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Knowledge Representation and Knowledge Reasoning in Square{most} and Square{all}

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Abstract

Making much of reduction operations, this paper firstly proves the validity of the generalized syllogism $EMO-1$, and then infers the other 14 valid generalized ones from $EMO-1$. That is to say that these syllogisms are reducible. The reason why these syllogisms are reducible is that the quantifiers in Square{all} can be mutually defined, and that so can the quantifiers in Square{most}. All proofs in this paper are deductive reasoning. All proofs in this paper are deductive reasoning, therefore this work is consistent. It is hoped that this paper not only benefits the study of modern logic, but also further development of inference machines.

Keywords: Aristotelian Quantifiers; Generalized Quantifiers; Generalized Syllogisms; Validity; Non-Trivial Generalized Syllogisms

Introduction

Generally speaking, there are lots of generalized quantifiers in natural language [1]. Noun phrases as well as their determiners are generalized quantifiers (such as his bag, most, both, fewer than half of the, infinitely many). In particular, Aristotelian quantifiers are special cases of generalized quantifiers, the latter being extensions of the former [2]. Hence, generalized syllogisms are extensions of Aristotelian syllogisms. The generalized quantifiers except Aristotelian ones are non-trivial generalized quantifiers, thus a non-trivial generalized syllogism includes at least one of them [3,4]. There are many domestic and international works on classical syllogisms, such as Aristotelian syllogisms [5-7] and Aristotelian modal syllogisms [8-10].

But there are few works on generalized syllogisms. This paper aims to promote their study. Since there are many generalized quantifiers in English language, this paper specifically concentrates on the generalized syllogisms with the generalized quantifiers in Square{most} which is composed of the following: most, at most half of the, fewer than half of the and at least half of the. Any one of them is the other three outer, inner, or dual negative quantifiers. Any of them can define the other three. Similarly, so can the quantifiers in Square{all} which is constituted of Aristotelian quantifiers (i.e. all, not all, some, no).

Preliminaries

In this paper, let g, r and u be lexical variables, and the sets composed of g, r and u be G, R, and U, respectively. '$|G \cap U|$' indicates the cardinality for the intersection of G and U. And D represents the domain of these variables. Q stands for any of the quantifiers in Square{all}, and $Q\neg$ and $\neg Q$ for its inner and outer negative quantifier, respectively. Let β, δ, ϕ and λ be well-formed formulas (shortened to wff). '$\vdash \beta$' states that the wff β can be proved, and '$\delta =_{\text{def}} \lambda$' that d can be defined by λ. The others are similar. All logical connectives (such as $\neg, \wedge, \rightarrow, \leftrightarrow$) in this paper are common operators in first-order logic [11].

The generalized syllogisms studied in the paper merely involve the quantifiers as follows: most, at least half of the, fewer than half of the, at most half of the, all, some, no, not all, and a non-trivial one at least involves one of the first four. Consequently, these syllogisms only involve the following propositions: $all(g,u)$, $not\,all(g,u)$, $some(g,u)$, $no(g,u)$, $most(g,u)$, $at\,least\,half\,of\,the(g,u)$, $at\,most\,half\,of\,the(g,u)$, $fewer$ than $half\,of\,the(g,u)$. They are respectively abbreviated as Proposition A, O, I, E, M, S, H, and F. For example, the first figure syllogism $no(r,u) \wedge most(g,r) \rightarrow not\,all(g,u)$ is abbreviated as EMO-1. Its instance is as follows:

Major premise: No student is a teacher.

Minor premise: Most of people in the classroom are students.

Conclusion: Not all people in the classroom are teachers.

Generalized Syllogism System with the Quantifier 'most'

This system includes the following parts: primitive symbols, relevant definitions, deductive rules and facts, and so on.

Primitive Symbols

i. brackets: (,)

ii. variables: g, r, u

iii. connectives: $\neg, \rightarrow$

iv. quantifiers: most, all

Formation Rules

i. If Q is a quantifier, g and u are variables, then Q(g, u) is a wff.

ii. If λ is a wff, then so is $\neg\lambda$.

iii. If λ and δ are wffs, then so is $\lambda \rightarrow \delta$.

iv. Only the formulas constructed by the above rules are wffs.

