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Smoothed Jackknife Empirical Likelihood for Weighted Rank Regression with Censored Data

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Abstract

To make inference for the semiparametric accelerated failure time (AFT) model with right censored data, which may contain outlying response or covariate values, we propose a smoothed jackknife empirical likelihood (JEL) method for the U -statistic obtained from a weighted smoothed rank estimating function. The jackknife empirical likelihood ratio is shown to be a standard chi-squared statistic. The new method improves upon the inference of the normal approximation method and possesses desirable important properties of easy computation and double robustness against influence of both outlying response and covariates. The advantages of the new method are demonstrated by simulation studies and data analyses. We illustrate our method by reanalyzing two data sets: the Stanford Heart Transplant Data and Multiple Myeloma Data.

Keywords: Accelerated failure time model; Jackknife empirical likelihood; Normal approximation; Outlying observations; Rank estimation; Robustness

Abbreviations: AFT: Accelerated Failure Time; JEL: Jackknife Empirical Likelihood; UWJEL: Unweighted JEL; WJEL: Weighted JEL; SSD: Sample Standard Deviation; AESD: Average Estimated Standard Deviation Of Estimators; MSE: Mean Squared Error Of Estimators; CP: Coverage Probability; LC: Length Of Confidence Intervals; JCP: Joint Coverage Probability; MCP: Marginal Coverage Probability; HGB: Haemoglobin; LogBUN: Logarithm Of Blood Urea Nitrogen; NSERC: National Science and Engineering Council

Introduction

A primary interest of survival analysis is often to understand the relationship between survival times and covariates measured on study participants, such as physical and biological measurements and medical conditions. Typically, survival data are not fully observed on all subjects, but rather some response values are censored. For $i=1,\ldots,n$, let T_i represent the survival time for the i^{th} subject, X_i be the associated p-dimensional vector of covariates, C_i denote the censoring time and δ_i denote the event indicator, i.e., $\delta_i = I(T_i \le C_i)$, which takes the value 1 if the event time is observed, or 0 if the event time is censored. Conditional on the covariates for the i^{th} subject, C_i is assumed to be independent of the failure times T_i. We define Y_i as the minimum of the survival time and the censoring time, i.e., $Y_i = \min(T_i, C_i)$. Then, the observed data are in the form $(Y_i,\delta_i,X_i), i=1,\ldots,n,$ which are assumed to be an independent and identically distributed (i.i.d.) sample from (Y,δ,X), here $Y=\min(T,C)$ and $\delta = I(T \le C)$.

The Cox proportional hazards (PH) model is the most prominent regression model used in survival analysis. However, when the proportional hazards assumption is not satisfied, the Cox PH model can produce incorrect regression parameter estimates. The accelerated failure time (AFT) model provides a useful alternative to the Cox PH model, where its covariate effects on the log transformed survival time T can be directly interpreted in terms of the regression coefficients. Because of this physical interpretability, the AFT model is more appealing than the PH model [1]. Following the definition in Kalbfleisch &Prentice [1] and Heller [2], the AFT model is defined as

$$\log(T_i) = \beta_0^{\tau} X_i + \varepsilon_i,\ \ i=1,\ldots,n, \quad (1)$$

Where the $\varepsilon_i' s$ are i.i.d random errors with an unknown distribution function and are independent of the covariates $X_i' s$, β_0 is the unknown true regression p-dimensional parameter to be estimated, and a^{τ} denotes the transpose of a vector or a matrix a. In data analysis, the log survival times from the regression residual, $\varepsilon^{\beta} = \log(T) - \beta^{\tau} X$, can be very large for small failure times, which is an indication that estimation and inference are sensitive to small failure times. Rank regression has been shown to be an effective method to regain robustness with respect to the outlying log survival times. For example, Fygenson & Ritov [3] proposed a monotonic rank estimating function

$$\tilde{S}_n(\beta) = n^{-3/2} \sum_{i=1}^{n}\sum_{j=1}^{n} \delta_i \left(X_i - X_j\right)\left[1 - I\left(r_i^{\beta} > r_j^{\beta}\right)\right], \quad (2)$$

Where, $r_i^{\beta} = \log(Y_i) - \beta^{\tau} X_i$. This estimating function is not continuous in β because of the indicator function $I(r_i^{\beta} > r_j^{\beta})$. This discontinuity creates difficulties in the derivation of the asymptotic distribution and computation of the estimator of β_0. To overcome these difficulties, Heller [2] developed a smoothed rank estimating function, which is monotonic and continuous with respect to the parameter vector. Moreover, to reduce the influence of outlying covariate values, he introduced a weight function in the smoothed rank estimating function. Heller's weighted smoothed rank estimating function for estimating β_0 is given by

$$S_n(\beta; w) = \left(S_{n1}(\beta; w), \ldots, S_{np}(\beta; w)\right)^{\tau}, \quad (3)$$

Where the k^{th} component is

$$S_n(\beta; w) = n^{-3/2} \sum_{i=j}^{n} \sum_{j=1}^{n} \delta_i (X_{ik} - X_{jk}) w_{ij} \times \left[1 - \Phi\left(\frac{r_i^{\beta} - r_j^{\beta}}{h}\right)\right], \ k = 1, \ldots, p.$$

The weight function, w_{ij}, is defined by

$$w_{ij} = \min\left\{1, \frac{1}{\max_k (X_{ik} - X_{jk})^2}\right\},$$

Which is symmetric and is chosen to reduce the influence of outlying covariate values on the estimator of β_0 and its asymptotic variance. The function, $\Phi(\cdot)$ is a local cumulative distribution function and is usually taken to be the standard normal distribution. It is a smooth approximation to the indicator function $I(r_i^{\beta} > r_j^{\beta})$ in (2) and ensures that $S_n(\beta; w)$ is differentiable in β and has bounded influence. The bandwidth, *h*, is used for smoothing purposes and satisfies conditions: $n \to \infty$, $nh \to \infty$, and $nh^4 \to 0$. In practice, as suggested by Heller [2], *h* can be set equal to $\hat{\sigma} n^{-.26}$, where $\hat{\sigma}$ is the sample standard deviation of the uncensored residuals $r_i^{\hat{\beta}}, \hat{\beta}$ is the zero solution of the estimating equation given in (3), and designated to be the estimator of the regression parameter β_0, the exponent, -0.26, provides the quickest rate of convergence while satisfying the above bandwidth conditions.

Heller [2] demonstrated that the weighted smoothed rank estimating function is not only robust to outlying survival times, but also to outlying covariate values, that is, it possesses the double robustness property. The computation of $\hat{\beta}$ becomes much easier than that of an estimator derived from a non-smoothed estimating equation, and it may be implemented through the standard Newton-Raphson algorithm. Heller [2] obtained the asymptotic normal distribution of $\hat{\beta}$ and established an inference procedure based on the normal approximation (NA) method. Many other methods have been suggested for censored regression. For example, Portnoy [4] proposed a censored regression quantile method, which is used to analyze conditional survival function without requiring specific distributional assumptions on the errors. Jin et al. [5] provided simple and reliable methods for implementing the Gehan rank estimator in the AFT model.

In this paper, we will develop an empirical likelihood based inference method and investigate its theoretical and numerical properties. We aim at improving the finite sample performance of the NA method. In the last two decades or so, the empirical likelihood (EL) method has become an attractive inference method for a number of statistical problems. In contrast to the NA method, the EL method has many nice features, for example, it combines the reliability of nonparametric methods with the effectiveness of the likelihood approach, and does not require variance calculations. Its application can be found in many publications. Owen [6,7] introduced EL confidence intervals and regions for parameters; Qin & Lawless [8] established Wilks' theorem for EL in an estimating equation seting; Owen [9] investigated EL for linear regression; and Qin & Zhou [10] suggested EL inference for the area under the ROC curve.

For the AFT model, Zhou [11] considered EL based on the censored empirical likelihood and Zhao [12] studied EL based on Fygenson & Ritov [3] estimating equation. Recently, Jing et al. [13] proposed the jackknife empirical likelihood (JEL) method, which combines the jackknife and the empirical likelihood for U-statistics. The most important property of the JEL method is its simplicity, which overcomes computational difficulty in an optimization problem with many nonlinear equations when sample size gets large. Based on the Gehan estimator studied by Jin et al. [5], Zhou [11] used EL method to obtain confidence regions. Our work is motivated by two survival data sets with outlying covariate values which have not been taken into account in the published analyses by researchers using both NA and EL methods.

Another strong motivation comes from both the double robustness property of the weighted smoothed rank estimator and the appealing finite sample properties of the JEL. In this paper, we will develop a new smoothed JEL method in the regression setting for the AFT model, where the parameters of interest may be multi-dimensional and are contained in some smoothed estimating functions, which involve a smoothing parameter and are multi-dimensional U statistics. Jing et al. [13] considered JEL for making inference about a one-dimensional parameter, whose estimator is directly defined by a U-statistic. Hence, their method cannot be immediately applied to our case. We will extend their method to smoothed estimating functions containing a multi-dimensional parameter vector and provide a rigorous justification for the proposed method. Via simulation studies and two real data examples, we will show that the proposed JEL method not only inherits the double robustness property of the NA method, but also contains some superior finite sample properties to the NA method. Hence, this new method provides a very useful and reliable tool for survival data analysis, which is easy for practitioners to adopt in their research work. The paper is organized as follows. In Section 2, we apply the JEL to the U-statistic derived from the weighted smoothed rank estimating function and show that the JEL ratio follows a standard chi-squared distribution. In Section 3, we conduct simulation studies to compare the performance of the JEL, NA and other competitors. In Section 4, we reanalyze two real data

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sets using our method: the Stanford Heart Transplant Data and the Multiple Myeloma Data, and compare the new method to other methods used in the literature. Section 5 includes our conclusion and discussion. Technical proofs are given in the Appendix.

Methodology and Main Result

NA method

Before we introduce our new inference method for the AFT model defined in (1), we state the result of the NA method given in Heller [2] Theorem 2 in the following paragraph. For the AFT model, under conditions $C1$-$C4$ given in Appendix, the weighted smoothed rank estimating function vector $S_n(\beta;w)$ given in (3) is a monotone field, is differentiable in β, and has bounded influence. The regression estimator $\hat{\beta}$ has an asymptotically normal distribution, i.e., $n^{1/2}(\hat{\beta}-\beta_0)$ converges in distribution to $N\left(0,A^{-1}(w)V(w)A^{-\tau}(w)\right)$, where

$$A(w)=\lim_{n\to\infty}E\left\{n^{-1/2}\partial S_n(\beta;w)/\partial\beta\right\}|\beta=\beta_0$$

And

$$V(w)=\lim_{n\to\infty}\operatorname{var}E\left\{S_n(\beta_0;w)\right\}.$$

To provide the NA based inference for β_0, one needs to estimate the variance-covariance Matrix $\Sigma=A^{-1}(w)V(w)A^{-T}(w)$. The estimator is given by

$$\Sigma_n=\left\{n^{-1/2}A_n\left(\hat{\beta};w\right)\right\}^{-1}V_n\left(\hat{\beta};w\right)\left\{n^{-1/2}A_n\left(\hat{\beta};w\right)\right\}^{-T},$$

Where, the (l,m) element of the second derivative matrix $A_n(\beta;w)$ is

$$A_{n(l,m)}(\beta;w)=n^{-3/2}\sum_{i=1}^{n}\sum_{j=1}^{n}\delta_i w_{ij}h^{-1}\left(X_{il}-X_{jl}\right)\left(X_{im}-X_{jm}\right)\phi\left(\frac{r_i^\beta-r_j^\beta}{h}\right),$$

Where, $\phi(u)=\partial\Phi(u)/\partial u$. And the (l,m) element of $V_n(\beta;w)$ is

$$V_{n(l,m)}(\beta;w)=n^{-3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1,k\neq j,}^{n}\left(X_{il}-X_{jl}\right)\left(X_{im}-X_{jm}\right)w_{ij}\left(e_{ij}^\beta-e_{ji}^\beta\right)w_{ik}\left(e_{ik}^\beta-e_{ki}^\beta\right),$$

