Biostatistics and Biometrics
Open Access Journal
ISSN: 2573-2633

Research Article
Volume 4 Issue 2 - December 2017
DOI: 10.19080/BBOAJ.2017.04.555631

Biostat Biometrics Open Acc J

Closed form Solutions of New Fifth Order Nonlinear Equation and New Generalized Fifth Order Nonlinear Equation via the Enhanced (G'/G)-expansion Method

A K M Kazi Sazzad Hossain[1]* and Md. Ali Akbar[2]

[1]*Department of Mathematics, Begum Rokeya University, Bangladesh*

[2]*Department of Applied Mathematics, University of Rajshahi, Bangladesh*

Submission: October 2, 2017; **Published:** December 08, 2017

***Corresponding author:** A K M Kazi Sazzad Hossain, Department of Mathematics, Begum Rokeya University, Rangpur, Bangladesh, Tel: 8.80171E+12; Email: kazi_bru@yahoo.com

Abstract

Closed form solutions of nonlinear evolution equations (NLEEs) are very imperative in order to better understand the inner mechanism and complexity of complex physical phenomena. The enhanced $(G'/G)-expansion$ method is a effectual and proficient mathematical tool which can be used to discover the closed form solutions of NLEEs arising in mathematical physics, applied mathematics and engineering. In this article, the enhanced $(G'/G)-expansion$ method is recommended and carry out to investigate the closed form solutions of the new fifth order non-linear equation and the new generalized fifth order non-linear equation. The performance of this method is reliable, proficient and possible to obtain a lot of new exact solutions than the existing other methods.

Keywords: The enhanced $(G'/G)-expansion$ method; New fifth order nonlinear equation; New generalized fifth order nonlinear equation; Nonlinear evolution equations (NLEEs); Closed form solutions

Introduction

Closed form solutions of nonlinear evolution equations (NLEEs) are getting importance to study of complex phenomena in the field of science and engineering. NLEEs are frequently appear in various fields, such as plasma physics, geophysics, nuclear physics, biomathematics optical fibers, biomechanics, gas dynamics, chemical reactions, geochemistry etc. Closed form solutions of NLEEs and its graphical representation reveal the inner mechanism of complex nonlinear phenomena. Therefore, it is a urgent issue and very important to search for more closed form solutions to NLEEs in order to better realization of the structure of nonlinear phenomena. But till now there is no distinctive method to inspect all kinds NLEEs. As a result diverse groups of mathematicians, physicist and engineers have been working vigorously to develop effective methods for which to solve all NLEEs.

For this reason, in the recent years there has been considerable progress in the development of finding effective methods to search exact solution, such as the nonlinear transform method [1], the first integration method [2], the F-expansion method [3], the Exp-function method [4-6], the Jacobi-elliptic function method [7,8], the Darboux transformation method [9], the complex hyperbolic function method [10,11], the auxiliary equation method [12], the Adomian decomposition method [13], the functional variable method [14], the sine cosine method [15], the Painleve expansion method [16], the $\exp(-\Phi(\eta))$ expansion method [17,18], the variational method [19,20], the simplified Hirota's method [21], the sine-cosine method [22], the Kudryashov method [23], the extended direct algebraic method [24,25], the modified simple equation method [26-30], the Lie group symmetry method [31], the $(G'/G)-expansion$ method [32-36], the enhanced $(G'/G)-expansion$ method [37,38], the improve $(G'/G)-expansion$ method, etc.

The recently developed the enhanced $(G'/G)-expansion$ method is getting popularity in use because of its directly advanced calculation procedure and there is possible to obtain large number of solution. The objective of this article is to introduce and make use of the enhanced $(G'/G)-expansion$ method to extract fresh and further general exact traveling wave solutions to the new fifth order nonlinear equation and new generalized fifth order nonlinear equation. The rest of the article is arranged as follows: In Section 2, enhanced $(G'/G)-expansion$ method is discussed. In Section 3, the enhanced $(G'/G)-expansion$ method is applied to examine the NLEEs point out above. In Section 4, results and physical explanations and In Section 5 conclusions are provided.

Algorithm of the enhanced $(G'/G)-expansion$

In this section, we explore the enhanced $(G'/G)-expansion$ method for finding traveling wave solutions to NLEEs. Let us consider a nonlinear evolution equation in two independent variables x and t in the form:

$$R\left(u,u_t,u_x,u_{tt},u_{xx},u_{xt},\ldots\right)=0, \quad (2.1)$$

Where $u=u(x,t)$ is an unknown function of x and t and R is a polynomial of $u(x,t)$ and its partial derivatives which contains the highest degree nonlinear terms. The essential steps concerning this method are offered in the following:

Step 1: Initiating a compound variable ξ with the combination of real variables x and t,

$$u\left(x,t\right)=u\left(\xi\right),\xi=x\pm\omega t, \quad (2.2)$$

Where, specify the speed of the traveling wave.

