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Investigation of Bending Vibrations in Circular Plates at Elasticity

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Abstract

In this literature, the vibration of circular plates in the presence of dynamic loads has been assessed and analyzed by Elasticity theory in dynamic state and also bending moments and shearing forces instantaneously have been discussed completely. It is necessary to mention that the afore-mentioned procedure has been done by utilizing the method of separation of variables. Then, attempts have been made to obtain modal frequencies by presenting a general formula for circular plates which can be considered as a novelty in the field of vibrations. A couple differential equations for vibrational plates by the separation equation has been solved and consequently modal mass and shearing force have been studied completely. Moreover, modal frequencies and modal shapes has generally been achieved and depicted graphically. In accordance with the obtained displacement charts, it is clear to say that displacement in the first mode of vibration is more than the next ones and also the same result is obtained for bending moment, shearing and stress forces. Furthermore, it is noteworthy that inasmuch as the mentioned circular plate is axisymmetric, the charts of modal shapes have been acquired symmetrically.

Keywords: Bending in Circular Page; Circular Plates Vibration; Modal Shapes; Polar Coordinates

Introduction

Generally speaking, all unknowns of the problems of theory of elasticity in dynamic state are consisted of three components of displacement, six stress components and six strain components along with time and then the mentioned problems will be solved by creating three equilibrium equations, six compatibility equations of strains and finally six equations which are related to the structural behavior. But a wide variety of problems due to their geometric forms and types of loading on them can easily be changed into simpler forms and solved. In the discussion of 2-D problems, it is considered that if the plate thickness is small in comparison with the plate dimensions and also the loading of the problem is limited to the internal forces of the plate, the problem can be solved in the state of 2-D stress with acceptable approximation [1-3]. In this paper, problems of the theory of elasticity in dynamic state have been investigated that they, in fact, are diffusion of stress waves in elastic media. By using Dynamic knowledge in building engineering and earthquake engineering, elastic theory in dynamic state is applied.

Investigating dynamic distributed loading on structural plates and reaction of plate structures as vibration is of great importance for users in civil engineering [4,5]. Plate vibrations in the presence of dynamic forces and loads exerted on them is, in fact, the investigation of the maximum instantaneous strength of the structural elements which the elements are dependent on some parameters such as stresses, bending, shearing forces, and instantaneous displacement due to the vibration of the structures [6-8]. In order to investigate the vibration of the circular plates in regard to dynamic loading [9-12], it is necessary to have the equation governing on the vibrational system which can be non-linear, so in this case, a specific method (AGM) to solve these equations solved [13-15].

Nomenclature

h	Thickness of the circular plate (m)
A	Cross section of the circular plate (m^2)
D	Bending rigidity (N/m^2)
υ	Poison ratio
ρ	Density (kg/m^3)
R	Radius of the circular plate (m)
$q(r)$	Distributed normal load exerted on the circular plate (N/m^2)
λ	Variable changing parameter
ω_n	Modal frequencies $(Rad/\sec)$
E	Modulus elasticity (N/m^2)
J_0, Y_0, J_1, Y_1	Bessel functions
i	Imaginary part of the complex number

Application

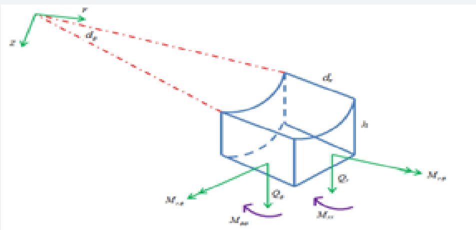

Figure 1: The arbitrary circular plate element of the mentioned physical model with all of the forces and moments exerted on it.

According to Figure 1, in order to gain the motion equations in terms of displacement function, it is essential to write Newton's first law for an optional volume element in a coordinate. Asregards Newton's second law for equilibrium of forces in accordance with Figure 1 which is a circular plate element, the vibrational differential equation governing on a circular plate with dynamic normal load in bending state is achieved as follows [16]:

$$D.\nabla^4 u_z + \bar{m}\frac{\partial^2 u_z}{\partial t^2} = q(r,\theta,t) \quad (1)$$

In partial differential equation (1), parameter is defined as the unit mass per area and is expressed as and $\bar{m} = \rho.h$ and $(D$ is bending rigidity of the structure that is written as:

