



# Improved Family of Ratio Estimators of Finite Population Variance in Stratified Random Sampling



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## Abstract

This paper introduces a new family of exponential ratio estimators of population variance in stratified random sampling and studies its properties. Based on Bahl & Tuteja [1], Kadillar & Cingi [2] and Solanki et al. [3], membership of the new family of estimators is identified. Analytical and numerical results show that under certain prescribed conditions, the new estimator has equal optimal efficiency with the regression estimator of population variance but always fares better than the classical ratio estimator of population variance by Isaki [4] and every identified existing estimator of its family.

**Keywords:** Optimum estimator; Large sample approximation; Optimal efficiency; Regression estimator

## Introduction

In sampling theory population information of the auxiliary variable such as the total, mean and variance are often used to provide additional information which help to increase the efficiency of the estimation of the population parameter(s) of interest. When the information on an auxiliary variable is known, one can use the ratio, product and regression estimators to improve the performance of the estimator of the study variable. When the correlation between the study variable and the auxiliary variable is positive, ratio method of estimation is quite effective. Many authors like [5-19] among others have proposed different ratio estimators (of total or mean) in sample surveys.

The estimation of the finite population variance has been of great significance in various fields such as Industry, Agriculture, Medical and Biological sciences. Various authors such as [4,20-35] have used auxiliary information to improve the efficiency of the estimator of population variance of the study variable. In this paper, the problem of constructing efficient estimator of the population variance is considered and a new family of exponential ratio estimators of population variance in stratified random sampling that is as efficient as the general regression estimator is introduced.

## Basic notations and definitions

Consider a finite population  $\Pi = (\pi_1, \pi_2, \dots, \pi_N)$  of size  $(N)$ . Let  $(X)$  and  $(Y)$  denote the auxiliary and study variables taking values  $X_i$  and  $Y_i$  respectively on the  $i$ -th unit  $\pi_i (i=1, 2, \dots, N)$  of the population. Let the population be divided into  $K$  strata with  $N_k$  units in the  $k^{th}$  stratum from which a simple random sample

of size  $n_k$  is taken without replacement. The total population size be  $N = \sum_{k=1}^K N_k$  and the sample size  $n = \sum_{k=1}^K n_k$ , respectively. Associated with the  $i$ th element of the  $h^{th}$  stratum are  $y_{hi}$  and  $x_{hi}$  with  $x_{hi} > 0$  being the covariate; where  $y_{hi}$  is the  $y$  value of the  $i$ th element in stratum  $k$ , and  $x_{hi}$  is the  $x$  value of the  $i$ th element in stratum  $k, k=1, 2, \dots, K$ , and  $i=1, 2, \dots, N_k$ . For the  $k^{th}$  stratum, let  $w_k = N_k/N$  be the stratum weights and  $f_k = n_k/N_k$  the sample fraction. Let the  $k^{th}$  stratum means of the study variable  $Y$  and auxiliary variable  $X$  ( $\bar{y}_k = \sum_{i=1}^{N_k} y_{ki}/N_k; \bar{x}_k = \sum_{i=1}^{N_k} x_{ki}/N_k$ ) be the unbiased estimator of the population stratum means ( $\bar{Y}_k = \sum_{i=1}^{N_k} y_{ki}/N_k; \bar{X}_k = \sum_{i=1}^{N_k} x_{ki}/N_k$ ) of  $Y$  and  $X$  respectively, based on  $n_k$  observations.

Let,

$$s_{ky}^2 = S_{ky}^2 (1 + e_{ky}), \quad s_{kx}^2 = S_{kx}^2 (1 + e_{kx})$$

So that,

$$E(e_{kx}) = E(e_{ky}) = 0, \quad E(e_{kx}^2) = \gamma_k [\beta_2(x) - 1], \quad E(e_{ky}^2) = \gamma_k (\lambda_{22} - 1),$$