The traveling wave transformations (2.2) permit us in dropping Eq. (2.1) to an ordinary differential equation (ODE) for $u=u(\xi)$ in the form:

$$S\left(u,u',u'',u'''\ldots\right)=0, \quad (2.3)$$

Where, S is a polynomial in $u(\xi)$ and its derivatives with respect to ξ.

Step 2: The solution of Eq. (2.3) can be expressed in the following form:

$$u(\xi)=\sum_{i=-n}^{n}\left(\frac{a_i(G'/G)^i}{\left(1+\lambda\left(G'/G\right)\right)^i}+b_i(G'/G)^{i-1}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}\right) \quad (2.4)$$

Where, $a_i,b_i(-n\le i\le n;n\in N)$ are constants to be determined later, $\sigma=\pm1$, $\mu\ne0$ and $G=G(\xi)$ satisfies the equation

$$G''+\mu G=0. \quad (2.5)$$

Step 3: Taking homogeneous balance between the highest order linear term and the nonlinear terms of the highest degree which appear in Eq. (2.3), we obtain the limiting value n.

Step 4: Substituting (2.4) into (2.3) together with (2.5) and then accumulating all terms of same powers of $(G'/G)^i$ and $(G'/G)^i\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$ and setting each coefficient to zero yields a system of algebraic equations for $a_i,b_i\left(-n\le i\le n;n\in N\right),\lambda$ and ω. Solving this system of equations supplies the values of the unknown parameters.

Step 5: From the general solution of equation (2.5), we obtain

When, $\mu<0$,

$$\frac{G'}{G}=\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right), \quad (2.6)$$

And

$$\frac{G'}{G}=\sqrt{-\mu}\coth\left(\xi_0+\sqrt{-\mu}\xi\right). \quad (2.7)$$

Again when μ>0,

$$\frac{G'}{G}=\sqrt{\mu}\tan\left(\xi_0-\sqrt{\mu}\xi\right), \quad (2.8)$$

And

$$\frac{G'}{G}=\sqrt{\mu}\cot\left(\xi_0+\sqrt{\mu}\xi\right). \quad (2.9)$$

Where, ξ_0 is an arbitrary constant. At last, substituting $a_i,b_i\left(-n\le i\le n;n\in N\right),\lambda$ and ω and solutions (2.6)-(2.9) into (2.4), we obtain more general and some fresh traveling wave solutions of (2.1).

Applications of the method

In this section, we inspect the closed form solutions of the new fifth order non-linear equation and new generalized fifth order non-linear equation with the help of the enhanced $(G'/G)-expansion$ method.

Example 1: In this subsection, we will use the enhanced $(G'/G)-expansion$ method search for the exact solution to the following new fifth order non-linear equation of the form [21]

$$u_{ttt}-u_{txxxx}-4\left(u_xu_t\right)_{xx}-4\left(u_xu_{xt}\right)_x=0 \quad (3.1)$$

The equation (3.1) transfer to ODE in the following form using wave transformation (2.2)

$$-\omega^3u'''+\omega u^{(v)}+4\omega\left(u'u'\right)''+4\omega\left(u'u''\right)'=0. \quad (3.2)$$

Integrating (3.2) with respect to twice and taking integration constant to zero, we get

$$u'''+6u'^2-\omega^2u'=0. \quad (3.3)$$

balancing the highest-order derivative term u''' and the highest-order nonlinear term u'^2 yields $n=1$. Thus, the solution structure of Eq. (3.3) becomes

$$u(\xi)=a_0+\frac{a_1\left(\frac{G'}{G}\right)}{1+\lambda\left(\frac{G'}{G}\right)}+\frac{a_{-1}\left(1+\lambda\left(\frac{G'}{G}\right)\right)}{\left(\frac{G'}{G}\right)}+b_0\left(\frac{G'}{G}\right)^{-1}\sigma\left(1+\frac{\left(\frac{G'}{G}\right)^2}{\mu}\right)+b_1\sigma\left(1+\frac{\left(\frac{G'}{G}\right)^2}{\mu}\right)+b_{-1}\left(\frac{G'}{G}\right)^{-2}\sigma\left(1+\frac{\left(\frac{G'}{G}\right)^2}{\mu}\right) \quad (3.4)$$

Where, $G=G(\xi)$ satisfies Eq. (2.5).

Substituting (3.4) with the equation (2.5) into equation (3.3), we attain a polynomial $(G'/G)^i$ of and $(G'/G)^i\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$. From this polynomial we get the coefficients $(G'/G)^i$ of and $G'/G)^i\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$. Equating them to zero, we achieve an over-determined system that contains thirty algebraic equations (for simplicity we skip to display them). Solving this system of algebraic equation, we get

Set 1: $\omega=\pm2\sqrt{-\mu},\lambda=\lambda,a_{-1}=0,a_0=a_0,a_1=1+\mu\lambda^2,b_{-1}=0,b_0=0,b_1=0.$

How to cite this article: A.K. M.K.S. Hossain., M. A. Akbar. Closed Form Solutions of New Fifth Order Nonlinear Equations and New Generalized Fifth Order Nonlinear Equations via the enhanced (G'/G)-expansion Method. Biostat Biometrics Open Acc J. 2017; 4(2): 555631. DOI: 10.19080/BBOAJ.2017.04.555631.

