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Bayes Estimation under a Finite Mixture of Truncated Generalized Cauchy Distributions Based On Censored Data with Application

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Abstract

In this paper, the Bayes estimates (BE's) of the parameters, reliability and hazard rate functions of a finite mixture of truncated generalized Cauchy distributions are obtained based on type-I, type-II and progressively type-II censored samples. A simulation study is carried out to study the behaviour of the mean squared errors (MSE's) of the estimates. All previous parameters and functions are obtained based on generated type-I, type-II and progressively type-II censored samples generated from a real data set as illustrative application.

Keywords: Truncated generalized Cauchy distribution; Bayes estimation; MCMC algorithm; Finite mixture models; Type-I censoring; Type-II censoring; Progressively type-II censoring

Abbreviations: BE's: Bayes Estimates; MSE's: Mean Squared Errors; TGCD: Truncated Generalized Cauchy Distribution; PDF: Probability Density; CDF: Cumulative Distribution Function; SF: Survival Function; HRF: Hazard Rate Function; LF: Likelihood Function

Introduction

The Cauchy distribution is a symmetric distribution with bell shaped density function as the normal distribution but with a greater probability mass in the tails. The distribution is often used in the cases which arise in outlier analysis. The Cauchy distribution has received applications in many areas, including biological analysis, clinical trials, stochastic modelling of decreasing failure rate life components, queuing theory, and reliability. For data from these areas, there is no reason to believe that empirical moments of any order should be infinite. Thus, the choice of the Cauchy distribution as a model is unrealistic since its moments of all orders are not infinite. The introduced truncated generalized Cauchy distribution (TGCD) can be a more appropriate model for the kind of data mentioned. For more details about Cauchy and truncated generalized Cauchy distributions, see the book by Johnson et al. [1] which covers the Cauchy distribution in many of its aspects starting from the history, properties, developments and applications up to the most recent research done in the subject matter, to the date of the book's publication. Also see, Ateya & AL-Hussaini [2] and Ahsanullah [3] which studied the TGCD extensively. The probability density (PDF), cumulative distribution function (CDF), survival function (SF) and hazard rate function (HRF) of the TGCD with parameters (α,β,γ) are given, respectively, by

$$f(t;\alpha,\beta,\gamma)=\frac{2}{\sqrt{\pi}}\frac{\Gamma\left(\alpha+\frac{1}{2}\right)}{r\Gamma(\alpha)}\left[1+\left(\frac{t-\beta}{r}\right)^2\right]^{-\alpha-\frac{1}{2}},\ t>\beta(\beta,\gamma,\alpha>0).\ (1.1)$$

$$\mathrm{F}(\mathrm{t};\alpha,\beta,\gamma)=\frac{2}{\sqrt{\delta}}\frac{\tilde{A}\left(\alpha+\frac{1}{2}\right)}{r\tilde{A}(\alpha)}\int_0^{\tan^{-1}(\mathrm{t}-\beta)/r}(\cos\varphi)^{2\alpha-1}d\varphi,\ (1.2)$$

$$\mathrm{S}(\mathrm{t};\alpha,\beta,\gamma)=1-\left(\left(\frac{2}{\sqrt{\pi}}\frac{\Gamma\left(\alpha+\frac{1}{2}\right)}{\mathrm{r}\Gamma(\alpha)}\int^{\tan^{-1}\left(\frac{t-\beta}{r}\right)}(\cos\varphi)^{2\alpha-1}\,d\varphi\right),\right.\ (1.3)$$

And

$$\mathrm{h}(\mathrm{t};\alpha,\beta,\gamma)=\frac{\left(\frac{2}{\sqrt{\pi}}\frac{\left(\alpha+\frac{1}{2}\right)}{r\Gamma(\alpha)}\left[1+\left(\frac{t-\beta}{r}\right)^2\right]^{\alpha-\frac{1}{2}}\right)}{\left(1-\frac{2}{\sqrt{\pi}}\frac{\left(\alpha+\frac{1}{2}\right)}{r\Gamma(\alpha)}\int^{\tan^{-1}\left(\frac{t-\beta}{r}\right)}(\cos\varphi)^{2\alpha-1}\,d\varphi\right)}$$

The mixture models are very important in the theoretical and applied fields especially in case of the heterogeneous population. For details about mixture models, see McLachlan & Peel [4], Titterington, Smith & Makov [5], Bozidar et al. [6] and Satheesh & Manju [7]. A random variable T is said to have a finite mixture of TGCD's with parameters $\theta_j = (\alpha_j, \beta_j, \gamma_j), j = 1,2,\ldots,k,$ if its PDF is given by

$$f_\Theta(t) = \sum_{j=1}^{k} p_j f_j(t;\theta_j), \quad (1.5)$$

Where, $\Theta = (\theta_1, \theta_2, \ldots, \theta_k, p_1, \ldots, p_k), p_j \geq 0$ and $\sum_{j=1}^{k} p_j = 1.$