Where,

$$\gamma_k = \left( \frac{1 - f_k}{n_k} \right) = \left( \frac{1}{n_k} - \frac{1}{N_k} \right); S_{kx}^2 = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (x_{ki} - \bar{X}_k)^2; S_{ky}^2 = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (y_{ki} - \bar{Y}_k)^2$$

$$S_{kxy} = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (x_{ki} - \bar{X}_k)(y_{ki} - \bar{Y}_k),$$

$$\beta_{2k}(y) = \left[ \mu_{k40}(y, y) / \mu_{k20}^2(y, y) \right],$$

$$\beta_{2k}(x) = \left[ \mu_{k40}(x, x) / \mu_{k20}^2(x, x) \right],$$

$$\lambda_{yx} = \left[ \mu_{k22}(y, x) / \mu_{k20}(y, x) \right],$$

$$\mu_{krs}(y, x) = N_k^{-1} \sum_{i=1}^{N_k} (y_{ki} - \bar{Y}_{ki})^r (x_{ki} - \bar{X}_{ki})^s,$$

$$\dot{U} = \left[ \frac{(\lambda_{22} - 1)}{[\beta_2(x) - 1]} \right]$$

$$\rho_k = (\lambda_{22} - 1) \left( [\beta_{2k}(x) - 1][\beta_{2k}(y) - 1] \right)^{\frac{1}{2}}$$

And,

$$\psi = \frac{S_{ky}^2}{S_{kx}^2}$$

### The suggested estimator

The suggested exponential ratio estimator of population variance is given by:

$$S_{pr}^2 = \sum_{k=1}^K W_k S_{ky}^2 \left\{ \tau_k - \alpha_k \left( \frac{S_{kx}^2}{S_{ky}^2} \right)^{\eta_k} \exp \left[ \frac{\omega_k (S_{kx}^2 - S_{ky}^2)}{(S_{kx}^2 + S_{ky}^2)} \right] \right\} \quad (1)$$

Where  $\alpha_k, \tau_k$  and  $\eta_k$  are suitably chosen scalars such that  $\alpha_k$  and  $\tau_k$  satisfies the condition  $\tau_k = 1 + \alpha_k; -\infty \leq \alpha_k \leq \infty$ . Expressing (1) in terms of the  $e$ 's gives

$$S_{pr}^2 = \sum_{k=1}^K W_k S_{ky}^2 \left( 1 + e_{kSy} \right) \left\{ \tau_k - \alpha_k \left( 1 + e_{kSx} \right)^{\eta_k} \exp \frac{\omega_k e_{kSx}}{2} \left( 1 + \frac{1}{2} e_{kSx} \right)^{-1} \right\}$$

Now, it is assumed that  $|e_{kSx}| < 1, |e_{kSy}| < 1$  so that expanding  $(1 + e_{kSx})^{\eta_k}, (1 + \frac{1}{2} e_{kSx})^{-1}$  and  $\exp \frac{\omega_k e_{kSx}}{2} (1 + \frac{1}{2} e_{kSx})^{-1}$  as a series in power of  $e_{kSx}$ , multiplying out and retaining terms of the  $e$ 's to the second degree gives

$$\hat{S}_{pr}^2 = \sum_{k=1}^K W_k S_{ky}^2 \left[ 1 - \frac{\alpha_k (2\eta_k + \omega_k) e_{kSx}}{2} - \alpha_k (2\eta_k + \omega_k) \frac{(2\eta_k + \omega_k - 2) e_{kSx}^2}{8} - \alpha_k (2\eta_k + \omega_k) e_{kSx} e_{kSy} + e_{kSx} \right]$$

So that

$$(S_{pr}^2 - S_y^2) = \sum_{k=1}^K W_k S_{ky}^2 \left[ \left\{ -\frac{\alpha_k (2\eta_k + \omega_k)}{2} \left( e_{kSx} + \frac{(2\eta_k + \omega_k - 2) e_{kSx}^2}{4} + e_{kSx} e_{kSy} \right) \right\} + e_{kSx} \right] \quad (2)$$

Taking the expectation of (2), the Bias of estimator  $S_{pr}^2$  is obtained as:

$$B(S_{pr}^2) = \sum_{k=1}^K W_k \gamma_k S_{ky}^2 \left[ -\frac{\alpha_k (2\eta_k + \omega_k)}{2} \left( \frac{1}{4} (2\eta_k + \omega_k - 2) [\beta_2(x) - 1] + (\lambda_{22} - 1) \right) \right] \quad (3)$$