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \quad (2)$$

Therefore, Eq.(1) is rewritten as follows[13]:

$$D.\nabla^4 u_{z(r,\theta,t)} + \rho h\frac{\partial^2 u_{z(r,\theta,t)}}{\partial t^2} = q(r,\theta,t) \quad (3)$$

$$or \quad D.\nabla^2\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}\right) + \rho h\frac{\partial^2 u_z}{\partial t^2} = q(r,\theta,t)$$

The operator ∇^2 in polar coordinates is defined as follows:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \quad (4)$$

In this paper, as regards Figure 2 in the following, a circular plate is considered which its surrounding is fixed that the distributed load of vibration is exerted on it. With regard to Figure 2a & 2b, since loading in different points of the plate is applied homogeneously on the circular plate, the forces exerted on the plate are axisymmetric. In axisymmetric state, all the terms of differential Eq.(3) are independent from the angular . So, all the derivatives in relevance to are equaled to zero and as a result, in axisymmetric state, the governing differential Eq.(3) will be changed as follows:

$$\frac{D}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_{z(r,t)}}{\partial r}\right)\right)\right\} + \rho h\frac{\partial^2 u_{z(r,t)}}{\partial t^2} = q(r,t) \quad (5)$$

Therefore in regard to Figure 2 a & b $q(r,t)$ is written as follows:

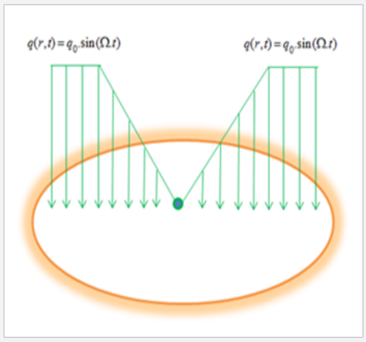

Figure 2a: The schematic diagram of the circular plate.

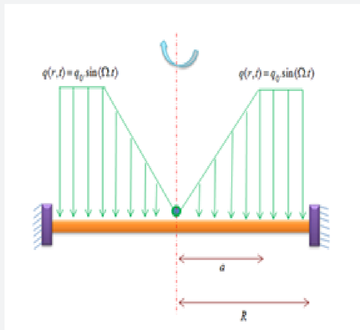

Figure 2b: The frontal view of the symmetric circular plate.

$$q(r,t) = \sin(\Omega t).\begin{cases} \left(\frac{r}{a}\right)q_0 & 0 \le r \le a \\ q_0 & a \le r \le R \end{cases} \quad (6)$$

5.1. Solving the differential equation

Differential equation(5) is solved by the separation equation [14-16] as follows:

$$u_z = V(r).z(t) \quad (7)$$

After substituting Eq.(7) into Eq.(5) and after arranging and substituting square of modal frequency, two differential equations have been yielded in the following forms:

$$\frac{D}{\rho h V(r)}\left\{\frac{d^4V(r)}{dr^4} + \frac{2}{r}\frac{d^3V(r)}{dr^3} - \frac{1}{r^2}\frac{d^2V(r)}{dr^2} + \frac{1}{r^3}\frac{dV(r)}{dr}\right\} = \omega_n^2 \quad (8)$$

$$\frac{1}{z(t)}\frac{d^2z(t)}{\partial t^2} - \frac{\sin(\Omega.t)}{\rho h V(v)z(t)}\left\{\begin{matrix} q_0\left(\frac{r}{a}\right) & 0 \le r \le a \\ q_0 & a \le r \le R \end{matrix}\right\} = -\omega_n^2 \quad (9)$$

And then, Eq.(9) can be rewritten as:

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$$\frac{d^2z(t)}{dt^2}+\omega_n^2 z(t)=\frac{1}{M_n}f_n(\tau) \quad (10)$$

In Eq. (10), parameters $f_n(\tau)$ and M_n are force function and modal mass of the plate vibration, respectively which are defined as equations bellow:

$$M_n=\int_0^{2\pi}\int_0^{R}\rho h V_n^2(r).rdrd\theta \quad (11)$$

$$f_n(\tau)=\int_0^{2\pi}\{(\int_0^{a}q_0(\frac{r}{a})+\int_a^{R}q_0)V_n(v).rdr\}\sin(\Omega t)d\theta \quad (12)$$

In Eq.(12), $V_n(r)$ is the function of modal shapes which by applying boundary conditions will be computed in the forgoing part of the paper.