With,

$$e_{ij}^\beta=\delta_i\left[1-\Phi\left(\frac{r_i^\beta-r_j^\beta}{h}\right)\right].$$

So the $(1-\alpha)100\%-level$ NA confidence region for β_0 is given by

$$R_{NA}=\left\{\beta:\left(\hat{\beta}-\beta\right)^\tau\left(\Sigma_n/n\right)^{-1}\left(\hat{\beta}-\beta\right)<\chi^2_{\alpha,p}\right\},$$

Where $\chi^2_{\alpha,p}$ is the α^{th} upper quantile of the chi-squared distribution with p degrees of freedom. When $p=1$, the confidence region R_{NA} becomes the confidence interval given by

$$CI_{NA}=\left\{\beta:\hat{\beta}-Z_{\alpha/2}\sqrt{\Sigma_n/n}<\beta<\hat{\beta}+Z_{\alpha/2}\sqrt{\Sigma_n/n}\right\},$$

Where, $Z_{\alpha/2}$ is the $(\alpha/2)^{th}$ upper quantile of the standard normal distribution. Taking $w_{ij}\equiv1$, we obtain the unweighted smoothed rank estimating function, $S_n(\beta)$, as follows:

$$S_n(\beta)=n^{-2/3}\sum_{i=1}^{n}\sum_{j=1}^{n}\delta_i\left(X_i-X_j\right)\left[1-\Phi\left(\frac{r_i^\beta-r_j^\beta}{h}\right)\right].$$

The estimator of β_0 based on this estimating function is its zero solution and Theorem 1 in Heller [2] provides the asymptotic distribution of this estimator. By imposing weights in the smoothed rank estimating function, Heller [2] Theorem 2 stated above indicates that its bounded influence provides stability to the regression estimator $\hat{\beta}$ in the presence of outlying survival times and covariate values, hence, the estimator posses the double robustness property.

Smoothed JEL method

Let $Z_i=(Y_i,\delta_i,X_i)$, $i-1,\ldots n.$ We rewrite the weighted smoothed rank estimating function, $S_n(\beta;w)$, given in (3) as a U-statistic with a symmetric kernel function in the following,

$$S_n(\beta;w)\equiv\frac{n-1}{2n^{1/2}}U_n(\beta),\quad(4)$$

Where, $U_n(\beta)$ is a U-statistic of degree 2 defined by

$$U_n(\beta)=\binom{n}{2}^{-1}\sum_{1\le j\le n}k(Z_i,Z_j;\beta)\quad(5)$$

With the kernel function

$$k(Z_i,Z_j;\beta)=(X_i-X_j)w_{ij}\left\{\delta_i\left[1-\Phi\left(\frac{r_i^\beta-r_j^\beta}{h}\right)\right]-\delta_j\left[1-\Phi\left(\frac{r_j^\beta-r_i^\beta}{h}\right)\right]\right\}.\quad(6)$$

The kernel has expectation of order $O(h^2)$ when evaluated at $\beta=\beta_0$, i.e., $E\left[k(Z_i,Z_j;\beta_0)\right]=O(h^2)\to0$, when $n\to\infty$. The U-statistic $U_n(\beta)$ is considered as a smoothed version of its analogue in $\tilde{S}_n(\beta)$ given by (2), which satisfies $E\tilde{S}_n(\beta_0)=0$. We will apply JEL to this U-statistic to get a smoothed JEL. To do that, we define jackknife pseudo-values as

$$\hat{V}_i(\beta)=nU_n(\beta)-(n-1)U_{n-1}^{(-i)}(\beta),\quad(7)$$

Where, $U_{n-1}^{(-i)}(\beta)=U(Z_1,\ldots,Z_{i-1},\ldots,Z_n)$ is the statistic $U_{n-1}(\beta)$ computed on the sample of $n-1$ variables formed from the original data set by deleting the i^{th} data value. Using these jackknife pseudo-values $\hat{V}_i(\beta)$ evaluated at β, we define the following jackknife empirical likelihood function,

$$L(\beta)=\max\left\{\prod_{i=1}^{n}p_i:\sum_{i=1}^{n}p_i=1,\sum_{i=1}^{n}p_i\hat{V}_i(\beta)=0,p_i\ge0\right\}.$$

Then the smoothed JEL ratio evaluated at β is given by

$$R(\beta)=\frac{L(\beta)}{n^{-n}}$$

$$=\max\left\{\prod_{i=1}^{n}(np_i):\sum_{i=1}^{n}p_i=1,\sum_{i=1}^{n}p_i\hat{V}_i(\beta)=0,p_i\ge0\right\}.$$

Using Lagrange multipliers, when 0 is contained in the convex hull of $\hat{V}_i(\beta)$'s, we have

$$p_i=\frac{1}{n\left\{1+\lambda^\tau\hat{V}_i(\beta)\right\}},\quad(8)$$

Where λ satisfies

$$f(\lambda)=\sum_{i=1}^{n}\frac{\hat{V}_i(\beta)}{1+\lambda^\tau\hat{V}_i(\beta)}=0.\quad(9)$$

The smoothed jackknife empirical log-likelihood ratio becomes

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$$\log R(\beta) = -\sum_{i=1}^{n} \log\left\{1+\lambda^{\tau}\hat{V}_i(\beta)\right\},$$

Then we get

$$-2\log R(\beta) = 2\sum_{i=1}^{n} \log\left\{1+\lambda^{\tau}\hat{V}_i(\beta)\right\}. \quad (10)$$

Jing et al. [13] proposed JEL for a single parameter using one-dimensional U-statistics and obtained several asymptotic results. We can easily extend their results to p-dimensional U-statistics and then to the smoothed JEL under the current setup. We state our main result in the following theorem, and relegate the proofs to the Appendix.

Theorem 1: Define $p\times 1$ vector $g(z;\beta)=E\left[k(z,Z_2;\beta)\right]$ and $p\times p$ variance-covariance matrix $\sigma_g^2 = Var\left[g(Z_1;\beta)\right]$. Under the conditions for the NA method stated in Heller (2007)'s Theorem 2, we have $E\left[k^2(z,Z_2;\beta)\right]<\infty$ (finite in element wise for a vector) and $\sigma_g^2>0$ (positive-definite for a matrix). Then, when $\beta=\beta_0$, we obtain

$$-2\log R(\beta) \xrightarrow{d} \chi_p^2,$$

As $n\to\infty$, where $\xrightarrow{d}$ denotes convergence in distribution. Based on this theorem, a $(1-\alpha)100\%-level$ JEL confidence region for β_0 is given by

$$R_{JEL} = \left\{\beta : -2\log R(\beta) \le \chi_{\alpha,p}^2\right\}.$$

To construct confidence intervals or regions for a single element or a q-dimensional $(1<q<p)$ subvector β_1 of the regression parameter vector $\beta=(\beta_1^{\tau},\beta_2^{\tau})^{\tau}$, let $W(\beta_1,\beta_2)=-2\log R(\beta)$, we propose profile JEL for β_1 defined as

$$\tilde{W}(\beta_1) = \inf_{\beta_2\in\mathbb{R}^{p-q}} W(\beta_1,\beta_2),$$

Where, $\mathbb{R}^{p-q}$ denotes the $(p-q)$ dimensional Euclidean space. The following theorem is similar to Theorem 1 for a full parameter vector. The proof of this theorem can be obtained by following that of Theorem 2.2 in Zhang & Zhao [14], we omit it here.

Theorem 2: Under the conditions for the NA method stated in Theorem 1, denote β_{01} as the true value of β_1, then the limiting distribution of $\tilde{W}(\beta_{01})$ is a chi-squared distribution with q degrees of freedom. That is

$$\tilde{W}(\beta_{01}) \xrightarrow{d} \chi_q^2.$$

A $(1-\alpha)100\%$-level profile JEL confidence region for β_{01} is given by

$$R_{PJEL} = \left\{\beta_1 : \tilde{W}(\beta_1) \le \chi_{\alpha,q}^2\right\}.$$

The idea of smoothed empirical likelihood is not new in the literature, for example, Whang [15] considered a smoothed empirical likelihood method to make inference for quantile regression models, where the estimating function is actually a U-statistics of degree one only and JEL is not needed. Here, our work further extends smoothed EL to smoothed JEL for the popular AFT model with possible outlying response or covariate values in survival analysis, and provides an exemplar of inference procedure for other types of rank regression models, where similar theoretical results can be established accordingly.

Simulation studies

In this section, two simulation studies are conducted to compare the performance of the NA method and other competitive methods to that of the smoothed JEL method under different contaminating conditions on survival times and covariate values. To evaluate sensitivity of the NA and JEL methods to the weights used in the smoothed rank estimating functions, results are also compared between the weighted and unweighted smoothed rank estimating functions under different censoring rates and sample sizes. In the first simulation study, we fit a model with a one-dimensional continuous covariate, and then compare the bias of estimators, sample standard deviation of estimators, average estimated standard deviation of estimators, mean squared error of estimators, the coverage probability and average length of the confidence intervals. In the second simulation study, a model with a two-dimensional covariate is fitted, and the performance of different methods is compared in terms of joint coverage probabilities and marginal coverage probabilities.

Simulation Study 1: AFT model with a one-dimensional co-variate

In the following AFT model with a one-dimensional covariate,

$$Y-\log(T)=\beta_0 X+\varepsilon,$$

where the true regression parameter $\beta_0=2$. The censoring time C was generated from a uniform distribution $U(0,c)$, here determines the censoring rate (cr). We chose different values of c to produce $\mathrm{cr}=25\%, 50\%, 75\%$, respectively. Sample size was taken to be $n=30$ and $n=100$, respectively. Various combinations of error distribution, censoring rate and sample size provide a comprehensive comparison between the NA and JEL methods. In order to compare the proposed method with its competitors in the literature, we also consider the quantile regression of Portnoy [4], the Gehan rank regression of Jin et al. [5] and the empirical likelihood method of Zhou [10] in simulation studies. For ease of presentation, we construct the error term ε as follows. Assume e be an arbitrary continuous random variable with cumulative distribution function $F_e(\mathrm{t})$, let $\mu_e=\mathrm{E}(e)$ and $\sigma_e=\sqrt{Var(e)}$. For any $0<\tau<1$, the τ^{th} quantile of e is given by $F_e^{-1}(\tau)$. For any $\sigma>0$, we define error term as $\varepsilon=g(e;\sigma,\tau)=\sigma\left\{e-F_e^{-1}(\tau)\right\}/\sigma_e$, which represents a (σ,τ) dependent transformation of e, then we have $E(\varepsilon)=\sigma\left\{E(\varepsilon)-F_e^{-1}(\tau)\right\}/\sigma_e$ and $\mathrm{Var}(\varepsilon)=\sigma^2$, the τ^{th} quantile of ε equals 0.

In the above AFT model, $\forall\ 0<k<1$, denote the k^{th} quantile of ε by t_k, then we have the k^{th} conditional quantile of $Y=\log(T)$ given X equals $\beta_0 X+t_k$ (where $k=\tau, t_k=0$) and $E(Y|X)=E(\varepsilon)+\beta_0 X$, i.e., the parameter β_0 of interest is the slope parameter in both the quantile regression and the log-linear AFT model, we can compare the performance of different methods by aiming

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at the same target parameter β. We choose four different error distributions corresponding to four types of skewness in simulation. Through the aforementioned process, they are generated from $e \sim N(0,1)$, $\chi^2(df=6)$, $\chi^2(df=1)$ and standard extreme value distribution (EV) with probability density function $f_e(t) = \exp\{t - \exp(t)\}$, $-\infty < t < \infty$, mean $\gamma = 0.577216$ and variance $\pi^2/6$, respectively, where $N(0,1)$ is standard normal distribution and symmetric, $\chi^2(df)$ is chi-squared distribution with df degrees of freedom and skewed to the right, smaller df results in larger skewness and vice versa, and EV is skewed to the left and can be generated from exponential distribution by $e = \log(u)$, u follows exponential distribution with rate equal to In this simulation setup, two different simulation scenarios were considered for four error distributions. In the first scenario, the univariate covariate X was generated from the standard normal distribution $N(0,1)$. For $e \sim N(0,1)$, EV, $\chi^2(df=6)$, and $\chi^2(df=1)$ respectively, the error term ε was generated from $g(e;\sigma=1,\tau=0.5)$, here $\tau = 0.5$ represents median regression.