The corresponding CDF, SF and HRF are given by

$$F_\Theta(t) = \sum_{j=1}^{k} p_j F_j(t;\theta_j), \quad (1.6)$$

$$S_\Theta(t) = \sum_{j=1}^{k} p_j S_j(t;\theta_j), \quad (1.7)$$

And,

$$h_\Theta(t) = \frac{f_\Theta(t)}{S_\Theta(t)}. \quad (1.8)$$

Where, $f_j(t;\theta_j), F_j(t;\theta_j)$ and $S_j(t;\theta_j)$ can be obtained from equations (1.1)-(1.3) after replacing $\theta = (\alpha, \beta, \gamma)$ by $\theta_j = (\alpha_j, \beta_j, \gamma_j)$.

Prior analysis and some important algorithms

In this section, a suggested prior and some important algorithms will be introduced.

Prior analysis: Suppose that the prior belief of the experimenter is measured by a prior PDF $\pi(p, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \beta)$ constructed as follows:

$$\pi(p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta) = \pi_1(p)\pi_2(\alpha_1,\gamma_1)\pi_3(\alpha_2,\gamma_2)\pi_4(\beta) = \pi_1(p)\pi_{21}(\gamma_1|\alpha_1)\pi_{22}(\alpha_1)\pi_{31}(\gamma_2|\alpha_2)\pi_{32}(\alpha_2)\pi_4(\beta) \quad (2.1.)$$

Suppose that $\pi_1(p)$ is Beta $(c_1, c_2), \pi_{21}(\gamma_1 | \alpha_1)$ is Gamma $(c_3, \alpha_1), \pi_{22}(\alpha_1)$ is Gamma $(c_4, c_5), \pi_{31}(\gamma_2 | \alpha_2),$ is Gamma $(c_4, c_5), \pi_{32}(\gamma_2 | \alpha_2)$ is Gamma (c_7, c_8) and finally $\pi_4(\beta)$ is Gamma (c_9, c_{10}) with respective densities

$$\pi_1(p) \propto p^{c_1-1}(1-p)^{c_2-1}, 0 \leq p \leq 1, (c_1, c_2 > 0), \quad (2.2)$$

$$\pi_{21}(\gamma_1|\alpha_1) \propto \alpha_1^{c_3}\gamma_1^{c_3-1}e^{-\gamma_1\alpha_1}, \alpha_1, \gamma_1 > 0, (c_3 > 0), \quad (2.3)$$

$$\pi_{22}(\alpha_1) \propto \alpha_1^{c_4-1}e^{-c_5\alpha_1}, \alpha_1 > 0, (c_4, c_5 > 0), \quad (2.4)$$

$$\pi_{31}(\gamma_2|\alpha_2) \propto \alpha_2^{c_6}\gamma_2^{c_6-1}e^{-\gamma_2\alpha_2}, \alpha_2, \gamma_2 > 0, (c_6 > 0) \quad (2.5)$$

$$\pi_{32}(\alpha_2) \propto \alpha_2^{c_7-1}e^{-c_8\alpha_2}, \alpha_2 > 0, (c_7, c_8 > 0), \quad (2.6)$$

$$\pi_4(\beta) \propto \beta^{c_9-1}e^{-c_{10}\beta}. \beta >, (c_7, c_8 > 0) \quad (2.7)$$

From equations (2.2) to (7.7) in equation (2.1), we can write the prior PDF of the parameters $(p, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \beta)$ as follows:

$$\pi(\rho,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta) \alpha \rho^{c_1-1}(1-p)^{c_2-1}\alpha_1^{c_3+c_4-1}\alpha_2^{c_6+c_7-1}\gamma_1^{c_3-1}\gamma_2^{c_6-1}\beta^{c_9-1}$$
$$\exp\{[\alpha_1(\gamma_1 + c_5) + \alpha_2(\gamma_2 + c_8) + c_{10}\beta]\},$$
$$0 \leq p \leq 1, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \beta > 0, (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10} > 0),$$

(2.8)

Where, $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9$ and are the prior parameters.