Set 2: $\omega=\pm\sqrt{-\mu},\lambda=0,a_{-1}=0,a_0=a_0,a_1=\frac{1}{2},b_{-1}=0,b_0=0,b_1=\pm\frac{\sqrt{\mu}}{2\sqrt{\sigma}}.$

Set 3: $\omega=\pm2\sqrt{-\mu},\lambda=\lambda,a_{-1}=-\mu,a_0=a_0,a_1=0,b_{-1}=0,b_0=0,b_1=0$

Set 4: $\omega=\pm4\sqrt{-\mu},\lambda=0,a_{-1}=-\mu,a_0=a_0,a_1=1,b_{-1}=0,b_0=0,b_1=0.$

Set 5: $\omega=\pm\sqrt{-\mu},\lambda=\lambda,a_{-1}=-\frac{\mu}{2},a_0=a_0,a_1=0,b_{-1}=0,b_0=\pm\frac{\mu}{2\sqrt{\sigma}},b_1=0.$

Now substituting solution set 1-5 with equation (2.5) into equation (3.4), we get sufficient traveling wave solution to Eq. (3.1) as follows:

When, ' < , we get the hyperbolic solution,

Profile-1: $$u_1(\xi)=a_0+\left(1+\mu\lambda^2\right)\frac{\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)}{\left(1+\lambda\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)} \quad (3.5)$$

$$u_1(\xi)=a_0+\left(1+\mu\lambda^2\right)\frac{\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)}{\left(1+\lambda\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)} \quad (3.6)$$

Where $\xi=x\pm2\sqrt{-\mu}t$,

Profile -2:

$$u_3(\xi)=a_0+\frac{1}{2}\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\pm\frac{1}{2}\sqrt{\mu\left(1-\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)} \quad (3.7)$$

$$u_4(\xi)=a_0+\frac{1}{2}\sqrt{-\mu}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\pm\frac{1}{2}\sqrt{\mu\left(1-\left(\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)} \quad (3.8)$$

Where $\xi=x\pm\sqrt{-\mu}t$

Profile-3:

$$u_5(\xi)=a_0-\left(\lambda\mu+\sqrt{-\mu}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)llioxet \quad (3.9)$$

$$u_6(\xi)=a_0-\left(\lambda\mu+\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right) \quad (3.10)$$

Where $\xi=x\pm2\sqrt{-\mu}t$

Profile-4:

$$u_7(\xi)=a_0\pm\sqrt{-\mu}\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)-\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right) \quad (3.11)$$

Where $\xi=x\pm4\sqrt{-\mu}t$

Profile-5:

$$u_8(\xi)=a_0-\frac{\sqrt{-\mu}}{2}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\left(\left(1+\lambda\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)\mp\sqrt{\left(1-\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)}\right)$$

(3.12)

$$u_9(\xi)=a_0-\frac{\sqrt{-\mu}}{2}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\left(\left(1+\lambda\sqrt{-\mu}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)\mp\sqrt{\left(1-\left(\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)}\right)$$

=(3.13)

Where $\xi=x\pm\sqrt{-\mu}t$

Again, for $\mu>0$, we get the following trigonometric solution:

Profile-6:

$$u_{10}(\xi)=a_0+(1+\mu\lambda^2)\frac{\sqrt{\mu}\tan\left(\xi_0-\sqrt{\mu}\xi\right)}{\left(1+\lambda\sqrt{\mu}\tan\left(\xi_0-\sqrt{\mu}\xi\right)\right)} \quad (3.14)$$

$$u_{11}(\xi)=a_0+(1+\mu\lambda^2)\frac{\sqrt{\mu}\cot\left(\xi_0+\right.}{\left(1+\lambda\sqrt{\mu}\cot\left(\xi\right.\right.} \quad (3.15)$$

Where $\xi=x\pm2\sqrt{-\mu}t$

Profile-7:

$$u_{12}(\xi)=a_0+\frac{\sqrt{\mu}}{2}\left(\tan(\xi_0-\sqrt{\mu}\xi)\pm\sqrt{\left(1+\left(\tan(\xi_0-\sqrt{\mu}\xi)\right)^2\right)}\right) \quad (3.16)$$

$$u_{13}(\xi)=a_0+\frac{\sqrt{\mu}}{2}\left(\cot(\xi_0+\sqrt{\mu}\xi)\pm\sqrt{\left(1+\left(\cot(\xi_0+\sqrt{\mu}\xi)\right)^2\right)}\right) \quad (3.17)$$

Where nnw $\xi=x\pm\sqrt{-\mu}t$

Profile-8:

$$u_{14}(\xi)=a_0-\left(\lambda\mu+\sqrt{\mu}\cot(\xi_0-\sqrt{\mu}\xi)\right) \quad 3.18$$

$$u_{15}(\xi)=a_0-\left(\lambda\mu+\sqrt{\mu}\tan(\xi_0+\sqrt{\mu}\xi)\right) \quad (3.19)$$