In the second scenario, a data set with a contaminated covariate and a contaminated error distribution was generated to test robustness of the NA and JEL methods. Specifically, 95% of the values of the covariate X were generated from the standard normal distribution $N(0,1)$ and 95% of the error term ε values were generated from $g(e;\sigma=1,\tau=0.5)$. The other 5% of the covariate X values were generated from the normal distribution $N(-5,1)$ and 5% of the error term ε values were generated from $g(e;\sigma=2,\tau=0.5)$, which contains a larger σ than in $g(e;\sigma=1,\tau=0.5)$. The details for the setup of this simulation study are displayed in Table 1.

Table 1: Design of Simulation Study 1.

Scenario	Distribution of e	Error $\varepsilon = g(e;\sigma,\tau)$	Covariate X
1. Not contaminated ε and X $(\sigma=1, \tau=0.5)$	$N(0,1)$	$\sigma\{e - F_e^{-1}(\tau)\}$	$N(0,1)$
	EV	$\sigma\{e - F_e^{-1}(\tau)\}/(\pi/\sqrt{6})$	$N(0,1)$
	$\chi^2(df=6)$	$\sigma\{e - F_e^{-1}(\tau)\}/\sqrt{12}$	$N(0,1)$
	$\chi^2(df=1)$	$\sigma\{e - F_e^{-1}(\tau)\}/\sqrt{2}$	$N(0,1)$
2. Contaminated ε and X $(\sigma=1, \text{or } 2\ \tau=0.5)$	$N(0,1)$	95% from $g(e;\sigma=1,\tau=0.5)$	$N(0,1)$
	EV	and	and
	$\chi^2(df=6)$	5% from $g(e;\sigma=2,\tau=0.5)$	5% from $N(-5,1)$
	$\chi^2(df=1)$		

We conducted simulation studies to compare the performance of the quantile regression method of Portnoy [4] (QR), the Gehan rank estimation method of Jin et al. [5] (JinNA), Heller's unweighted estimation method (UWNA), Heller's weighted estimation method (WNA), the empirical likelihood method of Zhou [10] (ZhouEL), and the proposed unweighted JEL (UWJEL) and weighted JEL (WJEL) methods, in terms of bias (Bias) of estimators for $\beta_0 = 2$, sample standard deviation of estimators (SSD), average estimated standard deviation of estimators (AESD), mean squared error of estimators (MSE), coverage probability (CP) of 95% confidence intervals, average length of confidence intervals (LC). QR, JinNA and ZhouEL were implemented using R functions crq, lss and RankRegTest in the R packages quantreg, lss and emplik, respectively.

There were $B = 1000$ replications in each simulation setting. We did 50^{th} quantile regression (i.e., median regression with $\tau = 05$) under low and medium censoring rates (cr = 25; 50%), results are denoted by $QR_{0.5}$, which guaranties that the estimates of coefficients could be obtained. When censoring rate is high (cr = 75%), we did 25^{th} quantile regression denoted by $QR_{0.25}$ for the same reason. Weighted and unweighted smoothed rank regression functions were investigated for both NA and JEL methods with both uncontaminated and contaminated data. Since Portnoy's, Jin's and Zhou's methods are available for unweighted cases only, and extending their methods to weighted cases is not trivial and beyond of the scope of the current research, we did not consider them in our simulation studies. The results are reported in Tables 2-5. We observe that QR method has relatively larger MSE, lower coverage probabilities and wider confidence intervals. Jin's Gehan rank estimation method is similar to Heller's UWNA. Zhou's EL approach is suited to the data with low censoring rate and large sample size. The coverage probabilities of JEL are closest to the nominal level 95% compared to those of NA and other competitors in most cases. This is particularly true for small sample sizes (e.g. n = 30) and high censoring rates (e.g. cr = 75%). It implies that JEL generally has better coverage probability than NA and other methods. On the other hand, the average length of JEL is slightly longer than that of NA and other methods. Also, when the sample size increases or the censoring rate decreases, the average length becomes shorter. In the cases of uncontaminated covariates, using weights or

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not does not affect results too much. However, in the cases of contaminated covariates, the weighted methods perform much better than the unweighted methods (to abuse terminologies in some way, we use weighted methods to refer to the JEL and NA methods based on the weighted smoothed estimating function, same thing for the unweighed methods). The weighted methods do not necessarily reduce the bias of the unweighted methods, but they provide more accurate approximation to the sampling distributions of the estimators, then result in better coverage probability.

Table 2: Summary results of the simulation study 1 in the first scenario when $e \sim N(0,1)$ or EV, covariate and error are not contaminated. $cr(\%)$: censoring rate; Bias: bias of estimators for $\beta_0 = 2$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; $CP(\%)$: coverage probability of 95% confidence intervals for β_0; LC: average length of confidence intervals; n: sample size. Results are based on 1000 simulation replicates.

			$\varepsilon = g(e;\sigma=1,\tau=0.5),\ e \sim N(0,1)$						$\varepsilon = g(e;\sigma=1,\tau=0.5),\ e \sim EV$					
n	**Cr**	**Method**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**
30	25	$QR_{0.5}$	-0.005	0.306	0.349	0.094	91.2	1.368	0.012	0.282	0.32	0.08	91.6	1.254
		JinNA	-0.003	0.248	0.239	0.062	92.7	0.939	0.003	0.203	0.207	0.041	93.6	0.81
		UWNA	0.01	0.248	0.231	0.061	91.5	0.906	0.013	0.203	0.197	0.041	91.6	0.774
		WNA	0.012	0.262	0.255	0.069	93.5	1	0.015	0.217	0.219	0.047	94.1	0.859
		ZhouEL	(The same as that of JinNA)				92	0.912	(The same as that of JinNA)				92.9	0.76
		UWJEL	(The same as that of UWNA)				95.2	0.973	(The same as that of UWNA)				94	0.846
		WJEL	(The same as that of WNA)				95.7	1.097	(The same as that of WNA)				95.5	0.949
	50	$QR_{0.5}$	0.03	0.601	0.524	0.362	91	2.053	-0.004	0.408	0.471	0.166	92.4	1.846
		JinNA	0.013	0.338	0.316	0.114	91.5	1.24	-0.003	0.257	0.263	0.066	92.8	1.032
		UWNA	0.041	0.342	0.301	0.118	90.2	1.18	0.019	0.259	0.245	0.067	90	0.961
		WNA	0.051	0.358	0.338	0.13	93.2	1.325	0.025	0.275	0.278	0.076	93.2	1.089
		ZhouEL	(The same as that of JinNA)				88.8	1.143	(The same as that of JinNA)				90.1	0.931
		UWJEL	(The same as that of UWNA)				92.5	1.384	(The same as that of UWNA)				94.3	1.159
		WJEL	(The same as that of WNA)				94.2	1.525	(The same as that of WNA)				96.1	1.272
	75	$QR_{0.25}$	0.02	1.043	0.907	1.087	89.1	3.558	-0.055	1.011	0.743	1.024	87.2	2.914
		JinNA	0.023	0.645	0.505	0.416	86.3	1.978	-0.068	0.582	0.412	0.343	86	1.615
		UWNA	0.077	0.659	0.413	0.439	79.1	1.619	-0.032	0.593	0.336	0.352	79.9	1.317
		WNA	0.131	0.653	0.486	0.443	83.9	1.904	0.057	0.565	0.398	0.323	85.3	1.561
		ZhouEL	(The same as that of JinNA)				76.4	1.49	(The same as that of JinNA)				77.8	1.235
		UWJEL	(The same as that of UWNA)				88.1	2.3	(The same as that of UWNA)				91.1	2.07
		WJEL	(The same as that of WNA)				92.3	2.539	(The same as that of WNA)				93.3	2.334

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		$QR_{0.5}$	0.005	0.162	0.178	0.026	90.5	0.698	0	0.149	0.159	0.022	89.5	0.625
		JinNA	0.003	0.13	0.129	0.017	94.4	0.504	0.002	0.109	0.106	0.012	93.3	0.417
		UWNA	0.01	0.13	0.128	0.017	94.1	0.502	0.007	0.109	0.106	0.012	93.2	0.415
	25	WNA	0.01	0.137	0.136	0.019	94.9	0.534	0.008	0.115	0.115	0.013	94.1	0.45
		ZhouEL	(The same as that of JinNA)				94.7	0.508	(The same as that of JinNA)				94.1	0.416
		UWJEL	(The same as that of UWNA)				95	0.512	(The same as that of UWNA)				94.3	0.422
		WJEL	(The same as that of WNA)				95.4	0.548	(The same as that of WNA)				95	0.46
		$QR_{0.5}$	0.008	0.224	0.258	0.05	89.2	1.011	0.006	0.207	0.229	0.043	87.7	0.897
		JinNA	0.004	0.166	0.166	0.027	94.2	0.652	0.004	0.135	0.133	0.018	95.4	0.524
		UWNA	0.019	0.166	0.166	0.028	94.4	0.649	0.014	0.135	0.133	0.018	94.6	0.521
100	50	WNA	0.022	0.174	0.179	0.031	95.8	0.701	0.018	0.144	0.145	0.021	95.7	0.57
		ZhouEL	(The same as that of JinNA)				94.8	0.657	(The same as that of JinNA)				94.9	0.52
		UWJEL	(The same as that of UWNA)				96	0.672	(The same as that of UWNA)				95.5	0.533
		WJEL	(The same as that of WNA)				96.7	0.728	(The same as that of WNA)				96	0.587
		$QR_{0.25}$	0.003	0.358	0.402	0.128	85.7	1.574	0.01	0.277	0.296	0.077	84.6	1.162
		JinNA	0.011	0.265	0.247	0.07	93	0.968	0.009	0.213	0.196	0.046	91.5	0.769
		UWNA	0.037	0.268	0.244	0.073	92.4	0.958	0.026	0.214	0.193	0.046	90.7	0.755
	75	WNA	0.049	0.282	0.269	0.082	93.3	1.055	0.035	0.225	0.213	0.052	92.9	0.833
		ZhouEL	(The same as that of JinNA)				92.1	0.953	(The same as that of JinNA)				91.2	0.746
		UWJEL	(The same as that of UWNA)				92.5	1.025	(The same as that of UWNA)				93.6	0.806
		WJEL	(The same as that of WNA)				93.3	1.126	(The same as that of WNA)				94.1	0.882

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Table 3: Summary results of the simulation study 1 in the second scenario when $e \sim N(0,1)$ or EV, both covariate and error are contaminated. $cr(\%)$: censoring rate; Bias: bias of estimators for $\beta_0 = 2$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; $CP(\%)$: coverage probability probability of 95% confidence intervals for β_0; LC: average length of confidence intervals; n: sample size. Results are based on 1000 simulation replicates.

