On A Boundary Value Problem for A Singularly Perturbed Differential Equation of Non-Classical Type

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Submission: February 27, 2018; Published: April 04, 2018

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Abstract

In a semi-infinite strip we consider a boundary value problem for a non-classical type equation of third order degenerating into a hyperbolic equation. The asymptotic expansion of the problem under consideration is constructed in a small parameter to within any positive degree of a small parameter.

Introduction

Boundary value problems for non-classical singularly perturbed differential equations were not studied enough. We can show the papers devoted to construction of asymptotic solutions to some boundary value problems for non-classical type differential equations [1-3].

In this note in the infinite semi-strip \(\Pi = \{(x, y) | 0 \leq x \leq 1, 0 \leq y < +\infty\}\) we consider the following boundary value problem

\[
\varepsilon^{-2} \frac{\partial^2}{\partial x^2} (\Delta u) - \varepsilon \Delta u + \varepsilon \frac{\partial u}{\partial y} + u = f(x, y)
\]

(1)

\[
u \bigg|_{x=1} = 0, \quad \varepsilon \frac{\partial u}{\partial y} \bigg|_{x=1} = 0, \quad (0 \leq y < +\infty)
\]

(2)

\[
u \bigg|_{x=0} = 0, \quad \lim_{y \to +\infty} u \bigg|_{x=1} = 0, \quad (0 \leq x \leq 1)
\]

(3)

Where \(\varepsilon > 0\) is a small parameter, \(\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\), \(f(t, x)\) is the given function.

The goal of the work is to construct the complete asymptotics in a small parameter of the solution of problem (1)-(3). When constructing the asymptotics we follow the M.I. Vishik L.A & Lusternik [4] technique.

The following theorem is proved.

Theorem

Let \(f(x, y) \in C^{n+1} (\Pi)\), and the following conditions be fulfilled:

\[
\varepsilon^i \frac{\partial^i f(x, y)}{\partial x^i \partial y^j} \bigg|_{x=1} = 0, \quad \varepsilon^i \frac{\partial^i f(x, y)}{\partial x^i \partial y^j} \bigg|_{x=0} \leq c \varepsilon^{-\gamma}, \quad k = k_1 + k_2, \quad k = 0, 1, ..., 2n + 3
\]

\(C_0 > 0, \gamma > 0\). Then for the solution of boundary value problem (1)-(3) the following asymptotic representation is valid:

\[
U = \sum_{i=0}^{n} \varepsilon^i W_i + \sum_{j=0}^{n+1} \varepsilon^j V_j + \varepsilon^{n+1} Z,
\]

where the functions \(W_i\) are determined by the first iterative process, \(V_j\) are the boundary layer type functions near the boundary \(x = 1\) determined by the second iterative process, \(Z\) is a remainder term, and for \(Z\) we have the estimation

\[
\varepsilon^2 \int_0^1 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f(t, x) \, dt \leq c \int_0^1 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f(t, x) \, dt + C_2 \int z^2 \, dx dy \leq C_2
\]

Where \(c_i > 0, c_i \geq 0\) are the constants independent of \(\varepsilon\).

References


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DOI: 10.19080/BBOAJ.2018.06.555676