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On A Boundary Value Problem for A Singularly Perturbed Differential Equation of Non-Classical Type

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Abstract

In a semi-infinite strip we consider a boundary value problem for a non-classical type equation of third order degenerating into a hyperbolic equation. The asymptotic expansion of the problem under consideration is construction is constructed in a small parameter to within any positive degree of a small parameter.

Introduction

Boundary value problems for non-classical singularly perturbed differential equations were not studied enough. We can show the papers devoted to construction of asymptotic solutions to some boundary value problems for non-classical type differential equations [1-3].

In this note in the infinite semi-strip $\Pi=\{(x,y)|0\le x\le 1, 0\le y<+\infty\}$ we consider the following boundary value problem

$$\varepsilon^2\frac{\partial}{\partial x}(\Delta u)-\varepsilon\Delta u+\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+u=f(x,y) \quad (1)$$

$$u|_{x=0}=u|_{x=1}=0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=1}=0, \qquad (0\le y<+\infty) \quad (2)$$

$$u|_{y=0}=0, \quad \lim_{y\to+\infty}u=0, \qquad (0\le x\le 1) \quad (3)$$

Where $\varepsilon>0$ is a small parameter, $\Delta\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $f(t,x)$ is the given function.

The goal of the work is to construct the complete asymptotics in a small parameter of the solution of problem (1)-(3). When constructing the asymptotics we follow the M.I. Vishik LA & Lusternik [4] technique.

The following theorem is proved.

Theorem

Let $f(x,y)\in C^{2n+3}(\Pi)$, and the following conditions be fulfilled:

$$\left.\frac{\partial^k f(x,y)}{\partial x^{k_1}\partial y^{k_2}}\right|_{x=y}=0; \quad \left|\frac{\partial^k f(x,y)}{\partial x^{k_1}\partial y^{k_2}}\right|\le ce^{-\gamma y}; \quad k=k_1+k_2; \quad k=0,1,\ldots,2n+3$$

$C>0$, $\gamma>0$. Then for the solution of boundary value problem (1)-(3) the following asymptotic representation is valid:

$$U=\sum_{i=0}^{n}\varepsilon^i W_i+\sum_{j=0}^{n+1}\varepsilon^j V_j+\varepsilon^{n+1}z,$$

where the functions W_i are determined by the first iterative process, V_j are the boundary layer type functions near the boundary x=1 determined by the second iterative process, ε^{n+1} Z is a remainder term, and for Z we have the estimation

$$\varepsilon^2\int_0^{+\infty}\left(\left.\frac{\partial z}{\partial x}\right|_{x=0}\right)^2 dy+\varepsilon\iint_\Pi\left[\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right]dxdy+C_1\iint_\Pi z^2dxdy\le C_2$$

Where $c_1>0$, $c_2>0$ are the constants independent of ε.

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