			$\varepsilon = g(e;\sigma=1,\tau=0.5),\ e \sim N(0,1)$						$\varepsilon = g(e;\sigma=1,\tau=0.5),\ e \sim EV$					
n	***Cr***	**Method**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**
30	25	$QR_{0.5}$	-0.002	0.255	0.262	0.049	82.4	1.028	-0.006	0.22	0.244	0.048	83.7	0.955
		JinNA	0.028	0.225	0.18	0.052	83.4	0.706	0.013	0.185	0.157	0.034	86.4	0.616
		UWNA	0.033	0.225	0.189	0.052	76.3	0.742	0.017	0.185	0.16	0.034	78.4	0.626
		WNA	0.021	0.239	0.234	0.058	95	0.918	0.024	0.201	0.2	0.041	94.7	0.786
		ZhouEL	(The same as that of JinNA)				78.8	0.619	(The same as that of JinNA)				81.9	0.521
		UWJEL	(The same as that of UWNA)				84.9	0.682	(The same as that of UWNA)				86.7	0.611
		WJEL	(The same as that of WNA)				95.2	1.027	(The same as that of WNA)				95	0.914
	50	$QR_{0.5}$	-0.003	0.298	0.337	0.089	83.5	1.323	-0.024	0.27	0.307	0.074	82.6	1.203
		JinNA	0.058	0.274	0.205	0.078	81.4	0.802	0.029	0.221	0.173	0.049	82.9	0.678
		UWNA	0.07	0.274	0.192	0.08	83.7	0.895	0.036	0.221	0.162	0.05	71.9	0.635
		WNA	0.073	0.321	0.3	0.108	93.4	1.176	0.056	0.246	0.245	0.064	93.4	0.959
		ZhouEL	(The same as that of JinNA)				77.1	0.674	(The same as that of JinNA)				77.3	0.56
		UWJEL	(The same as that of UWNA)				83.7	0.895	(The same as that of UWNA)				85.9	0.789
		WJEL	(The same as that of WNA)				94.4	1.426	(The same as that of WNA)				94.8	1.223
	75	$QR_{0.25}$	0.174	0.834	0.473	0.725	83.6	1.856	0.098	0.541	0.433	0.302	85.4	1.698
		JinNA	0.096	0.378	0.248	0.152	76.9	0.974	0.047	0.292	0.212	0.087	79.3	0.832
		UWNA	0.118	0.381	0.194	0.159	62.6	0.76	0.064	0.291	0.168	0.089	65.4	0.661
		WNA	0.183	0.518	0.384	0.301	86.7	1.508	0.132	0.409	0.305	0.184	87.5	1.196
		ZhouEL	(The same as that of JinNA)				65.6	0.744	(The same as that of JinNA)				68.9	0.645
		UWJEL	(The same as that of UWNA)				86.7	1.194	(The same as that of UWNA)				91.4	1.076
		WJEL	(The same as that of WNA)				91.5	1.72	(The same as that of WNA)				96.1	1.43
100	25	$QR_{0.5}$	-0.002	0.14	0.138	0.02	76.1	0.542	-0.012	0.125	0.127	0.016	79.7	0.5
		JinNA	0.024	0.13	0.118	0.018	89.8	0.461	0.013	0.103	0.1	0.011	91.2	0.391
		UWNA	0.028	0.131	0.121	0.018	88.6	0.474	0.016	0.103	0.102	0.011	90.2	0.399
		WNA	0.015	0.126	0.128	0.016	95.2	0.5	0.01	0.106	0.107	0.011	95.1	0.421
		ZhouEL	(The same as that of JinNA)				88.1	0.436	(The same as that of JinNA)				91	0.361
		UWJEL	(The same as that of UWNA)				90.6	0.482	(The same as that of UWNA)				94	0.41
		WJEL	(The same as that of WNA)				95.7	0.513	(The same as that of WNA)				95.6	0.43
	50	$QR_{0.5}$	-0.009	0.179	0.177	0.032	75.7	0.695	-0.016	0.159	0.158	0.026	76.3	0.62
		JinNA	0.054	0.161	0.138	0.029	87.2	0.543	0.034	0.123	0.113	0.016	87.7	0.442
		UWNA	0.063	0.161	0.144	0.03	86.1	0.566	0.04	0.122	0.115	0.017	87.4	0.452
		WNA	0.038	0.159	0.163	0.027	95	0.64	0.032	0.13	0.132	0.018	95.2	0.517
		ZhouEL	(The same as that of JinNA)				86.4	0.513	(The same as that of JinNA)				87.6	0.408
		UWJEL	(The same as that of UWNA)				88.6	0.56	(The same as that of UWNA)				91.4	0.457
		WJEL	(The same as that of WNA)				95.9	0.671	(The same as that of WNA)				95.5	0.536
	75	$QR_{0.25}$	0.109	0.254	0.218	0.076	66.3	0.856	0.073	0.18	0.17	0.038	69.1	0.667
		JinNA	0.119	0.219	0.167	0.062	80.8	0.655	0.073	0.156	0.134	0.03	82	0.524
		UWNA	0.132	0.221	0.174	0.066	77.4	0.685	0.084	0.157	0.138	0.032	79.8	0.541
		WNA	0.115	0.253	0.229	0.077	90.9	0.896	0.083	0.193	0.175	0.044	93	0.684
		ZhouEL	(The same as that of JinNA)				78.8	0.611	(The same as that of JinNA)				81.6	0.48
		UWJEL	(The same as that of UWNA)				83.9	0.67	(The same as that of UWNA)				86.6	0.536
		WJEL	(The same as that of WNA)				91.8	1.004	(The same as that of WNA)				95	0.788

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Table 4: Summary results of the simulation study 1 in the first scenario when $e \sim \chi^2(df=6)$ or $\chi^2(df=1)$ covariate and error are not contaminated. $cr(\%)$: censoring rate; censoring rate; Bias: bias of estimators for $\beta_0 = 2$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; $CP(\%)$: coverage probability coverage probability of 95% confidence intervals for β_0; LC: average length of confidence intervals; n: sample size. Results are based on 1000 simulation replicates.

			$\varepsilon = g(e;\sigma=1,\tau=0.5)$, $e \sim N(0,1)$						$\varepsilon = g(e;\sigma=1,\tau=0.5)$, $e \sim EV$					
n	**Cr**	**Method**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**
30	25	$QR_{0.5}$	0.014	0.279	0.324	0.078	91.8	1.269	0.008	0.185	0.207	0.034	92	0.81
		JinNA	0.008	0.201	0.201	0.04	93.3	0.788	0.002	0.114	0.127	0.013	97	0.497
		UWNA	0.018	0.202	0.19	0.041	91.5	0.746	0.009	0.13	0.117	0.017	94.1	0.46
		WNA	0.019	0.212	0.215	0.045	93.5	0.844	0.013	0.133	0.128	0.018	95.9	0.501
		ZhouEL	(The same as that of JinNA)				91.9	0.421	(The same as that of JinNA)				93.3	0.293
		UWJEL	(The same as that of UWNA)				92.8	0.823	(The same as that of UWNA)				92.8	0.592
		WJEL	(The same as that of WNA)				94.5	0.93	(The same as that of WNA)				94.9	0.646
	50	$QR_{0.5}$	0.04	0.424	0.468	0.181	90.9	1.834	0.012	0.317	0.311	0.101	91.6	1.218
		JinNA	0.02	0.265	0.255	0.07	92.9	0.998	-0.001	0.157	0.167	0.025	96.1	0.655
		UWNA	0.04	0.265	0.236	0.072	88.5	0.925	0.012	0.168	0.145	0.028	91.6	0.568
		WNA	0.046	0.281	0.269	0.081	92.5	1.056	0.019	0.173	0.159	0.03	94.4	0.625
		ZhouEL	(The same as that of JinNA)				89.1	0.909	(The same as that of JinNA)				88.3	0.542
		UWJEL	(The same as that of UWNA)				92.8	1.142	(The same as that of UWNA)				92.4	0.831
		WJEL	(The same as that of WNA)				94.4	1.232	(The same as that of WNA)				93.4	0.881
	75	$QR_{0.25}$	0.046	0.924	0.837	0.855	90.5	3.282	-0.006	0.854	0.504	0.728	92.3	1.976
		JinNA	0.049	0.538	0.427	0.291	88.2	1.673	0.001	0.358	0.311	0.128	92.6	1.218
		UWNA	0.079	0.529	0.338	0.285	80.7	1.326	0.017	0.357	0.239	0.128	83.5	0.937
		WNA	0.095	0.538	0.383	0.298	84.7	1.503	0.039	0.337	0.259	0.115	87.6	1.017
		ZhouEL	(The same as that of JinNA)				77.2	1.265	(The same as that of JinNA)				80.1	0.859
		UWJEL	(The same as that of UWNA)				91.5	2.098	(The same as that of UWNA)				92.1	1.562
		WJEL	(The same as that of WNA)				92	2.256	(The same as that of WNA)				94.1	1.654
100	25	$QR_{0.5}$	0.005	0.155	0.165	0.024	88.7	0.646	0	0.093	0.103	0.009	90.5	0.404
		JinNA	0.004	0.109	0.104	0.012	93.8	0.409	-0.001	0.043	0.049	0.002	99	0.193
		UWNA	0.009	0.11	0.104	0.012	93.5	0.407	0.003	0.055	0.055	0.003	96.5	0.214
		WNA	0.009	0.118	0.113	0.014	94.1	0.443	0.004	0.057	0.058	0.003	97.1	0.23
		ZhouEL	(The same as that of JinNA)				94.1	0.405	(The same as that of JinNA)				96.1	0.183
		UWJEL	(The same as that of UWNA)				94.1	0.413	(The same as that of UWNA)				95.3	0.225
		WJEL	(The same as that of WNA)				95	0.451	(The same as that of WNA)				96.2	0.236
	50	$QR_{0.5}$	0.009	0.214	0.233	0.046	88	0.912	0	0.137	0.413	0.019	87.8	0.56
		JinNA	0.001	0.133	0.129	0.018	93.6	0.506	-0.002	0.059	0.065	0.003	97.2	0.255
		UWNA	0.011	0.134	0.128	0.018	93.7	0.502	0.005	0.07	0.067	0.005	95	0.263
		WNA	0.015	0.142	0.141	0.02	93.9	0.552	0.008	0.073	0.072	0.005	96.2	0.283
		ZhouEL	(The same as that of JinNA)				86.4	0.513	(The same as that of JinNA)				93.1	0.24
		UWJEL	(The same as that of UWNA)				88.6	0.56	(The same as that of UWNA)				93.2	0.287
		WJEL	(The same as that of WNA)				95.9	0.671	(The same as that of WNA)				94.7	0.303
	75	$QR_{0.25}$	-0.002	0.263	0.286	0.069	85.4	1.122	-0.01	0.096	0.095	0.009	87.8	0.372
		JinNA	0.002	0.207	0.187	0.043	91	0.732	-0.007	0.097	0.109	0.009	97.3	0.426
		UWNA	0.109	0.209	0.182	0.044	91.2	0.715	0.003	0.107	0.103	0.011	95.4	0.402
		WNA	0.027	0.22	0.201	0.049	92.6	0.789	0.008	0.109	0.109	0.012	97.4	0.428
		ZhouEL	(The same as that of JinNA)				91.5	0.711	(The same as that of JinNA)				93.3	0.382
		UWJEL	(The same as that of UWNA)				91.9	0.768	(The same as that of UWNA)				94.9	0.483
		WJEL	(The same as that of WNA)				94	0.836	(The same as that of WNA)				96.3	0.507

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Table 5: Summary results of the simulation study 1 in the first scenario when $e \sim \chi^2(df=6)$ or $\chi^2(df=1)$ covariate and error are not contaminated. $cr(\%)$: censoring rate; censoring rate; Bias: bias of estimators for $\beta_0 = 2$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; $CP(\%)$: coverage probability coverage probability of 95% confidence intervals for β_0; LC: average length of confidence intervals; n: sample size. Results are based on 1000 simulation replicates.

