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Some Identities based on Success Runs of at Least Length k

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Abstract

In this research paper, an attempt has been made to obtain alternative formulas for distributions of order k based on runs of at least length k. First by using binomial scheme and 'balls-into-cells' technique an alternative formula for distribution of binomial distribution of order k as defined by Goldstein [1]. Inverse Bionomial scheme was then used with 'ball-into-cells technique to obtain alternate distribution of Geometric distribution of order k and negative binomial distribution of order k. The results were further extended to obtain Polya-Eggenberger distribution of order k and Inverse Polya-Eggenberger distributions of order k. All results were verified for exactness of probability.

Keywords : Polya-Eggenberger sampling scheme; Distributions of order k; 'Balls-into-cells' technique

Introduction

Distribution of runs and successions in various situations have been under considerable studies due to their applications to reliability theory of consecutive systems Grifitth [2]; Papastravridis & Sfkianakis [3], Sfkianakis, Kounias & Hillaris [4], Papastravridis & Koutras [5] & Cai [6], start-up demonstration tests [7], molecular biology [1], theory of radar detection, time sharing systems and quality control [8-12]. There are different ways of computations and enumeration of number of runs:

1. Feller [13] defined ways of counting the runs of exactly length k as counting the number from scratch everytime a run occurs. For example the sequence $SSS|SFSSS|SSS\mid F$ contains 3 success runs of length 3
2. Goldstein [7] proposed the distribution of the number of success runs of at least length k until the n-th trial. In this way of counting the number of runs of length 3 (or more), in the above example contains 2 success runs of length 3 (or more)
3. Schwager [14] and Ling [15,16] studied the distributions on the number of overlapping runs of length k. In the enumeration scheme $SSS|SFSSS|SSS\mid F$ contains 6 overlapping success runs of length 3.
4. Aki and Hirano [17] studied the distribution of success runs of exact length k. In the above example number of success runs of exact length 3 is 0.
5. Philippou[18] obtained the distribution of the number of trials until the first occurrence of consecutive k successes in Bernoulli trials with success probability as the geometric distribution of order $k, \left(G_k\left(x;p\right)\right)$

In this paper, we have suggested alternative formulas for Binomial distribution of order k based on success runs of based on atleast length k by using Balls-into-cells technique with direct sampling scheme with replacement. The same result was extended by using inverse sampling scheme to obtain alternative formula for Geometric distribution of order k and negative Bionomial distribution of order k. Finally, using Polya Eggenberger sampling scheme we have obtained alternative formulas for Polya-Eggenberger distributions of order k and Inverse Polya-Eggenberger distributions of order k [19,20].

Lemma: The number ways of distributing r indistinguishable balls in n cells such that each cell has atmost $(k-1)$ balls is given as:

$$F^{(k)}\left(n;r\right)=\sum_{i=0}^{\left[\frac{r}{k}\right]}\left(-1\right)^i\binom{n}{i}\binom{r+ik+n-1}{n-1}$$ (Riordan 1958) (1)

Binomial distribution of order k

Let X_n^k the number of success runs of length at least k and r be the number of success in n Bernoulli trials and $n-r$ be the number of failures.

Then,

Theorem 1:

$$P\left(X_n^k=x\right)=\sum_{r=xk}^{a}\binom{n-r+x}{x}\sum_{j=0}^{\left[\frac{r-xk}{k}\right]}\left(-1\right)^j\binom{n-r+1}{j}$$

$$\binom{n-xk-jk}{n-r}p^r q^{n-r},\quad x=0,1,\ldots,\left[\frac{n}{k}\right] \qquad (2)$$

Where,

$$a=\left[\frac{kx+(n+1)(k-1)}{k}\right]$$

Proof: First consider x runs of exact length k successes each. x successes and n-r failures can be arranged in $\binom{n-r+x}{x}$ ways. $r-xk$ remaining successes can be distributed into $n-r+1$ cells such that no cells have more than $k-1$ successes in

$$F^{(k)}(n-r-1;r-xk)=\sum_{j=0}^{\left[\frac{r-xk}{k}\right]}(-1)^j\binom{n-r+1}{j}\binom{n-xk-jk}{n-r}$$

ways.

Thus, leading to the result. This is an alternative representation of probability distribution function of

For the maximum possible value of r,

Let us assume that there are x successes of exact length $k+(k-1)$ followed by a failure each.