Squaring both sides of (2) and taking expectation, the Mean Square Error is obtained by Taylor's series approximation as:

$$SE(S_{pr}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \left[ [\beta_2(y) - 1] + \frac{\alpha_k (2\eta_k + \omega_k)}{4} (\alpha_k (2\eta_k + \omega_k - 2) [\beta_2(x) - 1] - 4(\lambda_{22} - 1)) \right] \quad (4)$$

The suggested estimator  $S_{pr}^2$  attains its optimal efficiency if  $(\eta_k, \omega_k) = (1, 1)$  and  $\frac{\partial MSE(S_{pr}^2)}{\partial \alpha_k} = 0$

So that

$$\alpha_k = \frac{2\Omega}{3} = \alpha_{k,opt} \quad (5)$$

Substituting the value of  $\alpha_{k,opt}$  in (5) for  $\alpha_k$  in (4), gives the MSE of an asymptotically optimum exponential ratio estimator (AOE)  $S_{pr}^2$  (or minimum MSE of  $S_{pr}^2$ ) as:

$$MSE_{opt}(S_{pr}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 [\beta_2(y) - 1] (1 - \rho_k^2) \quad (6)$$

### Alternatively

The suggested estimator  $S_{pr}^2$  would attain its optimal efficiency if:

$$\frac{\partial MSE(S_{pr}^2)}{\partial \omega_k} = \frac{\partial MSE(S_{pr}^2)}{\partial \eta_k} = \frac{\partial MSE(S_{pr}^2)}{\partial \alpha_k} = 0$$

So that

$$\alpha_k = \frac{2(\lambda_{22} - 1)}{(2\eta_k + \omega_k) [\beta_2(x) - 1]} \quad (7)$$

$$\eta_k = \frac{2(\lambda_{22} - 1) - \omega_k \alpha_k [\beta_2(x) - 1]}{2\alpha_k [\beta_2(x) - 1]} \quad (8)$$

$$\omega_k = \frac{2\{(\lambda_{22} - 1) - \eta_k \alpha_k [\beta_2(x) - 1]\}}{\alpha_k [\beta_2(x) - 1]} \quad (9)$$

Remark 1: Following from ((7), (8), (9)), it should be noted here that the proposed estimator  $S_{pr}^2$  would also attain its optimal efficiency with minimum MSE given as in (6), when either of the following optimality conditions is satisfied:

$$(i) \quad \eta_{k,opt} = \eta_k, \omega_{k,opt} = \omega_k \text{ and } \alpha_{k,opt} = \left( \frac{2\Omega}{2\eta_k + \omega_k} \right)$$

$$(ii) \quad \eta_{k,opt} = \eta_k, \alpha_{k,opt} = \alpha_k \text{ and } \omega_{k,opt} = \frac{2[\Omega - \eta_k \alpha_k]}{\alpha_k}$$

$$(iii) \quad \omega_{k,opt} = \omega_k, \alpha_{k,opt} = \alpha_k \text{ and } \eta_{k,opt} = \left( \frac{2\Omega - \omega_k \alpha_k}{2\alpha_k} \right)$$

### Membership of the proposed estimator

This section studies the properties of the proposed estimator and identifies some special members of its family and derives their mean square errors (MSEs) under certain prescribed conditions.

### Stratified random sampling estimator

When  $\tau_k = 1, \alpha_k = 0, \eta_k = \eta_k$  and  $\omega_k = \omega_k$ , the resulting family member of the proposed estimator is an unbiased stratified random sampling estimator of population variance given as:

$$S_{pr1}^2 = \sum_{h=1}^H w_h S_{ky}^2 \quad (10)$$

With MSE given as

$$MSE(S_{pr1}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 [\beta_2(y) - 1] \quad (11)$$

Remark 2: It should be noted here that irrespective of the values of  $\eta_k$  and  $\omega_k$ , whenever  $\tau_k = 1$  and  $\alpha_k = 0$ ; the resulting family member of the proposed estimator is always the unbiased stratified random sampling estimator of population variance given in (10).