			$\varepsilon = g(e;\sigma=1,\tau=0.5),\ e \sim N(0,1)$						$\varepsilon = g(e;\sigma=1,\tau=0.5),\ e \sim EV$					
n	**Cr**	**Method**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**	**Bias**	**SSD**	**AESD**	**MSE**	**CP**	**LC**
		$QR_{0.5}$	-0.008	0.22	0.249	0.048	84.6	0.974	-0.032	0.151	0.163	0.024	84.3	0.64
		JinNA	0.018	0.186	0.158	0.035	86.1	0.617	-0.014	0.117	0.11	0.014	89.7	0.431
		UWNA	0.021	0.186	0.157	0.035	77.3	0.615	-0.016	0.124	0.099	0.016	77.9	0.389
	25	WNA	0.027	0.197	0.198	0.04	93.8	0.776	0.014	0.128	0.12	0.017	96.6	0.469
		ZhouEL	(The same as that of JinNA)				80.8	0.521	(The same as that of JinNA)				79.6	0.312
		UWJEL	(The same as that of UWNA)				87.1	0.612	(The same as that of UWNA)				85.3	0.478
		WJEL	(The same as that of WNA)				95.9	0.917	(The same as that of WNA)				95.9	0.72
		$QR_{0.5}$	-0.012	0.283	0.315	0.08	83.8	1.236	-0.055	0.213	0.221	0.048	83.7	0.868
		JinNA	0.037	0.219	0.172	0.05	81.6	0.676	-0.015	0.137	0.123	0.019	88.8	0.483
		UWNA	0.044	0.219	0.158	0.05	72.2	0.619	-0.013	0.139	0.105	0.019	73.8	0.41
30	50	WNA	0.078	0.282	0.238	0.086	91.5	0.933	0.04	0.176	0.148	0.033	95.5	0.58
		ZhouEL	(The same as that of JinNA)				76.6	0.555	(The same as that of JinNA)				74.8	0.355
		UWJEL	(The same as that of UWNA)				84.3	0.802	(The same as that of UWNA)				87.2	0.609
		WJEL	(The same as that of WNA)				93.1	1.213	(The same as that of WNA)				94.8	0.988
		$QR_{0.25}$	0.086	0.388	0.453	0.158	86.7	1.776	-0.024	0.342	0.305	0.118	83.2	1.197
		JinNA	0.06	0.28	0.208	0.082	81	0.816	-0.027	0.229	0.151	0.053	86.1	0.595
		UWNA	0.078	0.28	0.164	0.084	65.6	0.644	-0.018	0.229	0.115	0.053	70.6	0.452
	75	WNA	0.151	0.375	0.297	0.164	86.6	1.164	0.068	0.276	0.196	0.081	91.5	0.767
		ZhouEL	(The same as that of JinNA)				72.1	0.626	(The same as that of JinNA)				69.4	0.429
		UWJEL	(The same as that of UWNA)				91.4	1.096	(The same as that of UWNA)				87.3	0.766
		WJEL	(The same as that of WNA)				93.9	1.441	(The same as that of WNA)				95.5	1.056
		$QR_{0.5}$	-0.008	0.135	0.133	0.018	78.7	0.522	-0.009	0.084	0.079	0.007	77.4	0.311
		JinNA	0.019	0.107	0.097	0.012	88.7	0.38	0.007	0.051	0.051	0.003	89.5	0.199
		UWNA	0.021	0.107	0.1	0.012	87.3	0.393	0.007	0.058	0.053	0.003	86	0.207
	25	WNA	0.011	0.111	0.106	0.012	94.1	0.416	0.007	0.056	0.056	0.003	96.3	0.221
		ZhouEL	(The same as that of JinNA)				88.2	0.349	(The same as that of JinNA)				86.8	0.165
		UWJEL	(The same as that of UWNA)				91.1	0.394	(The same as that of UWNA)				89.4	0.226
		WJEL	(The same as that of WNA)				94.6	0.423	(The same as that of WNA)				95.8	0.232
		$QR_{0.5}$	-0.018	0.174	0.159	0.031	74.1	0.621	-0.016	0.11	0.101	0.012	77.1	0.395
		JinNA	0.037	0.126	0.109	0.017	86.4	0.428	0.011	0.063	0.059	0.004	86.2	0.232
		UWNA	0.043	0.126	0.113	0.018	84.7	0.443	0.013	0.067	0.058	0.005	80.4	0.227
100	50	WNA	0.027	0.13	0.128	0.018	94	0.502	0.017	0.07	0.068	0.005	95.7	0.268
		ZhouEL	(The same as that of JinNA)				84.6	0.394	(The same as that of JinNA)				84.7	0.193
		UWJEL	(The same as that of UWNA)				87.5	0.438	(The same as that of UWNA)				88.4	0.255
		WJEL	(The same as that of WNA)				94.6	0.518	(The same as that of WNA)				95.1	0.301
		$QR_{0.25}$	0.075	0.181	0.146	0.039	63	0.573	0.024	0.073	0.057	0.006	55.6	0.222
		JinNA	0.075	0.155	0.127	0.029	80.8	0.5	0.016	0.081	0.074	0.007	84.5	0.289
		UWNA	0.085	0.153	0.129	0.031	76.6	0.505	0.021	0.082	0.069	0.007	77.4	0.269
	75	WNA	0.079	0.187	0.167	0.041	90.9	0.655	0.036	0.094	0.091	0.01	93.5	0.355
		ZhouEL	(The same as that of JinNA)				79.4	0.456	(The same as that of JinNA)				80.1	0.244
		UWJEL	(The same as that of UWNA)				85	0.513	(The same as that of UWNA)				86.6	0.315
		WJEL	(The same as that of WNA)				92.6	0.741	(The same as that of WNA)				95.4	0.483

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Simulation study 2: aft model with a two-dimensional co-variate

In this simulation study, we simulate the model with a two-dimensional covariate as follows:

$$\log(T) = \beta_0^{\tau} X + \varepsilon = \beta_{01} X_1 + \beta_{02} X_2 + \varepsilon,$$

Where $\beta_0 = (\beta_{01}, \beta_{02})^{\tau} = (2,-1)^{\tau}$. Similar to the one-dimensional simulation, this simulation setup also has two scenarios. We considered three different error distributions, i.e., $e \sim N(0,1)$, EV and $\chi^2(df=6)$, respectively. In the first scenario, X_{1i} and X_{2i} were generated from the standard normal distribution $N(0,1)$, X_{1i} and X_{2i} are independent, the error term ε was generated from $g(e;\sigma=1,\tau=0.5)$. In the second scenario, 95% of the covariates X_{1i} and X_{2i} were generated from the standard normal distribution $N(0,1)$, 95% of the error term ε values were generated from the other 5% of the X_{1i} and X_{2i} were generated from the normal distribution $N(-5,1)$, and 5% of the error term $g(e;\sigma=1,\tau=0.5)$. values were generated from All other information was the same as that in the one-dimensional case.

`Besides Bias, SSD and AESD, we also report the joint coverage probability (JCP) of 95% confidence regions for β_0 from JinNA, ZhouEL, NA and JEL methods respectively and marginal coverage probability (MCP) of 95% confidence intervals for each parameter component from all the methods. Again the results in Tables 6-11 show that in most cases, JEL has better performance than other methods. Especially when the censoring rate gets large and the sample size is small, QR was not stable and produced extraneous estimates, the JCP and MCP based on JEL is much closer to the nominal level than all the other methods. This again indicates that JEL has better inference precision than NA and other methods. When the covariates are contaminated, the weighted NA and JEL methods work better than the corresponding unweighted methods. The weighted methods perform better even when there is no contamination in the covariates. Hence, the weighted JEL method is the best choice in all the circumstances, and it is recommended for use in practice.

Table 6: Summary results of the simulation study 2 in the first scenario when covariates and error are not contaminated. cr: censoring rate; Bias: bias of estimators for SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; JCP: joint coverage probability of 95% confidence intervals for β_0; MCP: marginal coverage probability of 95% confidence intervals for each parameter component β_{0j}, j=1;2; n: sample size. Results are based on 1000 simulation replicates.

			Bias		SSD		AESD		MSE	JCP(%)	MCP(%)	
n	***Cr*(%)**	**Method**	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}	(β_{01},β_{02})	(β_{01},β_{02})	β_{01}	β_{02}
30	25	$QR_{0.5}$	0.014	-0.014	0.308	0.289	0.38	0.334	0.179	_	94.3	91.4
		JinNA	0.016	-0.014	0.257	0.236	0.244	0.224	0.122	87.4	92.3	90.8
		UWNA	0.029	-0.019	0.257	0.235	0.233	0.212	0.122	83.4	90.2	88.8
		WNA	0.025	-0.021	0.275	0.252	0.182	0.261	0.241	88.1	91.9	92.7
		ZhouEL	(The same as that of JinNA)							84.8	89.3	86.8
		UWJEL	(The same as that of UWNA)							88.6	91.4	90.9
		WJEL	(The same as that of WNA)							92.2	93.4	93.7
	50	$QR_{0.5}$	0.043	-0.041	0.619	0.427	0.56	0.456	0.568	_	94.2	92.5
		JinNA	0.042	-0.035	0.351	0.31	0.32	0.27	0.223	86.9	91.1	89.2
		UWNA	0.066	-0.047	0.354	0.311	0.297	0.253	0.228	79.4	88.1	86
		WNA	0.067	-0.049	0.372	0.326	0.341	0.294	0.251	85.9	91.4	90.5
		ZhouEL	(The same as that of JinNA)							77.5	83	80.3
		UWJEL	(The same as that of UWNA)							86.9	91.1	90.1
		WJEL	(The same as that of WNA)							90.5	92.9	92.7
	75	$QR_{0.25}$	0.106	0.062	3.601	2.318	1.185	0.923	18.335	_	89.5	89.5
		JinNA	0.013	-0.039	0.698	0.54	0.52	0.4	0.78	84.6	87	85.7
		UWNA	0.055	-0.061	0.707	0.545	0.415	0.326	0.803	70.4	82.2	81.5
		WNA	0.078	-0.068	0.732	0.568	0.493	0.402	0.869	78.9	87.8	86.5
		ZhouEL	(The same as that of JinNA)							58.5	61.9	63.6
		UWJEL	(The same as that of UWNA)							92.7	91.4	91.2
		WJEL	(The same as that of WNA)							93	92.1	92

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100	25	$QR_{0.5}$	0.009	-0.006	0.165	0.147	0.181	0.158	0.049	_	94.3	91.4
		JinNA	0.009	-0.006	0.129	0.124	0.128	0.116	0.032	92.5	90	90.5
		UWNA	0.015	-0.009	0.129	0.124	0.128	0.115	0.032	92	93.8	92.4
		WNA	0.016	-0.008	0.136	0.131	0.137	0.124	0.036	93.3	93.3	92.2
		ZhouEL	(The same as that of JinNA)							92.9	94.7	93.8
		UWJEL	(The same as that of UWNA)							93.4	94.3	92.7
		WJEL	(The same as that of WNA)							95	94.3	92.8
	50	$QR_{0.5}$	0.013	-0.007	0.231	0.183	0.26	0.21	0.087	_	94.2	92.5
		JinNA	0.011	-0.005	0.165	0.143	0.166	0.139	0.048	92.5	91	90.2
		UWNA	0.024	-0.012	0.166	0.144	0.165	0.138	0.049	91.7	94.1	93.5
		WNA	0.028	-0.014	0.174	0.152	0.178	0.151	0.054	94.3	93.2	93
		ZhouEL	(The same as that of JinNA)							93.1	95	94.5
		UWJEL	(The same as that of UWNA)							93.6	93.1	92.7
		WJEL	(The same as that of WNA)							95.3	94.3	94.1
	75	$QR_{0.25}$	-0.003	-0.004	0.34	0.259	0.406	0.304	0.183	_	88.3	89.1
		JinNA	0.018	-0.011	0.266	0.21	0.244	0.189	0.115	90.4	91.5	91.5
		UWNA	0.041	-0.023	0.269	0.213	0.241	0.188	0.119	88.8	91.3	90.6
		WNA	0.056	-0.029	0.278	0.226	0.267	0.21	0.133	91.4	93.3	92.8
		ZhouEL	(The same as that of JinNA)							86.7	87.8	88
		UWJEL	(The same as that of UWNA)							91.7	92.3	91.8
		WJEL	(The same as that of WNA)							93.4	94.1	93.8

Table 7: Summary results of the simulation study 2 in the second scenario when $\varepsilon = g(e;\sigma = 1, \tau = 0.5)$, $e \sim N(0,1)$, covariates and error are contaminated. cr: censoring rate; Bias: bias of estimators for $\beta_0 = (\beta_{01}\beta_{02})^T = (2,-1)^T \tau$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; JCP: joint coverage probability of 95% confidence intervals for β_0; MCP: marginal coverage probability of 95% confidence intervals for each parameter component β_{0j}, j = 1; 2; n: sample size. Results are based on 1000 simulation replicates.