Where, $\xi=x\pm2\sqrt{-\mu}t$

Profile-9:

$$u_{16}(\xi)=a_0+\sqrt{\mu}\left(\tan\left(\xi_0-\sqrt{\mu}\xi\right)-\cot\left(\xi_0-\sqrt{\mu}\xi\right)\right) \quad (3.20)$$

$$u_{17}(\xi)=a_0+\sqrt{\mu}\left(\cot\left(\xi_0+\sqrt{\mu}\xi\right)-\tan\left(\xi_0+\sqrt{\mu}\xi\right)\right) \quad (3.21)$$

Where, $\xi=x\pm4\sqrt{-\mu}t$

Profile-10:

$$u_{18}(\xi)=a_0-\frac{\sqrt{\mu}}{2}\cot\left(\xi_0-\sqrt{\mu}\xi\right)\left(\left(1+\lambda\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi)\right)\mp\sqrt{\left(1+\left(\tan(\xi_0-\sqrt{\mu}\xi)\right)^2\right)}\right)$$

(3.22)

$$u_{19}(\xi)=a_0-\frac{\sqrt{\mu}}{2}\tan\left(\xi_0+\sqrt{-\mu}\xi\right)\tan\left(\xi_0+\sqrt{-\mu}\xi\right)\left(\left(1+\lambda\sqrt{\mu}\cot(\xi_0+\sqrt{\mu}\xi)\right)\mp\sqrt{\left(1+\left(\cot(\xi_0+\sqrt{\mu}\xi)\right)^2\right)}\right)$$

(3.23)

Where, $\xi=x\pm\sqrt{-\mu}t$

Example 2: In this subsection, we will apply the given method in section 2 for the exact solution and then the solitary wave solution to the following generalized new fifth order non-linear equation of the form [21]

$$u_{ttt}-u_{txxxx}-\alpha\left(u_xu_t\right)_{xx}-\beta\left(u_xu_{xt}\right)_x=0 \quad (3.24)$$

Where, α and β are constant.

The traveling wave transformation $u(x,t)=u(\xi),\xi=kx-\omega t$, switches (3.24) to the ODE in the form

$$-\omega^3u'''+\omega k^4u^{(v)}+\alpha\omega k^3\left(u'u'\right)''+\beta\omega k^3\left(u'u''\right)'=0 \quad (3.2)$$

Integrating (3.2) with respect to ξ twice and taking integration constant to zero, we attain

$$k^4u'''+k^3(\alpha+\beta/2)u'^2-\omega^2u'=0 \quad (3.26)$$

How to cite this article: A.K. M.K.S. Hossain., M. A. Akbar. Closed Form Solutions of New Fifth Order Nonlinear Equations and New Generalized Fifth Order Nonlinear Equations via the enhanced (G'/G)-expansion Method. Biostat Biometrics Open Acc J. 2017; 4(2): 555631. DOI: 10.19080/BBOAJ.2017.04.555631.

Balancing the highest-order derivative term u''' and the highest-order nonlinear term u'^2, yields $n=1$.

Thus, the solution structure of Eq. (3.26) becomes

$$u(\xi)=a_0+\frac{a_1\left(\frac{G'}{G}\right)}{1+\lambda\left(\frac{G'}{G}\right)}+\frac{a_{-1}\left(1+\lambda\left(\frac{G'}{G}\right)\right)}{\left(\frac{G'}{G}\right)}+b_0\left(\frac{G'}{G}\right)^{-1}\sigma\left(1+\frac{\left(\frac{G'}{G}\right)^2}{\mu}\right)+b_1\sigma\left(1+\frac{\left(\frac{G'}{G}\right)^2}{\mu}\right)+b_{-1}\left(\frac{G'}{G}\right)^{-2}\sigma\left(1+\frac{\left(\frac{G'}{G}\right)^2}{\mu}\right)$$

(3.27)

Where, $G=G(\xi)$ satisfies Eq. (2.5).

Replacing (3.27) with the equation (2.5) into equation (3.26), we achieve a polynomial of $(G'/G)^i$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$. Equating the coefficient of these to zero, we achieve a system of algebraic equation which on solving, we get

Set 1: $\omega=\pm 2k^2\sqrt{-\mu},\lambda=\lambda,a_{-1}=0,a_0=a_0,a_1=\frac{12k(1+\mu\lambda^2)}{(2\alpha+\beta)},b_{-1}=0,b_0=0,b_1=0$.

Set 2: $\omega=\pm k^2\sqrt{-\mu},\lambda=0,a_{-1}=0,a_0=a_0,a_1=\frac{6k}{(2\alpha+\beta)},b_{-1}=b_0=0,b_1=\pm\frac{6k\sqrt{\mu}}{(2\alpha+\beta)\sqrt{\sigma}}.$.