Gibbs sampler

Gibbs Sampler is a method used to generate a random sample $\theta^1, \theta^2, \ldots, \theta^m$ from the posterior PDF $\pi^*(\theta|t)$ as follows:

Let $\theta^\circ = (\theta_1^\circ, \ldots, \theta_k^\circ)$ be an initial values [may be actual values of parameters, or may be the estimated values using any method]

2- Generate θ_1^1 from $\pi^*(\theta_1|\theta_2^\circ, \theta_3^\circ, \ldots, \theta_k^\circ, t)$.

3- Generate θ_1^1 from $\pi^*(\theta_2|\theta_1^1, \theta_3^\circ, \ldots, \theta_k^\circ, t)$.

4- Generate θ_1^1 from $\pi^*(\theta_i|\theta_1^1, \theta_2^1, \ldots, \theta_{i-1}^1, \theta_{i+1}^\circ, \ldots, \theta_k^\circ, t)$

5- Generate θ_k^1 from $\pi^*(\theta_k|\theta_1^1, \theta_2^1, \ldots, \theta_{k-1}^1, t)$ so we generate $\theta^1 = (\theta_1^1, \ldots, \theta_k^1)$

6- Repeat steps 1-5 m times we get $\theta^1, \theta^2, \ldots, \theta^m$.

Metropolis-Hastings algorithm

Is a method used to generate a number θ_i^j from the posterior PDF $\pi^*(\theta_i|\theta_1^j, \theta_2^j, \ldots, \theta_{i-1}^j, \theta_{i+1}^{j-1}, \ldots, \theta_k^{j-1}, x)$. This method can be summarized in the following steps:

1- Generate θ_i^* from a suitable PDF $f(\theta)$

2- $A^* = \min\{1, A\},$

$$A = \frac{\pi^*(\theta_1^j, \theta_2^j, \ldots, \theta_{i-1}^j, \theta_i^*, \theta_{i+1}^{j-1}, \ldots, \theta_k^{j-1}, t) f(\theta_i^*)}{\pi^*(\theta_1^j, \theta_2^j, \ldots, \theta_{i-1}^j, \theta_i^{j-1}, \ldots, \theta_k^{j-1}, t) f(\theta_i^{j-1})}$$

3- Generateu from $u(0,1)$

4- If $A < U$, then accept θ_i^* as θ_i^j, else $\theta_i^* \to \theta_i^{j-1}$ go to step1.

Markov chain monte carlo (MCMC) method

Let $\theta = (\theta_1, \theta_2, \ldots, \theta_k)$ be a parameters vector with a posterior PDF $\pi^*(\theta|t)$, $t = (t_1, t_2, \ldots, t_n)$ the vector of observations. If $\theta^1, \theta^2, \ldots, \theta^m$, where $\theta^i = (\theta_1^i, \theta_2^i, \ldots, \theta_k^i)$ is a random sample of size m generated from $\pi^*(\theta|t)$, then the BE of a function $u(\theta^i)$ based on squared error (SE) loss functions is given by

$$\hat{u}_{BS}(\theta) = \frac{1}{m}\sum_{i=1}^{m} u(\theta^i),$$

To generate from the posterior PDF $\pi^*(\theta|t)$, we will use Gibbs sampler and Metropolis–Hastings techniques. For more details about the MCMC method, see, for example Jaheen & Al-Harbi [8], Press[9], Upadhyaya et al. [10] and Upadhyaya & Gupta [11].

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Bayes estimation

In this section, the BE's of all parameters, survival and hazard rate functions will be estimated based on type-I, type-II and progressive type-II censoring schemes.

Bayes Estimation Based on type-I censoring scheme

Suppose that we have n items from a finite mixture TGCD's, with equal location parameters $(\beta_1=\beta_2=\beta)$. All items are put on a life testing experiment. Suppose that r units have failed during the interval $(0,t_0)$ and $(n\text{-}r)$ units are still active, where t_0 is a predetermined time. Let $t_1,\ldots,t_n$ be a random sample from the mixed population. The exact lifetime of an item will be observed only if $t_i \le t_0$, $i=1,2,\ldots,n$. This is known as type-I censored sample. The likelihood function(LF) based on type-I censored sample, see Lawless[12], may be written as

$$L(\theta\,|\,t)=\prod_{i=1}^{n}\left[f_{\Theta}\left(t_i\right)\right]^{\ddot{a}_i}\left[S_{\Theta}\left(t_0\right)\right]^{1-\ddot{a}_i} \qquad (3.1)$$