			Bias		SSD		AESD		MSE	JCP(%)	MCP(%)	
n	***Cr*(%)**	**Method**	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}	***n***	***Cr*(%)**	**Method**	β_{01}

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30	25	$QR_{0.5}$	0.012	-0.02	0.297	0.265	0.346	0.318	0.159	_	93.7	92.7
		JinNA	0.027	-0.012	0.244	0.224	0.217	0.203	0.11	80.5	89.6	90.9
		UWNA	0.038	-0.021	0.244	0.223	0.222	0.211	0.111	69.2	88.2	88.1
		WNA	0.035	-0.02	0.262	0.243	0.253	0.232	0.129	89.6	92.8	91.9
		ZhouEL	(The same as that of JinNA)							72.7	75	71.7
		UWJEL	(The same as that of UWNA)							80.9	90.2	90.4
		WJEL	(The same as that of WNA)							91.2	94.1	93.2
	50	$QR_{0.5}$	0.053	-0.055	0.588	0.406	0.539	0.446	0.515	_	96.5	95.4
		JinNA	0.067	-0.03	0.342	0.288	0.287	0.25	0.205	77.9	89.7	90
		UWNA	0.095	-0.05	0.343	0.289	0.269	0.245	0.212	63	83.7	86.3
		WNA	0.108	-0.054	0.38	0.322	0.334	0.285	0.262	85.6	91.3	90.6
		ZhouEL	(The same as that of JinNA)							64.3	60.6	62
		UWJEL	(The same as that of UWNA)							80	88.1	87.3
		WJEL	(The same as that of WNA)							90.7	92.5	93
	75	$QR_{0.25}$	0.785	-0.539	20.851	15.302	1.489	1.212	669.094	_	90	89.9
		JinNA	0.077	-0.044	0.734	0.552	0.527	0.409	0.851	74.2	84.3	84.3
		UWNA	0.125	-0.079	0.742	0.545	0.403	0.33	0.87	52.8	77.5	78.9
		WNA	0.22	-0.104	0.883	0.608	0.513	0.409	1.207	77.8	85.4	85.3
		ZhouEL	(The same as that of JinNA)							47.2	39.6	39.9
		UWJEL	(The same as that of UWNA)							90	87.1	89
		WJEL	(The same as that of WNA)							94.5	91.2	93.8
100	25	$QR_{0.5}$	0.008	-0.0007		0.159	0.137	0.169	0.151	_	90.2	90.6
		JinNA	0.022	0		0.13	0.116	0.122	0.112	88.7	92.9	93.4
		UWNA	0.028	-0.004		0.13	0.116	0.125	0.114	87	92.3	93.4
		WNA	0.02	-0.008		0.132	0.123	0.133	0.12	93.8	94.5	94.1
		ZhouEL	(The same as that of JinNA)							88.4	90.4	91.9
		UWJEL	(The same as that of UWNA)							90.1	93.5	94.9
		WJEL	(The same as that of WNA)							95.2	94.8	94.5
	50	$QR_{0.5}$	0.007	-0.008		0.219	0.176	0.242	0.198	_	89.5	89.1
		JinNA	0.046	0.005		0.17	0.138	0.158	0.134	86.5	91.7	93.2
		UWNA	0.06	-0.005		0.171	0.138	0.16	0.135	84.5	90.6	93
		WNA	0.044	-0.012		0.171	0.145	0.175	0.147	94	94.2	95
		ZhouEL	(The same as that of JinNA)							86	86.6	89.7
		UWJEL	(The same as that of UWNA)							89.7	91.8	94.4
		WJEL	(The same as that of WNA)							94.8	95.4	95.2
	75	$QR_{0.25}$	0.057	0.034		0.332	0.25	0.393	0.297	_	88.5	87.9
		JinNA	0.095	0.001		0.285	0.213	0.236	0.186	81.3	89.8	91.3
		UWNA	0.066	-0.053		0.408	0.299	0.294	0.241	65.4	82.3	86.8
		WNA	0.106	-0.024		0.291	0.227	0.265	0.204	90.5	92	91.7
		ZhouEL	(The same as that of JinNA)							78.1	71.5	72.7
		UWJEL	(The same as that of UWNA)							83.8	89.3	92
		WJEL	(The same as that of WNA)							92.3	92.2	93.5

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Table 8: Summary results of the simulation study 2 in the second scenario when $\varepsilon = g(e;\sigma=1,\tau=0.5)$, $e \sim EV$, covariates and error are contaminated. cr: censoring rate; Bias: bias of estimators for $\beta_0=(\beta_{01}\beta_{02})^T$ $\tau=(2,-1)^\tau$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; JCP: joint coverage probability of 95% confidence intervals for β_0; MCP: marginal coverage probability of 95% confidence intervals for each parameter component β_{0j}, j = 1; 2; n: sample size. Results are based on 1000 simulation replicates.

			Bias		SSD		AESD		MSE	JCP(%)	MCP(%)	
n	*Cr*(%)	Method	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}	*n*	*Cr*(%)	Method	β_{01}
30	25	$QR_{0.5}$	0.015	0.009	0.285	0.247	0.336	0.294	0.143	_	94.3	95
		JinNA	0.013	0.008	0.223	0.197	0.211	0.191	0.089	90.7	92.9	92.7
		UWNA	0.021	0.004	0.223	0.198	0.199	0.179	0.089	86.1	90	91.1
		WNA	0.024	0.003	0.235	0.21	0.226	0.203	0.099	90.6	93.6	93.7
		ZhouEL	(The same as that of JinNA)							88.2	89	89.8
		UWJEL	(The same as that of UWNA)							90.5	92.1	93.3
		WJEL	(The same as that of WNA)							94.2	94.4	95.2
	50	$QR_{0.5}$	0.035	-0.011	0.416	0.375	0.49	0.402	0.341	_	93.6	93.3
		JinNA	0.023	0.003	0.285	0.237	0.27	0.228	0.138	90.3	91.7	92.5
		UWNA	0.042	-0.006	0.286	0.239	0.247	0.207	0.14	82.7	88.6	89.1
		WNA	0.046	-0.008	0.303	0.252	0.285	0.24	0.157	88.7	92.4	93.3
		ZhouEL	(The same as that of JinNA)							81.4	85.5	84.8
		UWJEL	(The same as that of UWNA)							90.3	93.1	92.5
		WJEL	(The same as that of WNA)							92.4	93.8	95.1
	75	$QR_{0.25}$	-0.033	0.029	1.432	0.848	0.906	0.664	2.767	_	87.8	88.6
		JinNA	-0.037	0.044	0.678	0.415	0.442	0.334	0.634	85.5	87	84.9
		UWNA	-0.01	0.027	0.679	0.418	0.344	0.265	0.635	69.4	81.8	80.6
		WNA	-0.001	0.021	0.698	0.429	0.392	0.319	0.671	77.3	85.6	86.5
		ZhouEL	(The same as that of JinNA)							59.5	64	63.3
		UWJEL	(The same as that of UWNA)							93.9	91.8	92.3
		WJEL	(The same as that of WNA)							93.7	92.3	92.7
100	25	$QR_{0.5}$	0	-0.006	0.144	0.133	0.158	0.142	0.039	_	91.4	92
		JinNA	-0.001	-0.004	0.106	0.104	0.107	0.099	0.022	92.6	94.6	93.6
		UWNA	0.004	-0.006	0.106	0.104	0.106	0.098	0.022	92.1	94.3	93.2
		WNA	0.006	-0.008	0.113	0.11	0.115	0.106	0.025	93	95.3	94.1
		ZhouEL	(The same as that of JinNA)							93.2	95.1	93.3
		UWJEL	(The same as that of UWNA)							94	95.3	94.1
		WJEL	(The same as that of WNA)							94.3	96	94.7
	50	$QR_{0.5}$	0.005	-0.005	0.203	0.171	0.221	0.183	0.07	_	89.5	90
		JinNA	-0.001	-0.004	0.135	0.122	0.133	0.114	0.033	91.3	94.1	92.6
		UWNA	0.009	-0.009	0.135	0.123	0.132	0.112	0.033	91	93.3	92
		WNA	0.014	-0.011	0.144	0.13	0.144	0.122	0.038	92.3	94.7	93.4
		ZhouEL	(The same as that of JinNA)							91.2	93.8	91.3
		UWJEL	(The same as that of UWNA)							92.6	93.5	92
		WJEL	(The same as that of WNA)							93.5	95.6	93.9
	75	$QR_{0.25}$	-0.002	0.003	0.273	0.207	0.306	0.228	0.117	_	88.7	88.6
		JinNA	-0.005	-0.003	0.2	0.164	0.195	0.151	0.067	90	91.7	91.7
		UWNA	0.011	-0.011	0.2	0.165	0.19	0.148	0.067	86.9	90.7	90.8
		WNA	0.022	-0.016	0.21	0.173	0.21	0.164	0.075	91.6	93.1	92.5
		ZhouEL	(The same as that of JinNA)							86.5	88.4	87.4
		UWJEL	(The same as that of UWNA)							93.1	93.2	92.4
		WJEL	(The same as that of WNA)							94.8	95.2	94.4

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Table 9: Summary results of the simulation study 2 in the second scenario when $\varepsilon = g(e;\sigma=1,\tau=0.5)$, $e \sim EV$, covariates and error are contaminated. cr: censoring rate; Bias: bias of estimators for $\beta_0=(\beta_{01}\beta_{02})^{\tau}=(2,-1)^{\tau}$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; JCP: joint coverage probability of 95% confidence intervals for β_0; MCP: marginal coverage probability of 95% confidence intervals for each parameter component β_{0j}, j = 1; 2; n: sample size. Results are based on 1000 simulation replicates.