Then the remaining number of cells formed by $n-r$ failures and x successes will be $n-r+x+1$ each assumed to be having exactly $(k-1)$ successes.

So, $r\le\left[\frac{xk+(n+1)(k-1)}{k}\right]$

Some Examples:

For $n=2,k=2,x=0,1;a=\left[\frac{3}{2}\right]=1$

$$P(X_2^2=0)=\sum_{r=0}^{1}\sum_{j=0}^{\left[\frac{r}{2}\right]}(-1)^j\binom{3-r}{j}\binom{2-2j}{2-r}\binom{2-r}{2-r}p^r q^{2-r}$$

$$=q^2+2pq$$

$$P(X_2^2=0)=\sum_{r=2}^{2}\sum_{j=0}^{0}\binom{2-2}{0}\binom{2-2+1}{2-2}p^2$$

$$=p^2$$

$$P(X_2^2=0)+P(X_2^2=1)=(p+q)^2=1$$

For $n=3,k=2,x=0,1;a=\left[\frac{4}{2}\right]=2$

$$P(X_3^2=0)=\sum_{r=0}^{2}\sum_{j=0}^{\left[\frac{r}{2}\right]}(-1)^j\binom{4-r}{j}\binom{3-2j}{3-r}\binom{3-r}{3-r}p^r q^{3-r}$$

$$=q^3+3pq^2+p^2q$$

$$P(X_3^2=0)=\sum_{r=2}^{2}\sum_{j=0}^{\left[\frac{r-2}{2}\right]}(-1)^j\binom{4-r}{j}\binom{1-2j}{3-r}\binom{4-r}{3-r}p^r q^{3-r}$$

$$=2p^2q+p^3$$

$$\Rightarrow P(X_3^2=0)+P(X_3^2=1)=(q+p)^3=1$$

For $n=10,k=2,x=0,1,\ldots,5.$

For $x=0,a=\left[\frac{11}{2}\right]=5$

$$P(X_{10}^2=0)=\sum_{r=0}^{5}\sum_{j=0}^{\left[\frac{r}{2}\right]}(-1)^j\binom{11-r}{j}\binom{10-2j}{10-r}\binom{10-r}{10-r}p^r q^{10-r}$$

$$=q^{10}+\binom{10}{1}pq^9+\binom{10}{2}p^2q^8-9p^2q^8+\binom{10}{3}p^3q^7-64p^3q^7$$

$$+\binom{10}{4}p^4q^6-196p^4q^6+21p^4q^6+\binom{10}{5}p^5q^5-246p^5q^5$$

For $x=1,a=6$

$$P(X_{10}^2=1)=\sum_{r=2}^{6}\sum_{j=0}^{\left[\frac{r-2}{2}\right]}(-1)^j\binom{11-r}{j}\binom{8-2j}{10-r}\binom{11-r}{10-r}p^r q^{10-r}$$

$$=9p^2q^8+64p^3q^7+147p^4q^6+120p^5q^5+25p^6q^4$$

For $x=2,a=7$

$$P(X_{10}^2=2)=\sum_{r=4}^{7}\sum_{j=0}^{\left[\frac{r-4}{2}\right]}(-1)^j\binom{11-r}{j}\binom{6-2j}{10-r}\binom{12-r}{10-r}p^r q^{10-r}$$

$$=28p^4q^6+126p^5q^5+150p^6q^4+40p^7q^3$$

For $x=3,a=8$

$$P(X_{10}^2=3)=\sum_{r=6}^{8}\sum_{j=0}^{\left[\frac{r-6}{2}\right]}(-1)^j\binom{11-r}{j}\binom{4-2j}{10-r}\binom{13-r}{10-r}p^r q^{10-r}$$

$$=35p^6q^4+80p^7q^3+30p^8q^2$$

For $x=4,a=9$

$$P(X_{10}^2=4)=\sum_{r=6}^{9}\sum_{j=0}^{\left[\frac{r-8}{2}\right]}(-1)^j\binom{11-r}{j}\binom{2-2j}{10-r}\binom{14-r}{10-r}p^r q^{10-r}$$

$$=15p^8q^2+10p^9q$$

For $x=5,a=10$

$$P(X_{10}^2=5)=\sum_{r=10}^{10}\sum_{j=10}^{0}(-1)^0\binom{0}{0}\binom{5}{0}p^{10}=p^{10}$$