Computing modal frequencies and modal shapes

By solving Eq. (8) with regard to the boundary conditions of clamping ends, modal frequencies and shapes can be calculated. The answer of differential equation(8) is achieved as the Bessel function of the first and second kinds (J and Y) from zero order as follows:

$$V(r)=c_1 J_0(\lambda r)+c_2 Y_0(\lambda r)+c_3 J_0(i\lambda r)+c_4 Y_0(i\lambda r) \quad (13)$$

It is notable that in the above equation, parameter (i) is imaginary partof complex number and (λ) is defined as follows:

$$\lambda=\sqrt{\omega_n\sqrt{\frac{\rho h}{D}}} \quad (14)$$

Applying boundary conditions

In regard to the circular plate with fixed surrounding and also with axisymmetric loading, boundary conditions can be defined as follows:

The plane slope at the circle center $(r=0)$ is zero because of the axisymmetric state in the circular plane:

$$\frac{dV(r)}{dr}=0 \quad at \quad r=0 \quad (15)$$

Due to the characteristics of Bessel function from the second kind, (Y), in $(r=0)$ we have:

$$Y_0=\infty \quad (16)$$

According to Eq.(16) and the answer of differential equation(13), $(c_2$ and $c_4)$ coefficients will be zero in order to keep the stability of the system:

$$c_2=0 \quad , \quad c_4=0 \quad (17)$$

As a result, the answer of differential equation.(13) can be rewritten in accordance with Eq.(17) as follows:

$$V(r)=c_1 J_0(\lambda r)+c_3 J_0(i\lambda r) \quad (18)$$

Due to the fixed end of the circular plate, displacement and plane slope in points which joined to the body will be obtained zero as follows:

$$B.C\begin{cases} V(R)=0 & \text{therefore} \quad c_1 J_0(\lambda R)+c_3 J_0(i\lambda R)=0 \quad (19)\\ \frac{dV(R)}{dr}=0 & \text{and then} \quad -c_1\lambda J_1(\lambda R)-i\lambda c_3 J_1(i\lambda R)=0 \quad (20)\end{cases}$$

In order to compute non-zero answer from Eq.(19) and Eq.(20), the matrix determinant of the afore-mentioned set of equations should be zero as follows:

$$\det=\begin{vmatrix} J_0(\lambda R) & J_0(i\lambda R)\\ J_1(\lambda R) & iJ_1(i\lambda R)\end{vmatrix}=0 \quad (21)$$

The answer of Eq.(21) is yielded by introducing new variable $(\beta_n=\lambda.R)$ as:

$$\det=J_0(i\beta_n).J_1(\beta_n)-iJ_0(\beta_n).J_1(i\beta_n)=0 \quad (22)$$

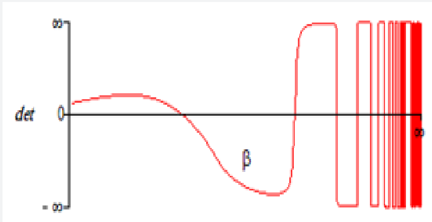

Figure 3: The roots of the determinant function.

If Eq.(22) is depicted as a function in Cartesian coordinates, the roots of the equation can be observed in the following chart. (Figure 3)

Table 1: The results of computing for different modal frequencies.

(n)	1	2	3	4	5	6	7	8	9	10
β_n	3.1962	6.306	9.439	12.577	15.716	18.856	21.997	25.137	28.278	31.420

Finally, the values of (β_n) from Eq.(22) is acquired as follows:

On the basis of the obtained values from Table 1 and changing variable $(\beta_n=\lambda.R)$, parameter (λ) can be gained as follows:

$$\lambda=\frac{\beta_n}{R}=(1.9974957n+0.0226)(\frac{\pi}{2R}) \quad , \quad n=1,2,3,... \quad (23)$$

By substituting Eq.(23) and variable changing $(\beta_n=\lambda.R)$ in to Eq.(14), modal frequencies can be achieved as follows (Figure 4-7):

$$\omega_n=\sqrt{\frac{D}{\rho h}}(\frac{\beta_n}{R})^2 \quad or \quad \omega_n=\sqrt{\frac{D}{\rho h}}(1.9974957n+0.0226)(\frac{\pi}{2R})^2 \quad (24)$$

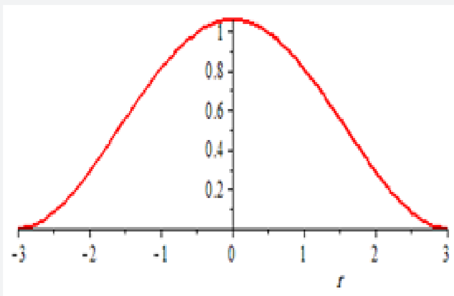

Figure 4: The first mode of vibration (n=1).

