]
24	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,15,16]+[17,18,19,20]+[21,22,23]
25	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,15]+[16,17,18]+[19,20,21]+[22,23,24]
26	5	4	[1,2,3,4]+[6,5,7,9]+[8,10,11,12]+[13,14,15,16]+[17,18,19]+[20,21,22]+[23,24,25]
27	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,15,14,16]+[17,18,20,19]+[21,22,23]+[24,25,26]
28	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,15,16]+[17,18,20,19]+[21,22,23,24]+[25,26,27]
29	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20]+[19,21,22]+[23,24,25]+[26,27,28]
30	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,12,11]+[13,14,17,15]+[16,18,20,19]+[21,22,23]+[25,24,26]+ [27,28,29]
31	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[14,13,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27]+ [28,29,30]
32	5	4	[1,2,3,4]+[6,5,7,8]+[9,10,11,12]+[15,14,17,13]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+ [29,30,31]
33	5	4	[1,2,3,4]+[6,5,7,8]+[11,9,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,24]+[23,25,26]+ [27,28,29]+[30,31,32]
34	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,20,18,19]+[21,22,23,24]+[25,26,27]+ [28,29,30]+[31,32,33]
35	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[33]+[34]
36	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[33,34,35]
37	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27]+ [28,29,30]+[31,32,33]+[34,35,36]
38	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,19,20]+[21,22,23,24]+[25,26,27,28]+ [29,30,31]+[32,33,34]+[35,36,37]
39	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,28,27]+ [29,30,31,32]+[33,34,35]+[36,37,38]
40	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[33,34,35,36]+[37,38,39]
41	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+ [29,30,31]+[32,33,34]+[35,36,37]+[38,39,40]
ν	p_1	p_2	Sets of Shifts
42	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,13]+[12,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[33,34,35]+[36,37,38]+[39,40,41]
43	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[33,34,35,36]+[37,38,39]+[40,41,42]
44	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[33,34,35,36]+[37,38,39,40]+[41,42,43]
45	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,17]+[16,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35]+[36,37,38]+[39,40,41]+[42,43,44]
46	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35,37]+[36,38,39]+[40,41,42]+[43,44,45]
47	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43]+[44,45,46]
48	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43,44]+[45,46,47]
49	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43]+[44,45,46]+[47,48,49]
50	5	4	[1,2,3,4]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43]+[44,45,46]+[47,48,49]

SBRM Designs in Linear Blocks of two unequal Sizes

In this Section SBRMDs are constructed for linear periods two different sizes (Table 3,4).

Table 3: BRMDs for $p_1 = 5$ and $p_2 = 3$ (Linear Periods).

v	p_1	p_2	Sets of Shifts
6	5	3	[0,1,3,2]+[4,5]
8	5	3	[0,1,2,3]+[4,5]+[6,7]
10	5	3	[0,1,2,3]+[4,5,6,7]+[8,9]
12	5	3	[0,1,2,3]+[4,5,6,7]+[8,9]+[10,11]
14	5	3	[0,1,2,3]+[4,5,6,7]+[8,9,10,11]+[12,13]
16	5	3	[0,1,2,3]+[4,5,6,7]+[8,9,10,11]+[12,13]+[14,15]
18	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17]
20	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17]+[18,19]
22	5	3	[0,1,2,3]+[5,4,6,8]+[7,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21]
24	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21]+[22,23]
26	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25]
28	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25]+ [26,27]
v	p_1	p_2	Sets of Shifts
30	5	3	[0,1,2,3]+[5,4,6,7]+[9,8,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29]
32	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29]+[30,31]
34	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,28]+[27,29,30,31]+[32,33]
36	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33]+[34,35]
38	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,12]+[11,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37]
40	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37]+[38,39]
42	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[13,12,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37,38,39]+[40,41]
44	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37,38,39]+[40,41]+[42,43]
46	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37,38,39]+[40,41,42,43]+[44,45]
48	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37,38,39]+[40,41,42,43]+[44,45]+[46,47]
50	5	3	[0,1,2,3]+[5,4,6,7]+[8,9,10,11]+[12,13,14,15]+[16,17,18,19]+[20,21,22,23]+[24,25,26,27]+[28,29,30,31]+[32,33,34,35]+[36,37,38,40]+[39,41,42,43]+[44,45,46,47]+[48,49]

Table 4: BRMDs for $p_1 = 5$ and $p_2 = 4$ (Linear Periods).