$$P(X_{10}^2=0)+P(X_{10}^2=1)+P(X_{10}^2=2)+P(X_{10}^2=3)+P(X_{10}^2=4)+P(X_{10}^2=5)$$

$$=(q+p)^{10}=1$$

Geometric distribution of order k

Let the first success run of length k occurs at N_k^{th} trial. Then,

Theorem 2

$$P(X_{10}^2=3)=\sum_{r=k}^{a}\sum_{j=0}^{\left[\frac{r-k}{k}\right]}(-1)^j\binom{n-r}{j}\binom{n-k-jk-1}{n-r-1}p^r q^{n-r}$$

$x\ge k$ (3)

Where, $a=\left[\frac{k+n(k-1)}{k}\right]$

Proof: Let there be r successes in

$$\underbrace{\ldots\ldots\ldots}_{n-k-1}F\underbrace{SS\ldots S}_{k}$$

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The number of ways of distributing $r-k$ successes preceding failures $n-r$ such that not more than $k-1$ successes occur proceeding the $n-r$ failures then by using (1) and replacing n by $n-r$ and r by $r-k$ is given by

$$F^{(k)}(n-r;r-k)=\sum_{j=0}^{\left[\frac{r-k}{k}\right]}(-1)^j\binom{n-r}{j}\binom{n-k-jk-1}{n-r-1}.$$

Hence, the result. This is thus an alternative formula for Geometric distribution of order k given by Philippou [18].

Remark 1: For $k=1$ (3) gives the probability mass function of Geometric Distribution.

An Example

For $n=10,k=3,a=\left[\frac{3+20}{3}\right]=\left[\frac{23}{3}\right]=7$

$$P(N_3=10)=\sum_{r=3}^{7}\sum_{j=0}^{\left[\frac{r-3}{3}\right]}(-1)^j\binom{10-r}{j}\binom{6-3j}{9-r}p^rq^{10-r}$$

$$=p^3q^7+6p^4q^6+15p^5q^5+16p^6q^4+6p^7q^3$$

Negative Binomial Distribution of order k

Let the x^{th} run of length k occurs at the N_k^x-th trial i.e.,

$$\underbrace{\overbrace{\underbrace{\ldots SS\ldots S}_{k}}^{1st}\ldots\overbrace{\underbrace{SS\ldots S}_{k}}^{2nd}\ldots\ldots\overbrace{\underbrace{SS\ldots S}_{k}}^{(x-1)th}\ldots F\,\overbrace{\underbrace{SS\ldots S}_{k}}^{(x-1)th}}_{N_k^x}$$

Theorem 3

$$P(N_k^x=n)=\sum_{r=xk}^{a}\sum_{j=0}^{\left[\frac{r-xk}{k}\right]}(-1)^j\binom{n-r}{j}\binom{n-xk-jk-1}{n-r-1}\binom{n-r+x-1}{x-1}p^rq^{n-r}$$

$x\ge nk$ (3)

Where, $a=\left[\frac{xk+n(k-1)}{k}\right]$

Proof

Let there be r successes and $n-r$ failures. $r-xk$ successes are to be distributed into $n-r$ cells preceding the $n-r$ failures such that no cell receives more than $k-1$ successes which is given by (1) replacing r by $r-xk$ and n by as

$$F^{(k)}(n-r;r-xk)=\sum_{j=0}^{\left[\frac{r-xk}{k}\right]}(-1)^j\binom{n-r}{j}\binom{n-xk-jk-1}{n-r-1}$$

Hence, the probability. Further, $xk+(n-r)(k-1)\ge r$. Hence, $r\le\left[\frac{xk+n(k-1)}{k}\right]=a$

This is thus an alternative formula for Negative Binomial distribution of order given by Philippou [18].

Some examples

For $k=1\to a=x$ and $r=x$

$P(N_1^x=n)=\binom{n-x-1}{n-x-1}\binom{n-1}{x-1}p^xq^{n-x}$ is the probability mass function of Negative Binomial Distribution

For $n=8,k=2,x=2,a=\left[\frac{4+8}{2}\right]=6$

$$P(N_2^2=8)=\sum_{r=4}^{6}\sum_{j=0}^{\left[\frac{r-4}{2}\right]}(-1)^j\binom{8-r}{j}\binom{3-2j}{7-r}\binom{9-r}{1}p^rq^{n-r}$$

$$=5p^4q^4+12p^5q^3+3p^6q^2$$

Polya-Eggenberger Distribution of order K

Let us assume an urn contains a white and b black balls. A ball is drawn, its colour is noted and it is returned to the urn with S additional balls of the same colour. The process is continued till n balls have been drawn.

