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# Is It Really Impossible To Divide By Zero?



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**Abstract**

It has been said that to define to divide by zero is impossible. But recent attempts show that it can be done very easily, which we would like to show below.

**Keywords:** Divide by zero; Impossible; Satisfying; Natural extension; Ordinary fraction; Real number; Amazing phenomena; Origin; Perpendicularity; Intersecting curves; Tangents; Intersection; Two curves; Infinity; Orthogonal; Slope; Horizontal line; Fixed radius; Initial point; Graph passes; Circle.

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**Definition**

Let us consider a function  $F : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  satisfying

$$\begin{cases} F(a, b) = \frac{a}{b} \text{ if } b \neq 0, \\ F(a, b)F(c, d) = F(ac, bd) \end{cases}$$

This is a natural extension of the ordinary fraction  $a/b$ .

While,

$$\begin{aligned} F(a, 0) &= \frac{2}{2} F(a, 0) = F(2, 2) F(a, 0) = F(2a, 0) \\ &= F(2, 1) F(a, 0) = \frac{2}{1} F(a, 0) = 2F(a, 0) \end{aligned}$$

implies  $F(a, 0) = 2F(a, 0)$ . Therefore  $F(a, 0) = 0$  for any real number  $a$  [1]. The fact shows that we have no choice but to define  $a/0 = 0$  for any real number  $a$ . Hence, we define

$$\frac{a}{0} = 0 \tag{1}$$

**For any real number  $a$ .**

Notice that we also get  $0/0 = 0$  by the definition [1-4]. This simple discussion was made by Sin-Ei Takahashi, an emeritus professor at Yamagata University. As you see the definition of the division by zero is very easy, but we have to notice that  $1/0$  is not the inverse of  $0$ , i.e.,

$$0 \times \frac{1}{0} = 0 \times 0 = 0.$$

The same definition was already given by Brahmagupta.

**What can be seen from the definition**

The definition (1) brings us several amazing phenomena. A typical thing is that most things we have been thinking as infinity without paying attention are exactly  $0$ . For example, for the function  $y = 1/x$ ,  $x = 0$  implies  $y = 0$ , i.e., its graph passes through the origin  $(0, 0)$ .

If the two lines  $a_i x + b_i y + c_i = 0 (i = 1, 2)$  are not parallel, they meet in the point

$$\left( \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}, \frac{a_2 c_1 - a_1 c_2}{a_1 b_2 - a_2 b_1} \right).$$

If they are parallel, i.e.,  $a_1 b_2 - a_2 b_1 = 0$ , the definition (1) shows that they meet in the origin  $(0, 0)$ . The other typical thing is that the two perpendicularly intersecting curves can be considered as they touch. Because we get  $\tan 90^\circ = 0$  by the definition (1). Therefore if two curves intersect perpendicularly, the slopes of the two tangents at the point of intersection are the same, i.e., we can still consider that the two curves touch at the point [5-8].

Let us consider a circle  $C$  of fixed radius  $r$  with center  $P$  on a half line with initial point  $O$  (Figure 1). Let  $t$  be one of the tangents of  $C$  from  $O$  and let  $\theta$  be the angle between  $l$  and  $t$ . Then we can easily get

$$|OP| = \frac{r}{\sin \theta}.$$

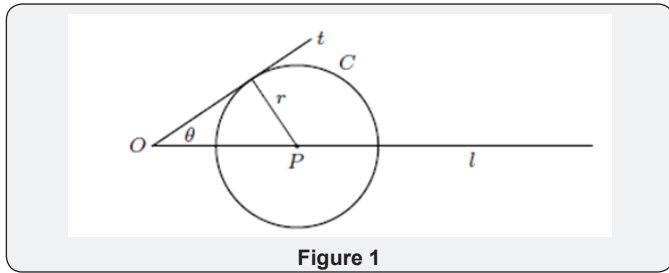


Figure 1

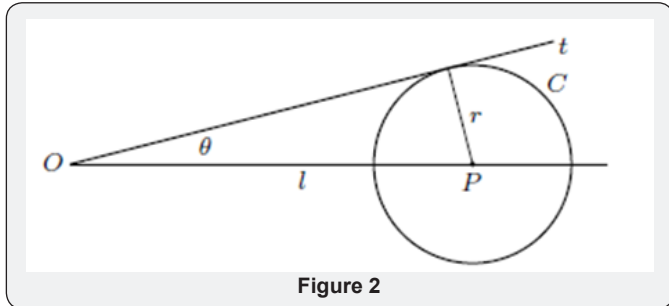


Figure 2

We now consider the angle  $\theta$  becomes smaller and smaller. Then  $P$  gets further away from  $O$  (Figure 2). In the case  $\theta = 0$ , the lines  $l$  and  $t$  coincide. Then one may think that the circle  $C$  has moved away into point at infinity (this is not an exact expression though). But the definition (1) implies

$$|OP| = \frac{r}{\sin 0^0} = \frac{r}{0} = 0.$$

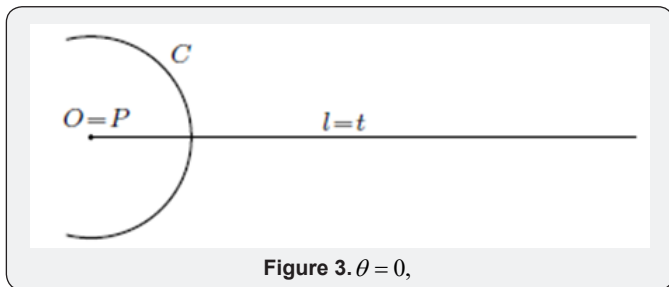


Figure 3.  $\theta = 0$ ,

Hence the point  $P$  coincides with  $O$  (Figure 3). The line  $t$  touches  $C$  in the case  $\theta > 0$ , but it is orthogonal to  $C$  in the case  $\theta = 0$ . But we can still consider that it touches  $C$  because the tangent at the point of intersection has slope  $\tan 90^0 = 0$  by the definition (1) if  $l$  is a horizontal line.

**Conclusion**

Mathematics is made upon the postulates. Any postulate should be taken into consideration, if it gives reasonable conclusions with new insights. The research of the division by zero was established by Saburo Saitoh [8], an emeritus professor at Gunma University, four years ago. Since the foundation of this topics are already made [1-8], it seems that the research is entering next stage as experts consider the division by zero in each of the special fields of Science.

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