Set 3: $\omega=\pm 2k^2\sqrt{-\mu},\lambda=\lambda,a_{-1}=-\frac{12k\mu}{(2\alpha+\beta)},a_0=a_0,a_1=0,b_{-1}=0,b_0=0,b_1=0$

Set 4: $\omega=\pm 4k^2\sqrt{-\mu},\lambda=0,a_{-1}=-\frac{12k\mu}{(2\alpha+\beta)},a_0=a_0,a_1=\frac{12k}{(2\alpha+\beta)},b_{-1}=b_0=0,b_1=0.$

Set 5: $\omega=\pm 2k^2\sqrt{-\mu},\lambda=\lambda,a_{-1}=-\frac{6k\mu}{(2\alpha+\beta)},a_0=a_0,a_1=b_{-1}=b_1=0,b_0=\pm\frac{6k\mu}{(2\alpha+\beta)\sqrt{\sigma}}.$

Now setting solution set 1-5 with equation (2.5) into equation (3.27), we get adequate traveling wave solution to Eq. (3.24) as follows:

When, $\mu<0,$, we get the hyperbolic solution,

Type-1:

$$u_{2_1}(\xi)=a_0+\frac{12k(1+\mu\lambda^2)}{(2\alpha+\beta)}\frac{\sqrt{-\mu}\tanh(\xi_0+\sqrt{-\mu}\xi)}{(1+\lambda\sqrt{-\mu}\tanh(\xi_0+\sqrt{-\mu}\xi))}\quad(3.28)$$

$$u_{2_2}(\xi)=a_0+\frac{12k(1+\mu\lambda^2)}{(2\alpha+\beta)}\frac{\sqrt{-\mu}\coth(\xi_0+\sqrt{-\mu}\xi)}{(1+\lambda\sqrt{-\mu}\coth(\xi_0+\sqrt{-\mu}\xi))}\quad(3.29)$$

Where, $\xi=x\pm 2k^2\sqrt{-\mu}t$

Type -2:

$$u_{2_3}(\xi)=a_0+\frac{6k}{(2\alpha+\beta)}\left(\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\pm\sqrt{\mu\left(1-\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)}\right)(3.30)\quad(3.30)$$

$$u_{2_3}(\xi)=a_0+\frac{6k}{(2\alpha+\beta)}\left(\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\pm\sqrt{\mu\left(1-\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)}\right)(3.30)\quad(3.31)$$

Where, $\xi=x\pm k^2\sqrt{-\mu}t$

Type-3:

$$u_{2_5}(\xi)=a_0-\frac{12k}{(2\alpha+\beta)}\left(\lambda\mu+\sqrt{-\mu}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)\quad(3.32)$$

$$u_{2_6}(\xi)=a_0-\frac{12k}{(2\alpha+\beta)}\left(\lambda\mu+\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)\quad(3.33)$$

Where, $\xi=x\pm 2k^2\sqrt{-\mu}t,$

Type-4:

$$u_{2_7}(\xi)=a_0\pm\frac{12k\sqrt{-\mu}}{(2\alpha+\beta)}\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)-\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)$$

Where,

Type-5:

$$u_{2_8}(\xi)=a_0-\frac{6k\sqrt{-\mu}}{(2\alpha+\beta)}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\left(\left(1+\lambda\sqrt{-\mu}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)\mp\sqrt{\left(1-\left(\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)}\right)$$

(3.35)

$$u_{2_9}(\xi)=a_0-\frac{6k\sqrt{-\mu}}{(2\alpha+\beta)}\tanh\left(\xi_0+\sqrt{-\mu}\xi\right)\left(\left(1+\lambda\sqrt{-\mu}\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)\mp\sqrt{\left(1-\left(\coth\left(\xi_0+\sqrt{-\mu}\xi\right)\right)^2\right)}\right)$$

(3.36)

Where, $\xi=x\pm k^2\sqrt{-\mu}t$

Again, for $\mu>0$, we get the following trigonometric solution:

Type-6:

$$u_{2_{10}}(\xi)=a_0+\frac{12k(1+\mu\lambda^2)}{(2\alpha+\beta)}\frac{\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi)}{(1+\lambda\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi))}\quad(3.37)$$

$$u_{2_{11}}(\xi)=a_0+\frac{12k(1+\mu\lambda^2)}{(2\alpha+\beta)}\frac{\sqrt{\mu}\cot(\xi_0+\sqrt{\mu}\xi)}{(1+\lambda\sqrt{\mu}\cot(\xi_0+\sqrt{\mu}\xi))}\quad(3.38)$$

Where, $\xi=x\pm 2k^2\sqrt{-\mu}t$

Type-7:

$$u_{2_{12}}(\xi)=a_0+\frac{6k\sqrt{\mu}}{(2\alpha+\beta)}\left(\tan\left(\xi_0-\sqrt{\mu}\xi\right)\pm\sqrt{\left(1+\left(\tan\left(\xi_0-\sqrt{\mu}\xi\right)\right)^2\right)}\right)\quad(3.39)$$

$$u_{2_{13}}(\xi)=a_0+\frac{6k\sqrt{\mu}}{(2\alpha+\beta)}\left(\cot\left(\xi_0+\sqrt{\mu}\xi\right)\pm\sqrt{\left(1+\left(\cot\left(\xi_0+\sqrt{\mu}\xi\right)\right)^2\right)}\right)\quad(3.40)$$