Let $X_{n;k}^s$ be the number of white ball runs of at least length k in n trials. Then,

Theorem 4: $$P(X_{n;k}^s=x)=\sum_{r=xk}^{a}\binom{n-r+x}{x}\sum_{j=0}^{\left[\frac{r-4xk}{k}\right]}(-1)^j\binom{n-r+1}{j}\binom{n-xk-jk}{n-r}\frac{a^{(r,s)}b^{(n-r,s)}}{(a+b)^{(n,s)}},\quad (5)$$

Where,

$a=\left[\frac{kx+(n+1)(k-1)}{k}\right]$

The proof follows from theorem 1. This is thus an alternative formula for Polya distribution of order given by Charalambides [18] Sen et al. [19].

Some examples

For $n=7,k=2,x=0,1,2,3.$

$$P(X_{7;2}^s=x)=\sum_{r=2x}^{a}\binom{7-r+x}{x}\sum_{j=0}^{\left[\frac{r-2x}{2}\right]}(-1)^j\binom{8-r}{j}\binom{7-2x-2j}{7-r}\frac{a^{(r,s)}b^{(7-r,s)}}{(a+b)^{(7,s)}}$$

$a=x+4$

$$P(X_{7;2}^s=x)=\sum_{r=0}^{4}(-1)^j\binom{8-r}{j}\binom{7-2x-2j}{7-r}\frac{a^{(r,s)}b^{(7-r,s)}}{(a+b)^{(7,s)}}$$

$$=\frac{b^{(7,s)}}{(a+b)^{(7,s)}}+7\frac{ab^{(6,s)}}{(a+b)^{(7,s)}}+15\frac{a^{(2,s)}b^{(5,s)}}{(a+b)^{(7,s)}}+10\frac{a^{(3,s)}b^{(4,s)}}{(a+b)^{(7,s)}}+\frac{a^{(4,s)}b^{(3,s)}}{(a+b)^{(7,s)}}$$

$$P(X_{7;2}^s=1)=\sum_{r=2}^{5}\binom{8-r}{1}\sum_{j=0}^{\left[\frac{r-2x}{2}\right]}(-1)^j\binom{8-r}{j}\binom{5-2j}{7-r}\frac{a^{(r,s)}b^{(7-r,s)}}{(a+b)^{(7,s)}}$$

$$=6\frac{a^{(2,s)}b^{(5,s)}}{(a+b)^{(7,s)}}+25\frac{a^{(3,s)}b^{(4,s)}}{(a+b)^{(7,s)}}+24\frac{a^{(4,s)}b^{(3,s)}}{(a+b)^{(7,s)}}+3\frac{a^{(5,s)}b^{(2,s)}}{(a+b)^{(7,s)}}$$

$$P(X_{7;2}^s=2)=\sum_{r=4}^{6}\binom{9-r}{2}\sum_{j=0}^{\left[\frac{r-4}{2}\right]}(-1)^j\binom{8-r}{j}\binom{3-2j}{7-r}\frac{a^{(r,s)}b^{(7-r,s)}}{(a+b)^{(7,s)}}$$

$$=10\frac{a^{(4,s)}b^{(3,s)}}{(a+b)^{(7,s)}}+18\frac{a^{(5,s)}b^{(2,s)}}{(a+b)^{(7,s)}}+3\frac{a^{(6,s)}b^{(1,s)}}{(a+b)^{(7,s)}}$$