Where, $f_{\grave{E}}\left(t_i\right)$ and $S_{\grave{E}}\left(t_o\right)$ are defined in (1.5) and (1.7) after replacing t by t_i and t_0 respectively,

$$t=(t_1,t_2,\ldots,t_n) \qquad (3.2)$$

And $\ddot{a}_i$ is an indicator function, given by

$$\ddot{a}_i=\begin{cases}1, & t_i\le t_0,\\ 0, & t_i>t_0.\end{cases} \qquad (3.3)$$

Using the LF (3.1) and the prior (2.8), the posterior PDF of the parameters $(p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta)$ can be written as

$$\pi^*(\theta\,|\,t)\,p^{c_1-1}(1-p)^{c_2-1}\,\alpha_1^{c_3+c_4-1}\alpha_2^{c_6+c_7-1}\gamma_1^{c_3-1}\gamma_2^{c_6-1}\beta^{c_9-1}$$
$$\exp\left\{\left[\alpha_1\left(\gamma_1+c_5\right)+\alpha_2\left(\gamma_2+c_8\right)+c_{10}\beta\right]\right\}\prod_{i=1}^{n}\left[f\left(t_i;\theta\right)\right]^{\delta_i}\left[S\left(t_0;\theta\right)\right]^{1-\delta_i} \qquad (3.4)$$

To estimate the parameters and functions $p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta$, survival and hazard rate functions at time t^*, we define a function $u(p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta)$ as

$$u\left(p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta\right)=p^{\delta_1}\alpha_1^{\delta_2}\alpha_2^{\delta_3}\gamma_1^{\delta_4}\gamma_2^{\delta_5}\beta^{\delta_6}(S(t^*))^{\delta_7}(h(t^*))^{\delta_8}. \qquad (3.5)$$

The BE of $u(p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta)$ is obtained in five cases:

When $\delta_1=1,\delta_2=0=\delta_3=\delta_4=\delta_5=\delta_6=\delta_7=\delta_8$, which is equivalent to estimating p.

When $\delta_2=1,\delta_1=0=\delta_3=\delta_4=\delta_5=\delta_6=\delta_7=\delta_8$, which is equivalent to estimating α_1.

When $\delta_3=1,\delta_1=0=\delta_2=\delta_4=\delta_5=\delta_6=\delta_7=\delta_8$, which is equivalent to estimating α_2.

When $\delta_4=1,\delta_1=0=\delta_2=\delta_3=\delta_5=\delta_6=\delta_7=\delta_8$, which is equivalent to estimating γ_1.

When $\delta_5=1,\delta_1=0=\delta_2=\delta_3=\delta_4=\delta_6=\delta_7=\delta_8$, which is equivalent to estimating γ_2.

When $\delta_6=1,\delta_1=0=\delta_2=\delta_3=\delta_4=\delta_5=\delta_7=\delta_8$, which is equivalent to estimating β.

When $\delta_7=1,\delta_1=0=\delta_2=\delta_3=\delta_4=\delta_5=\delta_6=\delta_8$, which is equivalent to estimating $S(t^*)$.

When $\delta_8=1,\delta_1=0=\delta_2=\delta_3=\delta_4=\delta_5=\delta_6=\delta_7$, which is equivalent to estimating $h(t^*)$.

Then, MCMC algorithm will be used to estimate all mentioned parameters and functions

Bayes estimation based on type-II censoring scheme

Assume that we put items from a finite mixture of TGCD's in a life testing experiment. Instead of continuing until all n items have failed, the experiment is terminated at the time of the r^{th} item failure. Such test can save time and money, since it could take a very long time for all items to fail. Suppose that $t_1<t_2<\ldots<t_r$ is a censored data of size r obtained from a life test on n items (type-II censored data) whose life times have a finite mixture of TGCD's. The likelihood function based on Type-II censored data, see Lawless [12], is given by

$$L(\theta\,|\,t)=\frac{n!}{(n-r)!}\left[\prod_{i=1}^{r}f_{\Theta}\left(t_i\right)[S_{\Theta}\left(t_r\right)\right]^{n-r}. \qquad (3.6)$$

Using the prior (2.8) and the LF (3.6), the posterior PDF of the parameters $(p,\alpha_1,\alpha_2,\gamma_1,\gamma_2,\beta)$ can be written as

$$\eth^*\left(\theta|t\right)=p^{c_1-1}\left(1-P\right)^{c_2-1}\alpha_1^{c_3+c_4-1}\alpha_2^{c_6+c_7-1}\tilde{a}_1^{c_3-1}\tilde{a}_2^{c_6-1}\beta^{c_9-1}$$
$$\exp\left(1-p\right)^{c_2-1}\alpha_1^{c_3+c_4-1}\alpha_2^{c_6+c_7-1}\gamma_1^{c_3-1}\gamma_2^{c_6-1}\beta^{c_9-1} \qquad (3.7)$$
$$\exp\left\{-\left[\alpha_1\left(\gamma_1+c_5\right)+\alpha_2\left(\gamma_2+c_8\right)+c_{10}\beta\right]\right\}\left[\prod_{i=1}^{r}f_{\Theta}\left(t_i\right)[S_{\Theta}\left(t_r\right)\right]^{n-r}.$$

and the same is done as 3.1.