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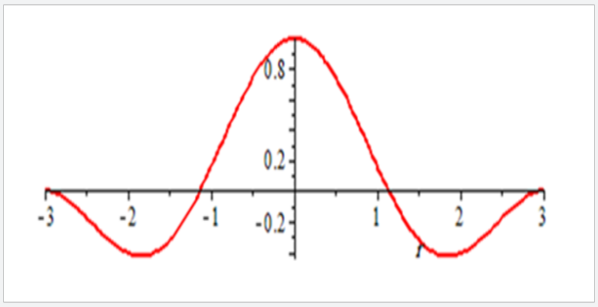

Figure 5: The second mode of vibration (n=2).

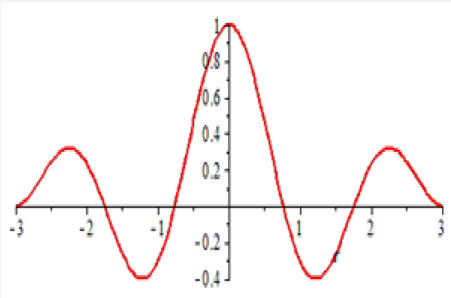

Figure 6: The third mode of vibration (n=3).

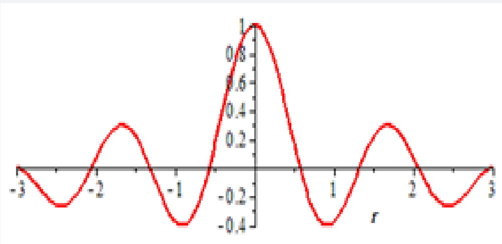

Figure 7: The fourth mode of vibration (n=4).

Computing the equation of modal shapes

By computing the coefficient c_3 in terms of the coefficient c_1 from Eq.(19) or Eq.(20) and substituting into Eq.(18), the equation of modal shapes can be calculated as follows:

$$V_n(r)=\frac{c_1}{J_1(i\beta_n)}\{J_0(\beta_n\frac{r}{R}).J_1(i\beta_n)+iJ_1(\beta_n).J_0(i\beta_n\frac{r}{R})\} \quad (25)$$

In regards to the values of β_n from Table 1 or Eq.(23), the modal shapes can be depicted in different modes for the circular plate as follows:

Solving the differential equation of time index

By determining the function of modal shape $u_n(r)$ from Eq.(25), the functions of modal mass M_n and modal force $f_n(\tau)$ from integrated equations(11,12) can easily be achieved. And also by determining modal mass and force, the differential equation of time index which is Eq.(10) can simply be computed.

It is notable that in order to compute the function of time index, Eq.(10), initial conditions (initial displacement and initial velocity) are needed which by supposing the lack of initial conditions, the answer of the time index differential equation can be yielded in the form of:

$$z_n(t)=\frac{1}{\omega_n M_n}\int_0^t f_n(\tau)\sin\{\omega_n(t-\tau)\}d\tau \quad (26)$$

By determining the displacement of time index $z_n(t)$ from Eq.(26) and the modal shape function $V_n(r)$ from Eq.(25), the vibrational displacement of structure element of the circular plate for every mode can be acquired as follows:

$$U_z(r,t)=u_n(r).z_n(t) \quad (27)$$

Afterwards the displacement of the structure can be gained for all of the modes according to the following equation:

$$U_z(r,t)=\sum_{n=1}^{\infty}u_n(r).z_n(r) \quad (28)$$

Calculating bending moments, stresses and shearing forces in every mode of vibration