Where, $\xi=x\pm k^2\sqrt{-\mu}t$

Type-8:

$$u_{2_{14}}(\xi)=a_0-\frac{12k}{(2\alpha+\beta)}\left(\lambda\mu+\sqrt{\mu}\cot\left(\xi_0-\sqrt{\mu}\xi\right)\right)\quad(3.41)$$

$$u_{2_{15}}(\xi)=a_0-\frac{12k}{(2\alpha+\beta)}\left(\lambda\mu+\sqrt{\mu}\tan(\xi_0+\sqrt{\mu}\xi)\right)\quad(3.42)$$

Where,

Type-9:

$$u_{2_{16}}(\xi)=a_0+\frac{12k\sqrt{\mu}}{(2\alpha+\beta)}\left(\tan\left(\xi_0-\sqrt{\mu}\xi\right)-\cot\left(\xi_0-\sqrt{\mu}\xi\right)\right)\quad(3.43)$$

$$u_{2_{17}}(\xi)=a_0+\frac{12k\sqrt{\mu}}{(2\alpha+\beta)}\left(\cot\left(\xi_0+\sqrt{\mu}\xi\right)-\tan\left(\xi_0+\sqrt{\mu}\xi\right)\right)\quad(3.44)$$

Where, $u_{2_{17}}(\xi)=a_0+\frac{12k\sqrt{\mu}}{(2\alpha+\beta)}\left(\cot\left(\xi_0+\sqrt{\mu}\xi\right)-\tan\left(\xi_0+\sqrt{\mu}\xi\right)\right)$

Type-10:

$$u_{2_{18}}=a_0+\frac{6k\sqrt{\mu}}{(2\alpha+\beta)}\cot\left(\xi_0-\sqrt{\mu}\xi\right)\left(\left(1+\lambda\sqrt{\mu}\tan\left(\xi_0-\sqrt{\mu}\xi\right)\right)+\sqrt{\left(1+\left(\tan\left(\xi_0-\sqrt{\mu}\xi\right)\right)^2\right)}\right)$$

(3.45)

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$$u_{2_{19}} = a_0 - \frac{6k\sqrt{\mu}}{(2\alpha+\beta)} \tan\left(\xi_0 + \sqrt{\mu}\xi\right)\left(\left(1+\lambda\sqrt{\mu}\cot\left(\xi_0+\sqrt{\mu}\xi\right)\right) \mp \sqrt{\left(1+\left(\cot\left(\xi_0+\sqrt{\mu}\xi\right)\right)^2\right)}\right)$$
(3.46)

Where, $\xi = x \pm k^2\sqrt{-\mu}t$.

Results and Physical Explanations

In this section, we have discussed about the obtained solution of new fifth order non-linear equation and new generalized fifth order non-linear equation. As of the above solution, it has been noticed that $\sigma = \pm 1$ and $\mu \neq 0$. The negative values of μ gives the hyperbolic solutions $u_1(\xi) - u_9(\xi)$ of the new fifth order non-linear equation through Profile 1 to 5 and positive values of μ gives, trigonometric solutions $u_{10}(\xi) - u_{19}(\xi)$ through Profile 6 to 10. The solutions $u_2(\xi)$ and $u_8(\xi)$ demonstrate the nature of kink wave. Solutions $u_1(\xi), u_5(\xi)$ and $u_7(\xi)$ display the nature of singular kink wave. Moreover, solutions $u_{10}(\xi) - u_{19}(\xi)$ represent the character of periodic traveling wave. The solutions $u_4(\xi)$ and $u_6(\xi)$ show the nature of soliton solution where $u_3(\xi)$ and $u_9(\xi)$ express the singular solution. The graphical illustrations of some obtained solutions are given below. The Figure 1 represents the soliton solution $u_4(\xi)$ in (3.8) for $\mu = -1, \sigma = -1, \xi_0 = 5, a_0 = 3, \lambda = 0$ within $-10 \le x, t \le 10$. Singular kink wave solution $u_5(\xi)$ for $\mu = -1, \xi_0 = 1, a_0 = 3, \lambda = 2$ and kink shape wave solution of $u_8(\xi)$ for $\mu = -2, \sigma = 1, \xi_0 = 3, a_0 = 2, \lambda = 1$ within the interval $-10 \le x, t \le 10$ have been shown in Figure 2 and Figure 3 respectively. Periodic wave solution $u_{19}(\xi)$ in (3.23) for $\mu = 1, \sigma = 1, \xi_0 = 1, a_0 = 2, \lambda = 1$ within the interval $-10 \le x, t \le 10$ and $-5 \le t \le 5$ are given by Figure 4.