Bayes estimation based on progressively type-II censoring scheme

The progressive type-II censored model is of importance in the field of reliability and life testing. Suppose nidentical units are placed on a lifetime test. At the time of the i^{th} failure, R_i surviving units are randomly withdrawn from the experiment, $1\le i\le r$. Thus, if r failures are observed then $R_1+R_2+\ldots+R_r$, units are progressively censored, hence $n=r+R_1+R_2+\ldots+R_r$ and $T_{1:r:n}^{M}<T_{2:r:n}^{M}<\ldots<T_{r:r:n}^{M}$ describe the progressively censored failure times, where $M=\left(R_1,R_2,\ldots,R_r\right)$ and $\sum_{i=1}^{r}R_i=n-r$. The likelihood function based on progressively type-II censored data $t=(t_{1:r:n}^{M},t_{(2:r:n)}^{M},\ldots,t_{(r:r:n)}^{M})$ which can be written for simplicity as $t=(t_1,t_2,\ldots,t_r)$ is given by

$$L(\theta\,|\,t)=c\prod_{i=1}^{r}f_{\grave{E}}\left(t_i\right)(S_{\Theta}(t_i))^{R_i}, \qquad (3.8)$$

Where,

$c=n\left(n-R_1-1\right)\left(n-R_1-R_2-2\right)\ldots(n-R_1-R_2-\ldots-R_{r-1}-r+1)$, See Balakrishnan & Aggarwala [13].

Using the LF (3.8) and the prior PDF (2.8), the posterior PDF will be given by

$$\pi^*\left(\theta|t\right)=C^*p^{c_1-1}\left(1-p\right)^{c_2-1}\alpha_1^{c_3+c_4-1}\alpha_2^{c_6+c_7-1}\gamma_1^{c_3-1}\gamma_2^{c_6-1}\beta^{c_9-1} \qquad (3.9)$$
$$\exp\left\{-\left[\alpha_1\left(\gamma_1+c_5\right)+\alpha_2\left(\gamma_2+c_8\right)+c_{10}\beta\right]\right\}\prod_{i=1}^{r}\left[f\left(t_i;\theta\right)\right]\left[S(t_0;\theta)\right]^{R_i}$$

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Where,

$C^* = n(n - R_1 - 1)(n - R_1 - R_2 - 1)\ldots(n - R_1 - R_2 - \ldots - R_{r-1} - r - 1)$, and also the same is done as 3.1 and 3.2

Simulation study and data analysis

In this section, all studied parameters and functions will be estimated based on type-I, type-II and progressive type-II censoring samples from generated and real data.

Simulation study

In this section, a simulation study is carried out to study the behavior of the MSE's. In case of $j = 2$, we can write $p_1 = p$, $p_2 = 1 - p$, then the vector of all parameters will be in the form $\theta = (\alpha_1, \alpha_2, \beta, \gamma_1, \gamma_2, p)$.

For different values of t_0 and r, follow the following steps:

Making use of the set of hyper parameters, the vector of the population parameters will be generated.

Making use of the generated vector of parameters $\theta = (\alpha_1, \alpha_2, \beta, \gamma_1, \gamma_2, p)$, samples of different sizes n (10, 20, 30) are generated from a mixture of two TGCD as follows:

- Generate U_1 and U_2 from the uniform distribution $U(0,1)$.
- If $u_1 < p$, generate from $F_1(t; \alpha_1, \beta, \gamma_1)$ using u_2, otherwise generate from $F_2(t; \alpha_2, \beta, \gamma_2)$ using u_2.

For a value t_0, we consider all values of the random variable T which are less than or equal $MSE(\hat{\theta}) = \frac{1}{m}\sum_{j=1}^{m}(\hat{\theta}_j - \theta)^2$, (type-I censored sample).

Based on this sample and for different values of t_0, we can use the Bayes method to obtain the estimate of the vector of parameters θ. survival and hazard rate functions.

Based on the sample $t_1 < t_2 < \ldots < t_r$ which is a type-II censored sample, we can use Bayes method, as done in case of type-I censoring, to obtain the BE's of the same vector of parameters and functions, for different values of r.