### Bahl & Tuteja [1] exponential ratio-type estimator

When  $\tau_k = 0, \alpha_k = -1, \eta_k = 0$  and  $\omega_k = -1$ , the resulting family member of the proposed estimator is the Bahl and Tuteja (1991)

exponential ratio-type estimator of population variance in stratified random sampling given as:

$$S_{pr2}^2 = \sum_{k=1}^K W_k s_{ky}^2 \exp \left[ \frac{(S_{kx}^2 - s_{kx}^2)}{(S_{kx}^2 + s_{kx}^2)} \right] \quad (12)$$

with MSE given as:

$$MSE(S_{pr2}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \left[ [\beta_2(y) - 1] + \frac{1}{4}([\beta_2(x) - 1] - 4(\lambda_{22} - 1)) \right] \quad (13)$$

**Kadilar & Cingi [2] chain ratio-type estimator**

When  $\tau_k = 0, \alpha_k = -1, \eta_k = -2$  and  $\omega_k = 0$ , the resulting family member of the proposed estimator is the Kadilar and Cingi (2003) chain ratio-type estimator of population variance in stratified random sampling given as:

$$S_{pr3}^2 = \sum_{k=1}^K W_k s_{ky}^2 \left( \frac{S_{hx}^2}{S_{kx}^2} \right)^2 \quad (14)$$

with MSE given as

$$MSE(S_{pr3}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \left[ [\beta_2(y) - 1] + 4[\beta_2(x) - 1] - (\lambda_{22} - 1) \right] \quad (15)$$

**Solanki et al. [3] class of ratio-type estimators**

When  $\tau_k = 2, \alpha_k = 1, \eta_k = \eta_k$  and  $\omega_k = \omega_k$ , the resulting family member of the proposed estimator is the Solanki et al. [3] class of ratio-type estimators of population variance in stratified random sampling given as:

$$S_{pr2}^2 = \sum_{k=1}^K W_k s_{ky}^2 \left\{ 2 - \left( \frac{S_{hx}^2}{S_{kx}^2} \right)^{\eta_k} \exp \left[ \frac{\omega_k (S_{kx}^2 - S_{kx}^2)}{(S_{kx}^2 + s_{kx}^2)} \right] \right\} \quad (16)$$

With MSE given as

$$MSE(S_{pr4}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \left[ [\beta_2(y) - 1] + \left[ \frac{(2\eta_k + \omega_k)}{4} \right] \times ((2\eta_k + \omega_k)[\beta_2(x) - 1] - 4(\lambda_{22} - 1)) \right] \quad (17)$$

**Analytical study**

**Efficiency comparisons**

This section compares the optimal MSE of the proposed ratio estimator of population variance ( $S_{pr}^2$ ) with the MSEs of some existing estimators of its family.

**Regression estimator**

The proposed ratio estimator would be more efficient than the general regression estimator of population variance ( $S_{REG}^2$ ) if:

$$MSE_{opt} (S_{pr}^2) - MSE_{opt} (S_{REG}^2) < 0$$

But

$$S_{REG}^2 = s_y^2 + \hat{A}(S_x^2 - s_x^2) \quad (18)$$

Where,

$$S_y^2 = \sum_{k=1}^K W_k S_{ky}^2, S_y^2 = \sum_{k=1}^K W_k s_{ky}^2, S_x^2 = \sum_{k=1}^K W_k S_{kx}^2, S_x^2 = \sum_{k=1}^K W_k s_{kx}^2$$

So that (18) becomes

$$S_{REG}^2 = \sum_{k=1}^K W_k s_{ky}^2 + B \sum_{k=1}^K W_k (S_{kx}^2 - s_{kx}^2) \quad (19)$$

Expressing (19) in terms of the  $e$ 's gives

$$S_{REG}^2 = \sum_{k=1}^K W_k S_{ky}^2 (1 + e_{kS_y}) + B \sum_{k=1}^K W_k S_{kx}^2 e_{kS_x}$$

Now, it is assumed that  $|e_{kS_x}| < 1$  and  $|e_{kS_y}| < 1$  so that

$$(S_{REG}^2 - S_y^2)^2 = \sum_{k=1}^K W_k^2 \left[ S_{ky}^4 e_{kS_y} + B^2 S_{kx}^4 e_{kS_x} - 2BS_{ky}^2 S_{kx}^2 e_{kS_y} e_{kS_x} \right] \quad (20)$$