			Bias		SSD		AESD		MSE	JCP(%)	MCP(%)	
n	***Cr*(%)**	**Method**	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}	***n***	***Cr*(%)**	**Method**	β_{01}
		$QR_{0.5}$	0.005	-0.015	0.273	0.232	0.313	0.284	0.128	_	95.9	94.2
		JinNA	0.012	0.002	0.205	0.187	0.19	0.177	0.077	83.7	92	91
		UWNA	0.02	-0.016	0.204	0.188	0.188	0.179	0.077	71.5	89	89.2
	25	WNA	0.028	-0.002	0.228	0.195	0.216	0.195	0.091	92.2	94.2	93.7
		ZhouEL	(The same as that of JinNA)							74.3	76.2	76.9
		UWJEL	(The same as that of UWNA)							81.6	92.8	91.5
		WJEL	(The same as that of WNA)							93.6	95.6	95.2
		$QR_{0.5}$	0.032	-0.042	0.414	0.364	0.468	0.388	0.306	_	94.3	94.2
		JinNA	0.032	-0.005	0.275	0.247	0.25	0.219	0.138	80.6	91.7	89.2
		UWNA	0.053	-0.023	0.279	0.252	0.229	0.206	0.144	66.1	86	85.9
30	50	WNA	0.066	-0.018	0.302	0.249	0.276	0.233	0.158	88.6	91.2	92.1
		ZhouEL	(The same as that of JinNA)							68	65.6	65.4
		UWJEL	(The same as that of UWNA)							83.9	91.7	89.1
		WJEL	(The same as that of WNA)							93.2	94.2	94.5
		$QR_{0.25}$	0.044	-0.014	1.603	1.358	0.905	0.666	4.412	_	89.2	87.9
		JinNA	0.038	-0.026	0.671	0.496	0.465	0.36	0.697	75.6	84.9	84
		UWNA	0.076	-0.053	0.672	0.493	0.342	0.281	0.703	53.7	76.2	75.6
	75	WNA	0.14	-0.049	0.776	0.502	0.421	0.334	0.874	78.1	83	85.7
		ZhouEL	(The same as that of JinNA)							48.1	37	40.9
		UWJEL	(The same as that of UWNA)							94	90.2	90.7
		WJEL	(The same as that of WNA)							95.8	93	94.2
		$QR_{0.5}$	0	-0.004	0.133	0.127	0.148	0.134	0.034	_	92.3	90.1
		JinNA	0.012	0.003	0.106	0.101	0.102	0.096	0.021	89.9	92.9	92.1
		UWNA	0.017	-0.001	0.106	0.101	0.103	0.096	0.022	86.7	92.2	91.9
	25	WNA	0.01	-0.006	0.11	0.106	0.112	0.102	0.023	93.8	94.5	94.4
		ZhouEL	(The same as that of JinNA)							88.6	91.3	91.3
		UWJEL	(The same as that of UWNA)							90.4	93.6	93.9
		WJEL	(The same as that of WNA)							95.2	95.5	95.3
		$QR_{0.5}$	0	-0.008	0.193	0.163	0.215	0.18	0.064	_	91.9	90.2
		JinNA	0.027	0.006	0.138	0.116	0.127	0.111	0.033	85.9	91.8	92.6
		UWNA	0.038	-0.002	0.138	0.117	0.128	0.111	0.034	83.3	90	92.7
100	50	WNA	0.026	-0.008	0.142	0.124	0.14	0.119	0.036	92.1	95.3	93.7
		ZhouEL	(The same as that of JinNA)							84.9	86.5	89.1
		UWJEL	(The same as that of UWNA)							87.9	91.9	93.6
		WJEL	(The same as that of WNA)							93.8	95	94.5
		$QR_{0.25}$	0.034	0.028	0.256	0.195	0.299	0.224	0.105	_	88.6	87.4
		JinNA	0.053	0.008	0.213	0.165	0.191	0.153	0.075	81.8	91.7	91.7
		UWNA	0.073	-0.006	0.213	0.165	0.189	0.151	0.078	77.5	89.4	91.1
	75	WNA	0.058	-0.013	0.209	0.164	0.207	0.16	0.074	91.6	93.4	92.1
		ZhouEL	(The same as that of JinNA)							77.7	75.5	76.5
		UWJEL	(The same as that of UWNA)							85.8	91.6	92.4
		WJEL	(The same as that of WNA)							93.7	94.6	94.1

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Table 10: Summary results of the simulation study 2 in the second scenario when $\varepsilon = g(e;\sigma = 1, \tau = 0.5)$, $e \sim \chi^2(df = 6)$, covariates and error are contaminated. cr: censoring rate; Bias: bias of estimators for $\beta_0 = (\beta_{01}\beta_{02})^\tau = (2,-1)^\tau$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; JCP: joint coverage probability of 95% confidence intervals for β_0; MCP: marginal coverage probability of 95% confidence intervals for each parameter component β_{0j}, j = 1; 2; n: sample size. Results are based on 1000 simulation replicates.

			Bias		SSD		AESD		MSE	JCP(%)	MCP(%)	
n	***Cr*(%)**	**Method**	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}	***n***	***Cr*(%)**	**Method**	β_{01}
30	25	$QR_{0.5}$	0.016	-0.022	0.28	0.256	0.329	0.29	0.145	_	93.4	92.6
		JinNA	0.003	-0.01	0.207	0.197	0.205	0.19	0.082	90.4	93.8	92.3
		UWNA	0.014	-0.013	0.207	0.196	0.192	0.178	0.082	86.4	91	89.9
		WNA	0.019	-0.015	0.22	0.205	0.218	0.201	0.091	90.3	93.8	91.9
		ZhouEL	(The same as that of JinNA)							88	89.9	88.1
		UWJEL	(The same as that of UWNA)							90.8	92.7	92.9
		WJEL	(The same as that of WNA)							93.6	94.6	94.1
	50	$QR_{0.5}$	0.029	-0.022	0.41	0.347	0.487	0.395	0.29	_	94.1	93
		JinNA	0.017	-0.008	0.259	0.229	0.261	0.222	0.12	90.3	93.5	91.6
		UWNA	0.035	-0.018	0.26	0.232	0.234	0.202	0.123	82.6	89.1	87.5
		WNA	0.047	-0.023	0.278	0.248	0.271	0.235	0.142	88.2	92.3	91.8
		ZhouEL	(The same as that of JinNA)							81.4	84.3	83
		UWJEL	(The same as that of UWNA)							91.1	93.7	91.6
		WJEL	(The same as that of WNA)							92.6	95	94.1
	75	$QR_{0.25}$	-0.041	0.016	0.891	0.815	0.862	0.677	1.46	_	88.1	88.3
		JinNA	-0.016	-0.001	0.63	0.458	0.464	0.351	0.606	85.5	87.3	86.2
		UWNA	0.011	-0.014	0.623	0.461	0.337	0.262	0.6	67.8	79.8	80.3
		WNA	0.025	-0.017	0.653	0.502	0.401	0.325	0.68	77.8	86	87.6
		ZhouEL	(The same as that of JinNA)							60.1	65.2	63.3
		UWJEL	(The same as that of UWNA)							95.8	92.8	93.3
		WJEL	(The same as that of WNA)							95.3	93.7	93.2
100	25	$QR_{0.5}$	0.003	0.003	0.148	0.135	0.167	0.147	0.04	_	91.9	90.8
		JinNA	0	0.002	0.106	0.098	0.105	0.097	0.021	93.1	94.7	94.2
		UWNA	0.005	-0.001	0.107	0.099	0.104	0.096	0.021	92.9	94.1	94.4
		WNA	0.007	-0.004	0.113	0.105	0.113	0.104	0.024	94.4	95.4	94
		ZhouEL	(The same as that of JinNA)							93.6	94.5	94.4
		UWJEL	(The same as that of UWNA)							94.5	94.9	94.6
		WJEL	(The same as that of WNA)							94.8	95.3	95.7
	50	$QR_{0.5}$	0.002	0.002	0.213	0.171	0.233	0.193	0.075	_	89.4	90.7
		JinNA	-0.004	0.003	0.133	0.111	0.129	0.11	0.03	92.7	93.4	93.2
		UWNA	0.006	-0.002	0.133	0.112	0.128	0.108	0.03	91.8	92.9	93.4
		WNA	0.009	-0.005	0.143	0.12	0.14	0.119	0.035	93.7	94.1	94.1
		ZhouEL	(The same as that of JinNA)							92.2	93.1	93.4
		UWJEL	(The same as that of UWNA)							93.6	93.8	94
		WJEL	(The same as that of WNA)							95.1	94.5	95.6
	75	$QR_{0.25}$	-0.001	0	0.255	0.187	0.283	0.214	0.1	_	88.2	88.5
		JinNA	0	0.001	0.198	0.149	0.188	0.145	0.061	90.8	92.7	92.7
		UWNA	0.016	-0.007	0.199	0.149	0.182	0.141	0.062	88	90.3	91.4
		WNA	0.023	-0.011	0.212	0.159	0.203	0.156	0.071	91.8	92.7	94.2
		ZhouEL	(The same as that of JinNA)							88.1	88.8	89.4
		UWJEL	(The same as that of UWNA)							93.7	93.9	93.8
		WJEL	(The same as that of WNA)							95.1	94.9	94.8

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Table 11: Summary results of the simulation study 2 in the second scenario when $\varepsilon = g(e;\sigma=1,\tau=0.5)$, $e \sim \chi^2(df=6)$, covariates and error are contaminated. cr: censoring rate; Bias: bias of estimators for $\beta_0=(\beta_{01}\beta_{02})^\tau=(2,-1)^\tau$; SSD: sample standard deviation of estimators; AESD: average estimated standard deviation of estimators; MSE: mean squared error of estimators; JCP: joint coverage probability of 95% confidence intervals for β_0; MCP: marginal coverage probability of 95% confidence intervals for each parameter component β_{0j}, j = 1; 2; n: sample size. Results are based on 1000 simulation replicates.

			Bias		SSD		AESD		MSE	JCP(%)	MCP(%)	
n	*Cr*(%)	Method	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}	*n*	*Cr*(%)	Method	β_{01}
30	25	$QR_{0.5}$	0.009	-0.021	0.254	0.234	0.306	0.278	0.12	_	93.6	95.1
		JinNA	0.013	-0.004	0.195	0.183	0.184	0.173	0.072	83.7	92.5	92.3
		UWNA	0.022	-0.013	0.196	0.183	0.183	0.173	0.072	71.8	87.2	90.4
		WNA	0.025	-0.017	0.215	0.197	0.21	0.193	0.086	90.8	93.3	93.4
		ZhouEL	(The same as that of JinNA)							76.7	77.9	76.5
		UWJEL	(The same as that of UWNA)							84.4	91.6	91.8
		WJEL	(The same as that of WNA)							94.2	94.3	95.4
	50	$QR_{0.5}$	0.017	-0.028	0.435	0.34	0.451	0.374	0.306	_	94.8	94.2
		JinNA	0.032	-0.006	0.259	0.228	0.237	0.209	0.12	79.4	91.4	91
		UWNA	0.05	-0.022	0.262	0.229	0.215	0.196	0.124	65	87.2	87.2
		WNA	0.064	-0.026	0.288	0.248	0.263	0.225	0.149	88.4	92.3	90.9
		ZhouEL	(The same as that of JinNA)							70.8	67.2	67
		UWJEL	(The same as that of UWNA)							85.9	91.2	88.9
		WJEL	(The same as that of WNA)							93.3	95.5	94.7
	75	$QR_{0.25}$	-0.02	0.053	0.96	0.671	0.911	0.663	1.373	_	88.6	89.8
		JinNA	0.011	-0.004	0.636	0.445	0.444	0.346	0.602	75.9	84.2	84
		UWNA	0.046	-0.029	0.645	0.452	0.327	0.269	0.622	53.1	78.4	77.5
		WNA	0.079	-0.023	0.672	0.466	0.403	0.32	0.675	80.9	85.8	85.1
		ZhouEL	(The same as that of JinNA)							50.9	40.4	41.9
		UWJEL	(The same as that of UWNA)							93.8	91.4	90.4
		WJEL	(The same as that of WNA)							95.7	92.8	93.7
100	25	$QR_{0.5}$	-0.005	-0.005	0.148	0.129	0.157	0.14	0.038	_	90.9	90.2
		JinNA	0.008	0.004	0.11	0.098	0.102	0.094	0.022	87.2	92.8	93.6
		UWNA	0.013	0	0.11	0.098	0.103	0.095	0.022	85.6	92.6	92.5
		WNA	0.011	-0.002	0.109	0.1	0.11	0.1	0.022	94.1	94.6	95.1
		ZhouEL	(The same as that of JinNA)							88	90.6	91.9
		UWJEL	(The same as that of UWNA)							89.3	93.4	94.6
		WJEL	(The same as that of WNA)							94.7	95.2	95.8
	50	$QR_{0.5}$	-0.009	-0.009	0.207	0.164	0.222	0.182	0.07	_	90.3	90.2
		JinNA	0.02	0.007	0.136	0.113	0.125	0.109	0.032	86	92	92.3
		UWNA	0.03	0	0.136	0.113	0.126	0.109	0.032	82.9	91	91.9
		WNA	0.021	-0.003	0.141	0.116	0.137	0.115	0.034	93	94.6	94.8
		ZhouEL	(The same as that of JinNA)							86	89.5	89.3
		UWJEL	(The same as that of UWNA)							89.5	93.3	93.3
		WJEL	(The same as that of WNA)							94	95.2	95.4
	75	$QR_{0.25}$	0.032	0.024	0.258	0.191	0.282	0.211	0.105	_	88.9	88.2
		JinNA	0.045	0.007	0.211	0.164	0.186	0.149	0.074	82.4	91.4	90.4
		UWNA	0.065	-0.007	0.212	0.165	0.182	0.146	0.076	75.6	88.7	88.8
		WNA	0.054	-0.004	0.215	0.157	0.201	0.153	0.074	91.2	93	93.5
		ZhouEL	(The same as that of JinNA)							78.9	77.1	75.4
		UWJEL	(The same as that of UWNA)							86.5	91.6	92.8
		WJEL	(The same as that of WNA)							93.5	94.5	94.9

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Application to real data analysis

In this section, two real data sets are used to illustrate our method and to compare with other methods in the literature. The data sets include the Stanford Heart Transplant Data and the Multiple Myeloma Data. Previous authors have analyzed these data sets in their work, for example, Jin et al. (2003) and Zhou (2005), but they didn't take outlying covariate values into consideration. Following their analyses, we consider a single continuous covariate in the first data set and two continuous covariates in the second data set. We first demonstrate that outlying covariate values exist in these data sets, and then we apply the proposed weighted JEL method for analysis.