Bending moments for each mode of vibration (n) are achieved according to the following equations:

$$\begin{cases} M_{rr}=-D(\frac{\partial^2 u_z}{\partial r^2}+\frac{\upsilon}{r}\frac{\partial u_z}{\partial r}) \\ M_{\theta\theta}=-D(\frac{1}{r}\frac{\partial u_z}{\partial r}+\upsilon\frac{\partial^2 u_z}{\partial r^2}) \end{cases} \quad (29)$$

And shearing forces for each mode of vibration are computed in regard to equations bellow:

$$\begin{cases} Q_r=-D\frac{\partial}{\partial r}(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}) \\ Q_\theta=0 \end{cases} \quad (30)$$

In this step, the stresses exerted on the plate for each mode of vibration are acquired in the forms of:

$$\begin{cases} \sigma_{rr}=-\frac{E.h}{1-\upsilon^2}(\frac{\partial^2 u_z}{\partial r^2}+\frac{\upsilon}{r}\frac{\partial u_z}{\partial r}) \\ \sigma_{\theta\theta}=-\frac{E.h}{1-\upsilon^2}(\frac{1}{r}\frac{\partial u_z}{\partial r}+\upsilon\frac{\partial^2 u_z}{\partial r^2}) \\ \sigma_{r\theta}=0 \end{cases} \quad (31)$$

By choosing the following physical values for the structure, we will have:

$$q_0=50(N/m^2) \quad , \quad E=3\times10^7(N/m^2) \quad , \quad \upsilon=0.3 \quad , \quad h=3(cm) \quad (32)$$
$$R=3(m) \quad , \quad a=2(m) \quad , \quad \Omega=2(Rad/sec)$$

On the basis of the afore-mentioned formulae and given physical values, the displacement of the circular plate, bending moment, shearing force and stress can be observed in the following charts for each mode of vibration. It is noteworthy that each time period, $T=\frac{2\pi}{\omega_n}$, is divided into 4 time domains as follows (Table 2):

Table 2: The obtained modal frequencies in different modes of vibrations.

n	1	2	3	4	5	6	7	8	9	10
ω_n	1.067	4.152	9.303	16.516	25.790	37.125	50.522	65.979	83.497	103.077

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In regard to the afore-mentioned table in the first mode of vibration, the following charts are depicted on the basis of $\omega_1 = 1.067$:

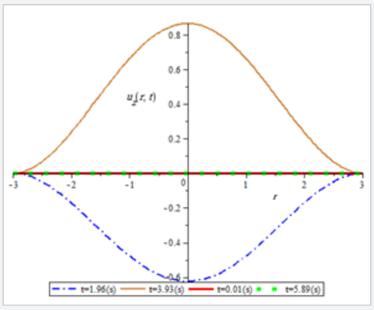

Figure 8: A comparison amongst various instantaneous displacements, $u_z(r,t)$, of the mentioned structure in the first mode of vibration for $t = 0.01, 1.96, 3.93, 5.89(s)$.

I. The displacement chart of the circular element (Figure 8)

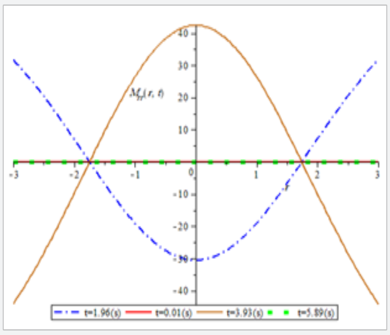

Figure 9: A comparison amongst various $M_{rr}(r,t)$ of the mentioned structure in the first mode of vibration for $t = 0.01, 1.96, 3.93, 5.89(s)$.

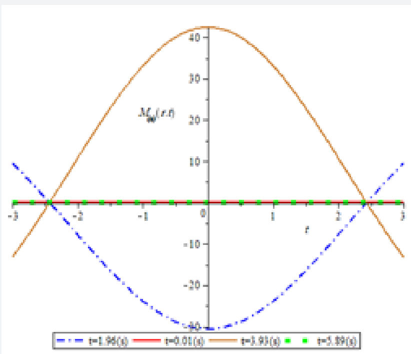

Figure 10: A comparison amongst various $M_{\theta\theta}(r,t)$ of the mentioned structure in the first mode of vibration for $t = 0.01, 1.96, 3.93, 5.89(s)$.