For different schemes, progressive type-II will be generated and the same is done based on the generated progressively type-II censored samples.

Repeat steps 1-6 (m) times for different samples.

The MSE's of $\hat{\theta}$ over the m samples is given by:

$$MSE(\hat{\theta}) = \frac{1}{m}\sum_{j=1}^{m}(\hat{\theta}_j - \theta)^2,$$

Where θ is the actual value of the vector of parameters $\hat{\theta}_j$ and θ is the estimate of the vector of parameters over the sample j.

In Tables 1-6 the BE's of all parameters and functions have been obtained based on Type-I, Type-II and progressively Type-II censored samples.

Table 1: MSE's of the BE's of $\alpha_1, \alpha_2, \beta, \gamma_1, \gamma_2$ and p based on type-I censored data of different sizes n, censoring values t_0 $(\alpha_1 = 2.5, \alpha_2 = 1.6, \beta = 1.5, \gamma_1 = 3.5, \gamma_2 = 1.7, p = 0.3)$.

n	t_0	$MSE(\hat{p})$	$MSE(\hat{\alpha}_1)$	$MSE(\hat{\alpha}_2)$	$MSE(\hat{\beta})$	$MSE(\hat{\gamma}_1)$	$MSE(\hat{\gamma}_2)$
10	2.5	0.0027	0.0652	0.0371	0.0291	0.04416	0.0819
	4.5	0.0024	0.0518	0.0351	0.0281	0.04194	0.0751
	6	0.0021	0.0501	0.0331	0.0221	0.03719	0.0681
20	2.5	0.0023	0.0441	0.0214	0.0104	0.03098	0.0916
	4.5	0.0019	0.0221	0.0201	0.0061	0.02997	0.0414
	6	0.0015	0.0179	0.0193	0.0042	0.02154	0.0251
30	2.5	0.0014	0.0192	0.0177	0.0037	0.02196	0. 0917
	4.5	0.0009	0.0104	0.0127	0.0021	0.02015	0.0421
	6	0.0004	0.0091	0.0105	0.0016	0.01947	0.0109

Table 2: MSE's of the BE's of $S(t^*)$ and $h(t^*)$ based on type-I censored data of different sizes n, censoring values t_0 $(\alpha_1 = 2.5, \alpha_2 = 1.6, \beta = 1.5, \gamma_1 = 3.5, \gamma_2 = 1.7, p = 0.3)$. $S(t^*) = 0.259437$, $h(t^*) = 0.964188$, $t^* = 3.0$.

n	t_0	$\overline{S(t^*)}$	$\overline{h(t^*)}$	$MSE(\overline{S(t^*)})$	$MSE(\overline{H(t^*)})$
10	2.5	0.21098	0.9612	0.002108	0.001412
	4.5	0.21008	0.9511	0.002011	0.001109
	6	0.22098	0.959	0.001901	0.001031
20	2.5	0.22156	0.9501	0.002013	0.000916
	4.5	0.23128	0.9591	0.001987	0.000519
	6	0.23891	0.961	0.001517	0.000318

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30	2.5	0.25125	0.959	0.001929	0.000615
	4.5	0.25192	0.9621	0.001615	0.000581
	6	0.26182	0.9615	0.001016	0.000505

Table 3: MSE's of the BE's of $\alpha_1,\alpha_2,\beta,\gamma_1,\gamma_2$ and p based on type-II censored data of different sizes n, censoring values $(\alpha_1 = 2.5, \alpha_2 = 1.6, \beta = 1.5, \gamma_1 = 3.5, \gamma_2 = 1.7, p = 0.3)$.

n	r	$MSE(\hat{p})$	$MSE(\hat{\alpha}_1)$	$MSE(\hat{\alpha}_2)$	$MSE(\hat{\beta})$	$MSE(\hat{\gamma}_1)$	$MSE(\hat{\gamma}_2)$
10	5	0.0031	0.0418	0.0331	0.0217	0.0424	0.1081
	10	0.0027	0.0397	0.0294	0.0211	0.0373	0.0715
20	5	0.0028	0.0407	0.0221	0.0061	0.0291	0.0712
	10	0.0021	0.0383	0.0219	0.0032	0.0261	0.0671
	20	0.0018	0.0311	0.0208	0.0028	0.0208	0.0409
30	15	0.0022	0.0209	0.0142	0.0018	0.0221	0.0441
	20	0.0012	0.0201	0.0116	0.0013	0.0183	0.0255
	30	0.0008	0.0143	0.0103	0.0009	0.0152	0.0106