Basic Axioms

A1: If β is a valid proposition in classical logic, then $\vdash \beta$.

A2: $\vdash no(r,u) \wedge most(g,r) \rightarrow not\ all(g,u)$ (that is, the syllogism EMO-1).

Rules of Deduction

Rule 1 (subsequent weakening): If $\vdash(\beta \wedge \delta \rightarrow \phi)$ and $\vdash(\phi \rightarrow \lambda)$, then $\vdash(\beta \wedge \delta \rightarrow \lambda)$.

Rule 2 (anti-syllogism): If $\vdash(\beta \wedge \delta \rightarrow \phi)$, then $\vdash(\neg\phi \wedge \beta \rightarrow \neg\delta)$.

Rule 3 (anti-syllogism): If $\vdash(\beta \wedge \delta \rightarrow \phi)$, then $\vdash(\neg\phi \wedge \delta \rightarrow \neg\beta)$.

Relevant Definitions

D1 (conjunction): $(\beta \wedge \delta) =_{def} \neg(\beta \rightarrow \neg\delta)$;

D2 (bicondition): $(\beta \leftrightarrow \delta) =_{def} \neg(\beta \rightarrow \delta) \wedge (\delta \rightarrow \beta)$;

D3 (inner negation): $(Q\neg)(g,u) =_{def} Q(g, D-u)$;

D4 (outer negation): $(\neg Q)(g,u) =_{def}$ It is not that $Q(g,u)$;

D5 (truth value): $all(g,u) =_{def} G \subseteq U$;

D6 (truth value): $some(g,u) =_{def} G \cap U \neq \phi$;

D8 (truth value): $no(g,u) =_{def} G \cap U \neq \phi$;

D9 (truth value): $not\ all(g,u) =_{def} G \nsubseteq U$;

D10 (truth value): $at\ least\ half\ of\ the(g,u)$ is true iff $|G \cap U| \leq 0.5|G|$ is true;

D11 (truth value): $most(g,u)$ is true iff $|G \cap U| \leq 0.5|G|$ is true;

D12 (truth value): $at\ most\ half\ of\ the(g,u)$ is true iff $|G \cap U| \leq 0.5|G|$;

D13 (truth value): $fewer$ than $half\ of\ the(g,u)$ is true iff $|G \cap U| \leq 0.5|G|$ is true.

Relevant Facts

Fact 1(inner negation):

i. $\vdash all(g,u) = no\neg(g,u)$;

ii. $\vdash no(g,u) = all\neg(g,u)$;

iii. $\vdash some(g,u) = not\ all\neg(g,u)$;

iv. $\vdash not\ all(g,u) = some\neg(g,u)$;

v. $\vdash most(g,u) = fewer$ than $half\ of\ the\neg(g,u)$;

vi. $\vdash$ $fewer$ than $half\ of\ the(g,u) = most\neg(g,u)$;

vii. $\vdash at\ least\ half\ of\ the(g,u) = at\ most\ half\ of\ the\neg(g,u)$;

viii. $\vdash at\ most\ half\ of\ the(g,u) = at\ least\ half\ of\ the\neg(g,u)$.

Fact 2(outer negation):

i. $\vdash \neg all(g,u) = not\ all(g,u)$;

ii. $\vdash \neg not\ all(g,u) = all(g,u)$;

iii. $\vdash \neg no(g,u) = some(g,u)$;

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iv. $\vdash \neg some(g,u) = no(g,u)$;

v. $\vdash \neg most(g,u) = at\ most\ half\ of\ the(g,u)$;

vi. $\vdash \neg at\ most\ half\ of\ the(g,u) = most(g,u)$;

vii. $\vdash \neg fewer$ than $half\ of\ the(g,u) = at\ least\ half\ of\ the(g,u)$;

viii. $\vdash \neg at\ least\ half\ of\ the = fewer$ than $half\ of\ the(g,u)$.

Fact 3 (symmetry):

i. $\vdash some(g,u) \leftrightarrow some(u,g)$;

ii. $\vdash no(g,u) \leftrightarrow no(u,g)$.