II. The charts of bending moment (Figure 9 &10)

III. Shearing force charts (Figure 11)

IV. Stress charts (Figure 12 &13)

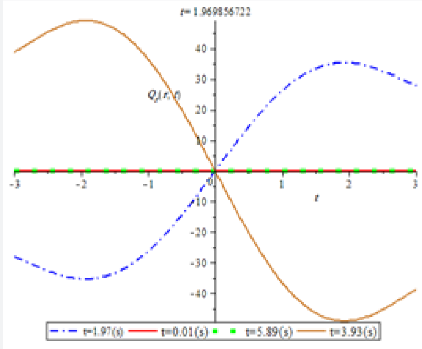

Figure 11: A comparison amongst various $Q_r(r,t)$ of the mentioned structure in the first mode of vibration for $t = 0.01, 1.97, 3.93, 5.89(s)$.

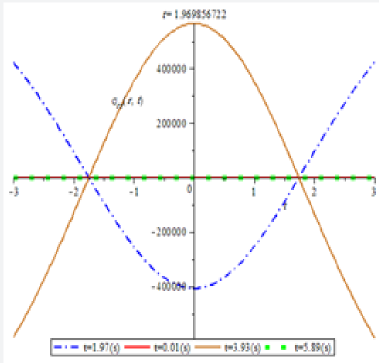

Figure 12: A comparison amongst various $\sigma_{rr}(r,t)$ of the mentioned structure in the first mode of vibration for $t = 0.01, 1.97, 3.93, 5.89(s)$.

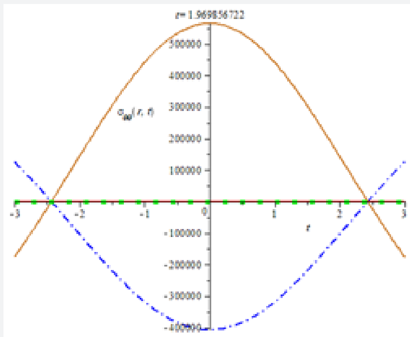

Figure 13: A comparison amongst various $\sigma_{\theta\theta}(r,t)$ of the mentioned structure in the first mode of vibration for $t = 0.01, 1.96, 3.93, 5.89(s)$.

Eventually, in the second mode of vibration which $\omega_2 = 4.152$, the following charts are illustrated:

V. The displacement chart of the circular element (Figure 14)

It is notable that the other charts of the second mode and upper modes of vibration such as stress, shearing force and bending moment will easily be computed like the above procedure.

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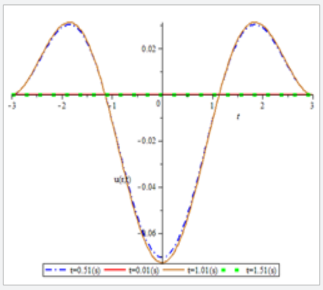

Figure 14: A comparison amongst various instantaneous displacements, $u_z(r,t)$, of the mentioned structure in the second mode of vibration for $t = 0.01, 0.51, 1.01, 1.51(s)$.

Conclusion

In the present paper, the vibration of circular plates in regard to the dynamic loading has been investigated by the method of separation of variables. In order to perform the afore-mentioned procedure, the Newton's first law for an arbitrary circular plate element and the Newton's second law for equilibrium of forces have been applied to achieve the vibrational differential equation governing on a circular plate with dynamic normal load in bending state. Since loading in different points of the mentioned plate is applied homogeneously, the forces exerted on the plate are axisymmetric and as everyone knows in axisymmetric state, all the terms of the obtained differential Equations are independent from the angular . So, all the derivatives in relevance to are equaled to zero and as a result, the equation governing on the present system simplified significantly.

After that by utilizing Separation law, modal frequencies and modal shapes have been acquired in regard to the boundary conditions. Consequently, the functions of modal mass and modal frequencies have been achieved. In this step, the time index function has been yielded in order to compute the vibrational displacement of the circular plate for each mode of vibration on the basis of the most general solution which is the sum of all solutions (Separation Law). Eventually, the values of bending moment, shearing force and shearing stress for the first and second mode of vibrations have been acquired and depicted completely. Results show that displacement in the first mode of vibration is more than the next ones and also the same result is obtained for bending moment, shearing and stress forces. Furthermore, it is noteworthy that in as much as the mentioned circular plate is axisymmetric, the charts of modal shapes have been acquired symmetrically.

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