Table 4: MSE's of the BE's of $S(t^*)$ and $h(t^*)$ based on type-II censored data of different sizes n, censoring values r $(\alpha_1 = 2.5, \alpha_2 = 1.6, \beta = 1.5, \gamma_1 = 3.5, \gamma_2 = 1.7, p = 0.3)$. $S(t^*) = 0.259437,\ h(t^*) = 0.964188,\ t^* = 3.0.$.

n	r	$\overline{S(t^*)}$	$\overline{h(t^*)}$	$MSE(\overline{S(t^*)})$	$MSE(\overline{h(t^*)})$
10	5	0.2812	0.9501	0.0025	0.0108
	10	0.2781	0.9521	0.0022	0.0101
20	5	0.2412	0.9491	0.0016	0.0008
	10	0.2421	0.9503	0.0009	0.0004
	15	0.2398	0.958	0.0005	0.0001
30	15	0.2536	0.9501	0.0002	0.00009
	20	0.2543	0.9605	0.0001	0.00004
	30	0.2589	0.9601	0.00006	0.00002

Table 5: MSE's of the BE's of $\alpha_1,\alpha_2,\beta,\gamma_1,\gamma_2$ and p based on progressively type-II censored data of different sizes n, censoring schemes M.
$M_1 = (0,5,3,2,2,1,0,0,2,0)$
$M_2 = (0,3,1,0,1,0,0,2,1,0,0,1,0,1,0)$
$M_3 = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)$.
$(\alpha_1 = 2.5, \alpha_2 = 1.6, \beta = 1.5, \gamma_1 = 3.5, \gamma_2 = 1.7, p = 0.3)$.

n	(r, M)	$MSE(\hat{p})$	$MSE(\hat{\alpha}_1)$	$MSE(\hat{\alpha}_2)$	$MSE(\hat{\beta})$	$MSE(\hat{\gamma}_1)$	$MSE(\hat{\gamma}_2)$
25	$(10, M_1)$	0.0027	0.0501	0.0351	0.0207	0.05162	0.08142
	$(15, M_2)$	0.0019	0.0481	0.0321	0.0183	0.04417	0.07916
	$(25, M_3)$	0.0011	0.0448	0.0305	0.0034	0.03012	0.07011

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Table 6: MSE's of the BE's of $S(t^*)$ and $h(t^*)$ based on progressively type-II censored data of different sizes n, censoring schemes M.

$M_1 = (0,5,3,2,2,1,0,0,2,0)$

$M_2 = (0,3,1,0,1,0,0,2,1,0,0,1,0,1,0)$

$M_3 = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)$.

$(\alpha_1 = 2.5, \alpha_2 = 1.6, \beta = 1.5, \gamma_1 = 3.5, \gamma_2 = 1.7, p = 0.3)$.

$S(t^*) = 0.259437, \ h(t^*) = 0.964188, \ t^* = 3.0.$

n	(r, M)	$\overline{S(t^*)}$	$\overline{h(t^*)}$	$MSE(\overline{S(t^*)})$	$MSE(\overline{h(t^*)})$
25	$(10, M_1)$	0.2471	0.9512	0.0042	0.0027
	$(15, M_2)$	0.2506	0.9445	0.0038	0.0021
	$(25, M_3)$	0.2522	0.9517	0.0028	0.0018

Data analysis

In this section, a mixture of two real data sets from Ateya & Madhagi [14] is introduced. These data are (after ordering)