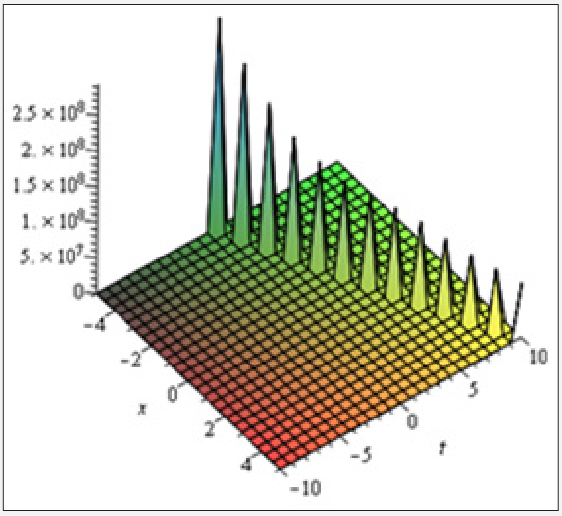

Figure 1: Soliton solution $u_4(\xi)$ in (3.8) for $\mu = -1, \sigma = -1, \xi_0 = 5, a_0 = 3, \lambda = 0.$

Again from the solutions of the new generalized fifth order non-linear equation, it is observed that the negative values of μ suggest the hyperbolic solutions $u_{2_1}(\xi) - u_{2_9}(\xi)$ through Type 1 to 5 and the positive values of μ , recommend the trigonometric solutions $u_{2_{10}}(\xi) - u_{2_{19}}(\xi)$ through Type 6 to 10. The solutions $u_{2_2}(\xi)$ and $u_{2_6}(\xi)$ demonstrate the kink wave. The solutions $u_{2_1}(\xi), u_{2_5}(\xi), u_{2_7}(\xi)$ and $u_{2_8}(\xi)$ are singular kink wave solution. Furthermore, the solutions $u_{2_{10}}(\xi) - u_{2_{19}}(\xi)$ are the periodic traveling wave solution where the solutions $u_{2_3}(\xi)$ and $u_{2_9}(\xi)$ represent singular solution. The solutions $u_{2_4}(\xi)$ express the well-known singular soliton. Figure 5 shows the graphical illustrations of singular soliton $u_{2_4}(\xi)$ in (3.31) for $\mu = -2, \sigma = -1, \xi_0 = 2, a_0 = 3, k = 1, \alpha = 1, \beta = 2, \lambda = 0$ within the interval $-5 \le x \le 5$ and $-3 \le t \le 3$. The Singular kink wave solution $u_{2_7}(\xi)$ in (3.34) for $\mu = -2, \sigma = -1, \xi_0 = 1, a_0 = 3, k = 1, \alpha = 1, \beta = 2, \lambda = 1$ within the interval $-5 \le x, t \le 5$ is given in Figure 6. For simplicity we ignored the others figures.

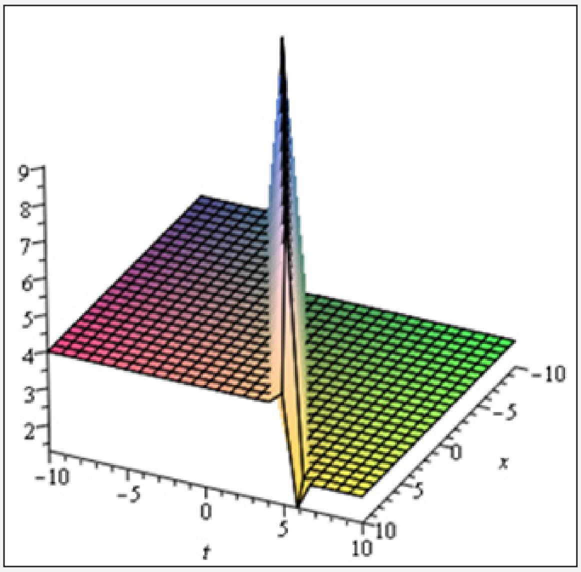

Figure 2: Singular kink wave solution $u_5(\xi)$ in (3.9) for $\mu = -1, \sigma = 1, \xi_0 = 1, a_0 = 3, \lambda = 2.$

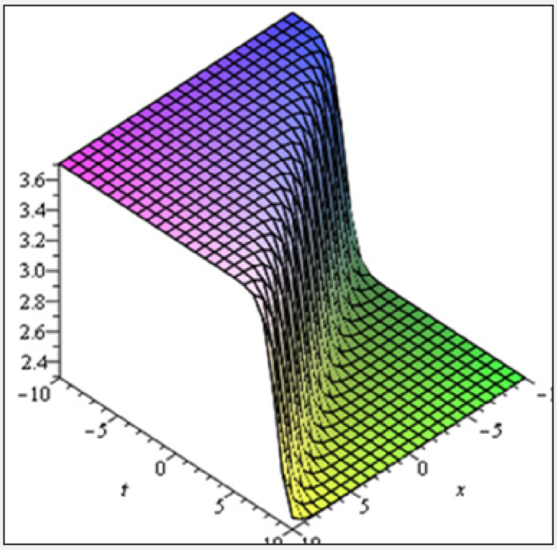

Figure 3: Kink shape wave solution $u_8(\xi)$ in (3.12) for $\mu = -2, \sigma = 1, \xi_0 = 3, a_0 = 2, \lambda = 1.$

How to cite this article: A.K. M.K.S. Hossain., M. A. Akbar. Closed Form Solutions of New Fifth Order Nonlinear Equations and New Generalized Fifth Order Nonlinear Equations via the enhanced (G'/G)-expansion Method. Biostat Biometrics Open Acc J. 2017; 4(2): 555631. DOI: 10.19080/BBOAJ.2017.04.555631.