Taking the expectation of (20), the Mean Square Error is obtained by Taylor's series approximation as:

$$MSE(S_{REG}^2) = \sum_{k=1}^K W_k^2 \gamma_k \left[ S_{ky}^4 [\beta_{2k}(y) - 1] + B^2 S_{kx}^4 [\beta_{2k}(x) - 1] - 2BS_{ky}^2 S_{kx}^2 (\lambda_{22k} - 1) \right] \quad (21)$$

Where the value of B that minimizes (21) is given by

$$B_{opt} = \psi \Omega \quad (22)$$

Substituting the value of  $B_{opt}$  in (22) for B in (21), gives the optimal MSE ( $MSE_{opt}$ ) of the general regression estimator ( $S_{REG}^2$ ) as:

$$MSE_{opt} (S_{REG}^2) = \sum_{k=1}^K W_k^2 S_{ky}^4 \gamma_k \left[ \beta_2(y) - 1 \right] (1 - \rho_k^2) \quad (23)$$

**Remark 3:** Following from (6) and (23), it is evidenced that under certain prescribed optimality conditions, the proposed ratio estimator ( $S_{pr}^2$ ) has equal optimal efficiency with the general regression estimator ( $S_{REG}^2$ ). The implication is that the proposed exponential ratio estimator of population variance is as good as the general regression estimator of population variance.

**Stratified random sampling estimator**

The proposed estimator would be more efficient than the stratified random sampling estimator of population variance if:

$$MSE_{opt} (S_{pr}^2) - MSE(S_{pr1}^2) < 0$$

$$\Rightarrow \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (\lambda_{22k} - 1)^2 [\beta_2(x) - 1]^{-1} > 0 \quad (24)$$

Certainly, for (24) to hold  $\beta_2(x) > 1$

**Bahl & Tuteja [1] exponential ratio-type estimator**

The proposed estimator would be more efficient than the Bahl & Tuteja [1] exponential ratio-type estimator of population variance if:

$$MSE_{opt} (S_{pr}^2) - MSE(S_{pr2}^2) < 0$$

$$\Rightarrow \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (\beta_2(x) - 2\lambda_{22k} + 1)^2 > 0$$

So that either

$$(i) \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \beta_2(x) > \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (2\lambda_{22k} - 1)$$

Or

$$(ii) \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \lambda_{22k} < \frac{1}{2} \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 [\beta_2(x) + 1]$$

**Kadilar & Cingi [2] chain ratio-type estimator**

The proposed estimator would be more efficient than the Kadilar & Cingi [2] chain ratio-type estimator of population variance if:

$$MSE_{opt}(S_{pr}^2) - MSE(S_{pr3}^2) < 0$$

$$\Rightarrow \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (2\beta_2(x) - \lambda_{22k} - 1)^2 > 0$$

So that either

$$(i) \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \beta_2(x) > \frac{1}{2} \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (\lambda_{22k} + 1)$$

Or

$$(ii) \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \lambda_{22k} < \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 [2\beta_2(x) - 1]$$

**Solanki et al. [3] class of ratio-type estimators**

The proposed estimator would be more efficient than the Kadilar & Cingi [2] chain ratio-type estimator of population variance if:

$$MSE_{opt}(S_{pr}^2) - MSE(S_{pr4}^2) < 0$$

$$\Rightarrow \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (3\beta_2(x) - 2\lambda_{22k} - 1)^2 > 0 : (\eta_k, \omega_k) = (1, 1)$$

**Empirical study**

**Table 1:** Data Statistics.

Parameter	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5
$N_k$	40	20	32	26	42
$n_k$	14	8	10	12	16
$\bar{X}_k$	24.62	48.63	32.85	64.36	41.43
$\bar{Y}_k$	35.26	42.08	16.49	22.33	36.45
$\beta_{2x}(y)$	21.28	35.32	18.45	42.83	28.37
$\beta_{2x}(x)$	10.25	16.82	22.42	12.06	18.67
$\lambda_{22k}$	8.86	18.56	16.2	14.25	10.17
$S_{kr}^2$	90.25	64.83	105.38	86.44	58.38
$S_{ky}^2$	125.26	82.08	78.48	92.06	112.42
$W_k^2$	0.06	0.02	0.04	0.03	0.07
$\gamma_k$	0.05	0.08	0.07	0.05	0.04
$\eta_k$	1	-2	-1	2	1
$\omega_k$	-1	2	1	-2	-2