2.3707, 2.4282, 2.4743, 2.4858, 2.4858, 2.5088, 2.5663, 2.6239, 2.6239, 2.6814,2.762, 3.0266, 3.1187, 3.1509, 3.5905, 3.6825, 3.6825, 3.7136, 3.7373, 3.7799, 3.8322, 3.8603, 3.8981, 3.9644, 4.059, 4.1583, 4.2577, 4.2719, 4.3326, 4.3846, 4.4275, 4.4275, 4.532, 4.554, 4.5856, 4.6172, 4.6172, 4.6488, 4.7121, 4.7355, 4.807, 5.1518, 5.166, 5.1944, 5.2369, 5.6201, 5.9607, 6.5568, 7.1529, 7.644, 7.81, 7.84, 7.938, 8.0044, 8.134, 8.526, 8.82, 8.82, 9.31, 9.31, 9.506, 9.8, 10, 10.001, 10.1, 10.3, 10.9504, 11.3302, 11.3935, 11.6, 11.8394, 12.6457, 12.9286, 13.1, 13.169, 13.2, 13.3246, 13.4, 13.7, 13.7914, 13.8, 14, 14, 14, 14.0177, 14.0885, 14.1, 14.1733, 14.4, 14.6, 14.6118, 15.0645, 15.5, 15.5454, 15.9698, 16.1, 16.479, 16.5, 16.9, 16.9, 17.0306, 17.1, 17.2, 17.3, 17.3, 17.3, 17.3, 17.4, 17.7, 17.8, 17.8793, 17.9, 18.2, 18.9, 19.2357, 19.4, 20.0812, 21.5, 22.7587, 23.4, 23.6, 24.1, 25, 26.4226, 26.5, 26.7, 26.7, 26.8, 26.9, 27.6, 28.4, 28.9, 29.4, 29.8, 30, 30.157, 30.4, 30.9, 31.2, 31.8, 32.9, 33.9, 34.4551, 35.5, 37.6963, 37.7, 39.8, 40.5852, 41.1, 42.5, 44.6719, 46.2, 48.829, 51.9, 54.3249, 55.5, 58.2, 60.3141, 62.1, 65.5281, 67.2192, 67.7, 72.4151, 76.7, 79.8316, 85.5389, 86.3, 92.0212, 94, 97.6, 99.2082, 101.3, 105.3, 106, 107.6, 107.9, 109.3, 110, 110.5, 112.8, 115.4, 118.5, 120, 120.3, 120.5, 121.9, 124.1, 126.1, 131.3, 133.8, 135.4, 137.9, 139.5, 141.9, 142.8, 145.8, 146.5, 149.7, 150.6, 153.5, 158.6, 161.3, 163.9, 168.3, 174, 176.7, 180.9, 184.3, 190.3, 196.8

The BE's of the parameters, survival and hazard rate functions based on type-I, type-II and progressively type-II under the previous real data set are summarized in Tables 7-12.

Table 7: BE's of the population parameters based on type-I censoring scheme from real data set.

n	t_0	$\hat{p}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
210	30	0.28162	4.97078	10.2716	2.3231	3713.72	4132.99
	90	0.28618	5.16162	10.3415	2.3651	3733.21	4267.65
	200	0.29017	5.12739	10.3651	2.4015	3731.87	4254.87

Table 8: BE's of the survival and hazard rate functions based on type-I censoring scheme from real data set at .

n	t_0	$\overline{S(t^*)}$	$\overline{h(t^*)}$
210	30	0.937385	0.000858
	90	0.940918	0.000887
	200	0.939715	0.000898

Table 9: BE's of the population parameters based on type-II censoring scheme from real data set.

n	r	$\hat{p}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
210	70	0.271260	4.93372	10.55165	2.4098	3689.32	4217.09
	140	0.27701	5.32415	10.41524	2.3908	3701.76	4231.82
	210	0.28971	5.11902	10.29081	2.3991	3781.18	4271.93

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Table 10: BE's of the survival and hazard rate functions based on type-II censoring scheme from real data set at .

n	$\hat{\gamma}_2$	$\overline{S(t^*)}$	$\overline{h(t^*)}$
210	70	0.929412	0.000839082
	140	0.938919	0.000861827
	210	0.936725	0.000885243

Table 11: BE's of the population parameters based on progressively type-II censoring scheme from real data set.

n	(r, M)	$\hat{p}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
210	(75, M1)	0.26901	4.87159	10.71287	2.3291	3644.95	4180.15
	(150, M2)	0.27109	5.21782	10.51527	2.3781	3741.98	4261.54
	(210, M3)	0.28017	5.10082	10.33162	2.4055	3750.15	4197.92

M1= (12, 017, 30, 020, 27, 015, 22, 07, 35, 06, 9, 04)

M2= (10, 030, 20, 020, 20, 030, 5, 020, 2, 030, 3, 014)

M3= (0210)

Table 12: BE's of the survival and hazard rate functions based on progressively type-II censoring scheme from real data set.

n	(r, M)	$\overline{S(t^*)}$	$\overline{h(t^*)}$
210	(75, M1)	0.918273	0.000821625
	(150, M2)	0.925612	0.000832154
	(210, M3)	0.932012	0.000884428

data set.

M1= (12, 017, 30, 020, 27, 015, 22, 07, 35, 06, 9, 04)

M2= (10, 030, 20, 020, 20, 030, 5, 020, 2, 030, 3, 014)

M3= (0210)

In most cases, observe the following:

- For fixed and and by increasing the sample size n, we often get smaller MSE' s.
- For fixed sample size n and by increasing the censoring values fixed t_0and r, we often get smaller MSE' s.
- The largest values of and in each case represent the complete sample case.
- In progressively type II, for fixed n, we often get smaller MSE' s by increasing r.

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