v	p_1	p_2	Sets of Shifts
7	5	4	[0,1,2,3]+[4,5,6]
10	5	4	[0,1,2,3]+[4,5,6]+[7,8,9]
11	5	4	[0,1,2,3]+[4,5,7,8]+[6,9,10]
14	5	4	[0,1,2,3]+[4,5,6,7]+[8,9,10]+[11,12,13]
15	5	4	[0,1,2,3]+[4,5,7,9]+[6,8,10,11]+[12,13,14]
16	5	4	[0,1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+[13,14,15]
17	5	4	[0,1,2,3]+[4,5,6,7]+[8,10,9]+[11,12,13]+[14,15,16]
18	5	4	[0,1,2,3]+[4,5,6,8]+[7,9,10,11]+[12,13,14]+[16,15,17]
19	5	4	[0,1,2,3]+[4,5,6,7]+[8,9,11,12]+[10,13,14,15]+[16,17,18]
20	5	4	[0,1,2,3]+[4,5,6,7]+[8,9,10]+[11,12,13]+[14,15,16]+[17,18,19]
21	5	4	[0,1,2,3]+[4,6,5,7]+[8,9,10,13]+[11,12,14]+[15,16,17]+[18,19,20]
22	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,13]+[12,14,15,4]+[16,17,18]+[19,20,21]

23	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,13]+[12,14,15,16]+[4,17,18,19]+[20,21,22]
24	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[13,14,15]+[4,16,17]+[18,19,20]+[21,22,23]
25	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[13,14,15,4]+[16,17,18]+[19,20,21]+[22,23,24]
ν	p_1	p_2	Sets of Shifts
26	5	4	[0,1,2,3]+[6,5,7,9]+[8,10,11,12]+[13,14,15,16]+[17,18,19,4]+[20,21,22]+[23,24,25]
27	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[13,15,14,16]+[17,18,20,19]+[21,23,22,4]+[24,25,26]
28	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[13,14,15,16]+[17,18,20]+[4,19,21]+[22,23,24]+[25,26,27]
29	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[13,14,17,15]+[16,18,20,19]+[4,21,22]+[23,24,25]+ [26,27,28]
30	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,12,11]+[13,14,17,15]+[16,18,20,19]+[21,22,23,4]+[25,24,26]+ [27,28,29]
31	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[14,13,17,15]+[16,18,20]+[4,19,21]+[22,23,24]+[25,26,27]+[28,29,30]
32	5	4	[0,1,2,3]+[6,5,7,8]+[9,10,11,12]+[15,14,17,13]+[16,18,20,19]+[21,22,23]+[4,24,25]+ [26,27,28]+[29,30,31]
33	5	4	[0,1,2,3]+[6,5,7,8]+[11,9,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,24,4]+[23,25,26]+ [27,28,29]+[30,31,32]
34	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,20,18,19]+[21,22,23,24]+[25,26,27,4]+ [28,29,30]+[31,32,33]
35	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,28]+ [29,30,31,32]+[4,33,34]
36	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27]+ [4,28,29]+[30,31,32]+[33,34,35]
37	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,20,19]+[21,22,23,24]+[25,26,27,4]+ [28,29,30]+[31,32,33]+[34,35,36]
38	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,18,19,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31,4]+[32,33,34]+[35,36,37]
39	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,28,27]+[29,30,31,32]+[4,33,34,35]+[36,37,38]
40	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31]+[4,32,33]+[34,35,36]+[37,38,39]
41	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+[4,29,30,31]+[32,33,34]+[35,36,37]+[38,39,40]
42	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,13]+[12,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[4,33,34,35]+[36,37,38]+[39,40,41]
43	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,17,15]+[16,19,18,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34,35,36]+[4,37,38,39]+[40,41,42]
44	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,24]+[25,26,27,28]+[29,30,31,32]+[33,34,35]+[4,36,37]+[38,39,40]+[41,42,43]
45	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,17]+[16,19,18,20]+[21,22,23,25]+[24,26,27,28]+ [29,30,32,31]+[33,34,35,4]+[36,37,38]+[39,40,41]+[42,43,44]
46	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,37]+[4,36,38,39]+[40,41,42]+[43,44,45]
47	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[4,41,42,43]+[44,45,46]
ν	p_1	p_2	Sets of Shifts
48	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39]+[4,40,41]+[42,43,44]+[45,46,47]
49	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[4,41,42]+[43,44,45]+[46,47,48]
50	5	4	[0,1,2,3]+[6,5,7,8]+[9,11,10,12]+[13,14,15,16]+[17,19,18,20]+[21,22,23,25]+[24,26,27,28]+[29,30,32,31]+[33,34,35,36]+[37,38,39,40]+[41,42,43,4]+[44,45,46]+[47,48,49]

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