So that either

$$(i) \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \beta_2(x) > \frac{1}{3} \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (2\lambda_{22k} + 1)$$

Or

$$(ii) \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \lambda_{22k} < \frac{3}{2} \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 [3\beta_2(x) - 1]$$

**Isaki [4] ratio-type estimator**

The proposed estimator would be more efficient than the Isaki [4] ratio-type estimator of population variance if:

$$MSE_{opt}(S_{pr}^2) - MSE(S_{Ratio}^2) < 0$$

$$\Rightarrow \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 (\beta_2(x) - \lambda_{22k})^2 > 0$$

So that

$$\sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \beta_2(x) > \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 \lambda_{22k}$$

**The percent relative efficiency (PRE)**

The percent relative efficiency (PRE) of an estimator  $\phi$  with respect to the stratified random sampling estimator of population variance ( $S_{pr1}^2$ ) is defined by:

$$PRE(\phi, S_{pr1}^2) = \frac{MSE(S_{pr1}^2)}{MSE(\phi)} \times 100$$

$$MSE(S_{pr1}^2) = \sum_{k=1}^K W_k^2 \gamma_k S_{ky}^4 [\beta_2(y) - 1] = 3,125.78$$

In this section, the performance of the proposed ratio estimator is assessed with every identified existing estimators of its family and the classical ratio estimator of population variance by Isaki [4]. The merits of the suggested ratio estimator over other existing estimators were judged using the Data Statistics in Table 1.

**Discussion of Results**

Analytical comparisons showed that the proposed ratio estimator of population variance under certain realistic conditions is more efficient than the unbiased stratified sampling estimator of population variance, classical ratio estimator of population variance by Isaki [4], Bahl & Tuteja [1] ratio-type estimator, Solanki et al [3] ratio-type estimator and Kadillar & Cingi [2] ratio-type estimator but has equal optimal efficiency with the regression estimator of population variance. Numerical results for the percent relative efficiency (PREs) in Table 2 reveals that the proposed estimator ( $S_{pr}^2$ ) has 153 percent gains in efficiency while the conventional ratio estimator of population variance by Isaki (1983) has 142 percent gains in efficiency; this shows that the proposed ratio estimator ( $S_{pr}^2$ ) is 11 percent more efficient than the Isaki 4] ratio estimator of population variance. Similarly, the proposed estimator ( $S_{pr}^2$ ) is 14 percent and 65 percent more efficient than the Bahl & Tuteja [1] and the Solanki et al. [3] ratio-type estimators of population variance respectively. Also, in using the proposed ratio estimator, one will have 117 percent efficiency over the Kadillar & Cingi [2] ratio-type estimator of population variance. Generally, the proposed ratio estimator fares better than every identified existing estimators of its family and has equal optimal efficiency with the regression estimator of population variance.

**Table 2:** MSEs and PREs for the estimators.

New York:	New York:	New York:	New York:
1	$S_{pr}^2$	2,045.09	152.84
2	$S_{REG}^2$	2,045.09	152.84
3	$S_{Ratio}^2$	2,200.83	142.03
4	$S_{pr2}^2$	2,243.21	139.34
5	$S_{pr4}^2$	3,571.96	87.51
6	$S_{pr3}^2$	8,776.33	35.66

**Conclusion**

Sequel to the discussion of results above, it is concluded that the proposed ratio estimator ( $S_{pr}^2$ ), fares better than every identified existing estimators of its family and has equal optimal efficiency with the general regression estimator ( $S_{REG}^2$ ) which has always been preferred because of its efficiency. Therefore, the new ratio estimator of population variance in stratified random

sampling should be preferred in practical situations by survey researchers as it provides consistent and more precise estimates of the population parameters of interest.

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