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$$P\left(X_{7;2}^{s}=3\right)=\Sigma_{r=6}^{7}\binom{10-r}{3}\Sigma_{j=0}^{\left[\frac{r-6}{2}\right]}(-1)^{j}\binom{8-r}{j}\binom{1-2j}{7-r}\frac{a^{(r,s)}b^{(7-r,s)}}{(a+b)^{(7,s)}}$$

$$=4\frac{a^{(6,s)}b^{(1,s)}}{(a+b)^{(7,s)}}+\frac{a^{(7,s)}}{(a+b)^{(7,s)}}$$

$$\xrightarrow{yields}P\left(X_{7;2}^{s}=0\right)+P\left(X_{7;2}^{s}=1\right)+P\left(X_{7;2}^{s}=2\right)+P\left(X_{7;2}^{s}=x\right)$$

$$=\frac{b^{(7,s)}}{(a+b)^{(7,s)}}+7\frac{ab^{(6,s)}}{(a+b)^{(7,s)}}+21\frac{a^{(2,s)}b^{(5,s)}}{(a+b)^{(7,s)}}+35\frac{a^{(3,s)}b^{(4,s)}}{(a+b)^{(7,s)}}+35\frac{a^{(4,s)}b^{(3,s)}}{(a+b)^{(7,s)}}$$

$$+21\frac{a^{(5,s)}b^{(2,s)}}{(a+b)^{(7,s)}}+7\frac{a^{(6,s)}b^{(1,s)}}{(a+b)^{(7,s)}}+\frac{a^{(7,s)}}{(a+b)^{(7,s)}}=\sum_{x=0}^{7}\binom{7}{x}\frac{a^{(x,s)}b^{(7-x,s)}}{(a+b)^{(7,s)}}=1$$

Inverse polya-eggenberger distribution of order k

Let us assume an urn contains a white and b black balls. Balls are drawn one-by-one with s replacement along with additional balls of the same colour of ball drawn. The process is continued till n balls have been drawn [21].

Let $N_k^{x,s}$ be the number of trials required for x^{th} run of k white balls to occur $\left(N_k^{x,0}=N_k^{x}\right)$. Then,

Then, using theorem 3 we have,

Theorem 5

$$\left(N_k^{x,s}=n\right)=\sum_{r=k}^{a}\sum_{j=0}^{\left[\frac{r-xk}{k}\right]}(-1)^{j}\binom{n-r}{j}\binom{n-k-jk-1}{n-r-1}\binom{n-r+x-1}{x-1}\frac{a^{(r,s)}b^{(n-r,s)}}{(a+b)^{(n,s)}}$$

$x \geq nk$ (6)

Where, $a=\left[\frac{xk+n(k-1)}{k}\right]$ This is thus an alternative formula for Inverse Polya distribution of order k given by

Some examples

For $n=9,k=3,x=2$

$$\left(N_3^{2,s}=9\right)=\sum_{r=6}^{8}\sum_{j=0}^{\left[\frac{r-6}{3}\right]}(-1)^{j}\binom{9-r}{j}\binom{2-3j}{8-r}\binom{10-r}{x-1}\frac{a^{(r,s)}b^{(n-r,s)}}{(a+b)^{(n,s)}}$$

$=4\frac{a^{(6,s)}b^{(3,s)}}{(a+b)^{(9,s)}}+6\frac{a^{(7,s)}b^{(2,s)}}{(a+b)^{(9,s)}}+2\frac{a^{(8,s)}b^{(1,s)}}{(a+b)^{(9,s)}}$ which include following arrangements of white and black balls:

$$\underbrace{WWBWWWWWW,WWWWWBWWW,}_{two\ arrangements\ of\ 1black\ \&8white\ balls}$$

$$\underbrace{BWBWWWWWW,WBBWWWWWW,WBWWWBWWW,WWWWBBWWW,BWWWWBWWW,WWWBWBWWW}_{six\ arrangements\ of\ 2black\ \&7white\ balls}$$

$$\underbrace{,BBBWWWWWW,BBWWWBWWW,WWWBBBWWW,BWWWBBWWW}_{four\ arrangements\ of\ 3black\ \&6white\ balls}$$

For $n=11,k=2,x=3$

$$\left(N_3^{2,s}=11\right)=\sum_{r=6}^{8}\sum_{j=0}^{\left[\frac{r-6}{3}\right]}(-1)^{j}\binom{11-r}{j}\binom{4-2j}{10-r}\binom{13-r}{2}\frac{a^{(r,s)}b^{(n-r,s)}}{(a+b)^{(n,s)}}$$

$$=21\frac{a^{(6,s)}b^{(5,s)}}{(a+b)^{(11,s)}}+60\frac{a^{(7,s)}b^{(4,s)}}{(a+b)^{(11,s)}}+30\frac{a^{(8,s)}b^{(3,s)}}{(a+b)^{(11,s)}}$$

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