Stanford heart transplant data

The Stanford Heart Transplant Data can be found in Miller & Halpern [16], and is obtained by using attach(stanford2) inside the R survival package. In short, the Stanford heart transplant program began in October 1967 and a total of 184 patients received heart transplants. The information contained in the data set include: survival time in days; an indicator of whether the patient was dead or alive by February 1980; the age of the patient in years at the time of transplant; and the T5 mismatch score, which makes a distinction between deaths primarily due to rejection of the donor heart by the recipient's immune system and non-rejection related deaths. For 27 of the 184 transplant patients, the T5 mismatch scores were missing because the tissue typing was never completed. Following Miller & Halpern [16] suggestion, the five patients with survival times less than 10 days were deleted in order to compare our new methods with existing methods. In the end, there were 152 cases with a complete data record, which we will use to fit the following

model: $\log_{10}(T_i) = \beta X_i + \varepsilon_i$,

Where, T_i is the survival time and X_i is the age of the i^{th} patient at their heart transplant. In this data set, the censoring rate is cr = 36% with 55 people still alive at the end of the observation period and 97 deceased individuals. At first, we use box plots to check the outliers of the observed response Y_i and covariate X_i. Figure 1 clearly shows that there are some outliers (small values) of the ages of the patients, therefore, the weighted JEL is desirable. The results from the fitted model based on weighted approaches are: $\hat{\beta} = -0.0537$ with $se(\hat{\beta}) = 0.0171$. The 95% WJEL and WNA confidence intervals for β are (-0.0890,-0.0210) with length=0.068 and (-0.0872,-0.0202) with length=0.067, respectively. They are very close except for a lightly longer length by JEL. Both confidence intervals indicate a significant negative association between age and survival time in this patient population.

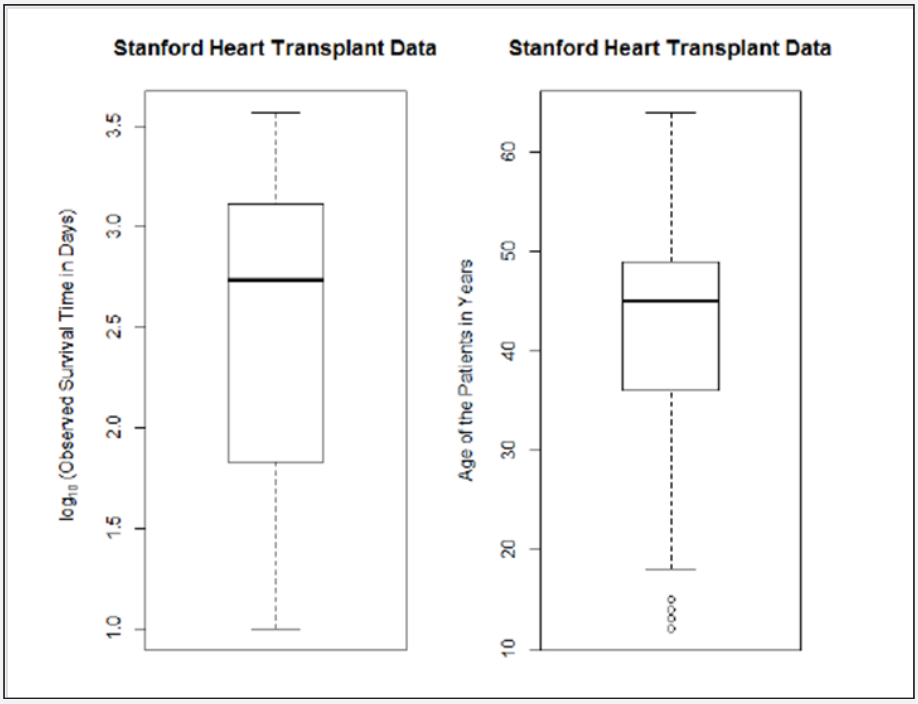

Figure 1 : Box Plots of the Stanford Heart Transplant Data.

When we ignore the outliers and use the unweighted methods, we obtain $\hat{\beta} = -0.0255$ with $se(\hat{\beta}) = 0.0171$. The 95% UWJEL confidence interval for β (-0.0436,-0.0035) and 95% UWNA confidence interval for β is (-0.0465,-0.0044). These results are quite different from those based on the weighted methods, however, they are very similar to those of Zhou [10] which show that $\hat{\beta} = -0.0253$ with $se(\hat{\beta}) = 0.0107$. obtained from the resampling method, the 95% confidence interval for β based on the censored empirical likelihood ratio is (-0.0446,-0.0030) and the 95% Wald confidence interval is (-0.0462,-0.0044). This is not surprising, since Zhou's method does not take the outliers into consideration and is an unweighted method.

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Multiple myeloma data

The Multiple Myeloma Data were reported by Krall et al. [17], and can be obtained from SAS [18]. The data set contains information for survival times and nine covariates: survival time, censoring status, logarithm of blood urea nitrogen (LogBUN), haemoglobin (HGB), Platelet, Age, LogWBC, Frac, LogPBM, Protein, SCalc. Out of total of 65 observations, 17 were censored, and the censoring rate is cr=26%. Following Jin et al. [5], we fit an AFT model with these two covariates: Log(BUN) and HGB. To improve numerical efficiencies, we also standardize the original covariates to have zero mean and unit variance. The fitted model is

$$\log(T_i) = \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i,$$

Where X_{1i} is the standardized score of Log(BUN), and X_{2i} is the standardized score of HGB.

From Figure 2 we notice that the standardized scores of Log(BUN) have outliers (large values). Using the weighted methods, we obtain weighted NA estimates of the regression coefficients, $(\hat{\beta}_1, \hat{\beta}_2,) = (-0.4622, 0.2714)$, with estimated standard errors (0.1626, 0.1753), respectively. The chi-square test statistics with 2 degrees of freedom for testing $(\beta_1, \beta_2,) = (0,0)$ based on the WJEL method and the WNA method are 11.34 and 11.21, and the corresponding p-values are 0.003 and 0.004, respectively, both indicating a jointly significant effect of Log(BUN) and HGB on survival time. The estimates obtained from the unweighted methods are $(\hat{\beta}_1, \hat{\beta}_2,) = (-0.5142, 0.2839)$, with estimated standard errors (0.1399, 0.1732). The chi-square test statistics with 2 degrees of freedom for testing $(\beta_1, \beta_2,) = (0,0)$ based on the UWJEL method and the UWNA method are 16.909 and 16.666, and the corresponding p-values are 0.00021 and 0.00024, respectively. The estimated regression coefficients are similar to Jin et al. (2003)'s results, which did not consider outliers and weights: their estimates were (-0.532, 0.292) with estimated standard errors (0.146, 0.169). These results once again substantiate our claim that when covariates have outliers (e.g. the standardized score of LogBUN), the rank estimates of regression coefficients under the weighted and unweighted methods may be quite different [19-21].

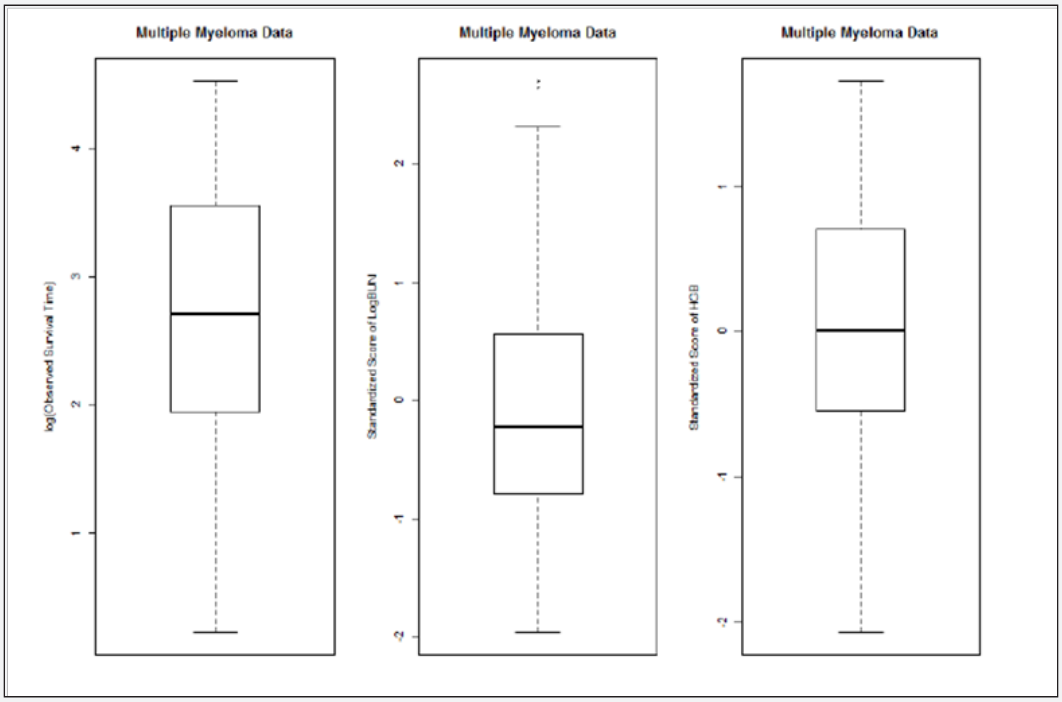

Figure 2 : Box Plots of the Multiple Myeloma Data.

Conclusion and Discussion

In this paper, we have developed the smoothed JEL method, a new inference method for the regression parameters in the AFT model with right censored data containing outlying response or covariate values. Based on the weighted smoothed rank estimation function proposed by Heller [2], jackknife and empirical likelihood are integrated to yield the new method. The proposed smoothed JEL method preserves not only important features of empirical likelihood, but also the double robustness of the weighted smoothed rank estimation. Another advantage of the new method is that it can be easily implemented in a standard software environment and used by practitioners, for example, the R package such as emplik already exists to maximize the empirical likelihood functions and to obtain the values of the test statistics. Simulation studies indicate that the smoothed JEL method outperforms the NA and other competitors in the sense of improving accuracy of inferences for regression parameters. This is especially evident when sample size is small or censoring rate is high. Two real data sets with outlying covariate values

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are reanalyzed using the new method and the results show superiority of the new method over those in the literature. Therefore, in practice, when the AFT model is used for survival data analysis, especially when there are outlying observations in either responses or covariates, the proposed method deserves first consideration. Our method can also be extended to other types of rank regression models, we leave this for future research.

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