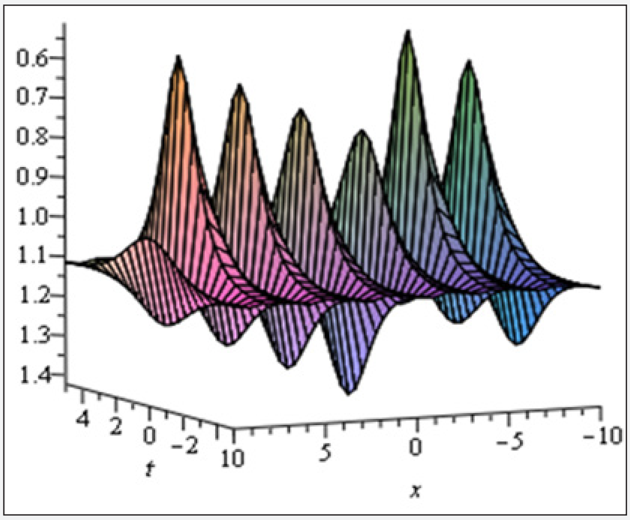

Figure 4: Periodic wave solution $u_{19}(\xi)$ in (3.23) for $\mu=1, \sigma=1, \xi_0=1, a_0=2, \lambda=1$.

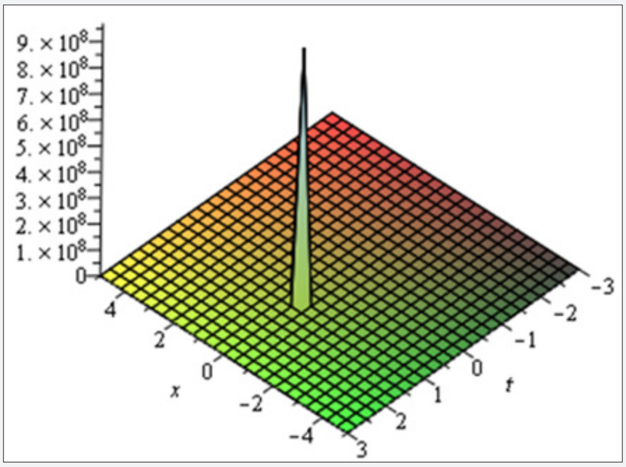

Figure 5: Soliton solution $u_{2_4}(\xi)$ in (3.31) for $\mu=-2, \sigma=-1, \xi_0=2, a_0=3, k=1, \beta=2, \lambda=0$.

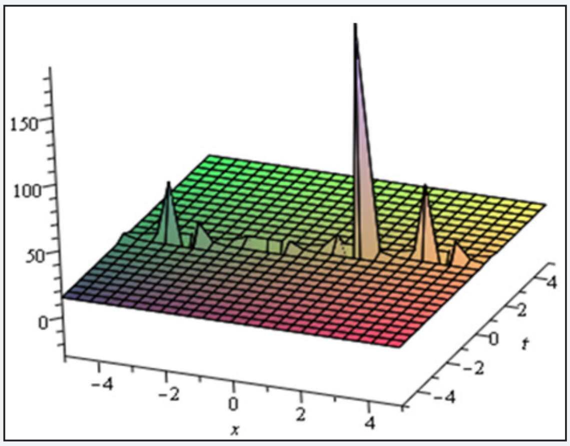

Figure 6: Singular kink solution $u_{(27)}(\xi)$ in (3.34) for $u_{2_7}(\xi)\ \mu=-1, \sigma=-1, \xi_0=1, a_0=3, k=1, \beta=2, \lambda=1$.

Conclusion

In this article, enhanced $(G'/G)-expansion$ method has been successfully applied to find the closed form wave solutions of new fifth order nonlinear equation and new generalized fifth order nonlinear equation. The solutions are verified to check the correctness of the solutions by putting them back into the original equation and found correct. The key advantage of the enhanced $(G'/G)-expansion$ method against other methods is that the method provides more general and huge amount of new closed form wave solutions. The closed form solutions have its great importance to interpretation the inner mechanism of the complex physical phenomena. Therefore this method is very concise and straightforward to handling and can be applied for finding closed form solutions of other NLEEs arising in science and engineering.

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DOI: 10.19080/BBOAJ.2017.04.555631

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