Fact 4(Subordination):

i. $\vdash all(g,u) \to some(g,u)$;

ii. $\vdash no(g,u) \to not\ all(g,u)$;

iii. $\vdash all(g,u) \to most(g,u)$;

iv. $\vdash most(g,u) \to some(g,u)$;

v. $\vdash at\ least\ half\ of\ the(g,u) \to some(g,u)$;

vi. $\vdash all(g,u) \to at\ least\ half\ of\ the(g,u)$;

vii. $\vdash at\ most\ half\ of\ the(g,u) \to not\ all(g,u)$;

viii. $\vdash fewer$ than $half\ of\ the(g,u) \to not\ all(g,u)$.

The above facts can be proved in generalized quantifier theory [12,13] or first-order logic [11].

The Reducible Relationships between/among Generalized Syllogisms

The validity of the syllogism $EMO-1$ is proved in Theorem 1. The following (2.1) '$EMO-1 \to EAH-2$' illustrates the syllogisms $EAH-2$ derived from $EMO-1$. In other words, $EAH-2$ is valid. It can be concluded that there is a reducible relationship between the two syllogisms. Other situations are similar to this.

Theorem 1 (EMO-1): The syllogism $no(r,u) \wedge most(g,r) \to not\ all(g,u)$ is valid.

Proof: Suppose that $no(r,u)$ and $most(g,r)$ are true, then $R \cap U = \phi$ and $|G \cap R| > 0.5|G|$ are true according to Definition D8 and D11, respectively. Hence it shows that $|G \cap U| \le 0.5|G|$. And it shows that $at\ most\ half\ of\ the(g,u)$ is true in the light of Definition D11. Thus, it gives that $not\ all(g,u)$ is true in line with Fact (4.7).

Theorem 2: There are at least the following 14 valid generalized syllogisms obtained from $EMO-1$:

i. $EMO-1 \to EMO-2$

ii. $EMO-1 \to EAH-2$

iii. $EMO-1 \to EAH-2 \to EAH-1$

iv. $EMO-1 \to AMI-3$

v. $EMO-1 \to AMI-3 \to MAI-3$

vi. $EMO-1 \to AMI-1$

vii. $EMO-1 \to AMI-1 \to MAI-4$

viii. $EMO-1 \to EMO-2 \to AFO-2$

ix. $EMO-1 \to EMO-2 \to AFO-2 \to AAS-1$

x. $EMO-1 \to EMO-2 \to AFO-2 \to FAO-3$

xi. $EMO-1 \to EAH-2 \to AEH-2$

xii. $EMO-1 \to EAH-2 \to AEH-2 \to AEH-4$

xiii. $EMO-1 \to AMI-3 \to EMO-3$

xiv. $EMO-1 \to AMI-3 \to EMO-3 \to EMO-4$

Proof:

[1] $\vdash no(r,u) \wedge most(g,r) \to not\ all(g,u)$ (i.e. $EMO-1$, Basic Axiom A2)

[2] $\vdash no(u,r) \wedge most(g,r) \to not\ all(g,u)$ (i.e. $EMO-2$, by [1] and Fact (3.2))

[3] $\vdash \neg not\ all(g,u) \wedge no(r,u) \to \neg most(g,r)$ (by [1] and Rule 2)

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[4] $\vdash all(g,u) \wedge no(r,u) \rightarrow at\ most\ half\ of\ the(g,r)$ (i.e. $EAH-2$, by [3], Fact (2.2) and (2.5))

[5] $\vdash all(g,u) \wedge no(u,r) \rightarrow at\ most\ half\ of\ the(g,r)$ (i.e. $EAH-1$, by [4] and Fact (3.2))

[6] $\vdash \neg not\ all(g,u) \wedge most(g,r) \rightarrow \neg no(r,u)$ (by [1] and Rule 3)

[7] $\vdash all(g,u) \wedge most(g,r) \rightarrow some(r,u)$ (i.e. $AMI-3$, by [6], Fact (2.2) and (2.3))

[8] $\vdash all(g,u) \wedge most(g,r) \rightarrow some(u,r)$ (i.e. $MAI-3$, by [4] and Fact (3.1))

[9] $\vdash all\neg(r,u) \wedge most(g,r) \rightarrow some\neg(g,u)$ (by [1], Fact (2.1) and (2.4))

[10] $\vdash all(r,D-u) \wedge most(g,r) \rightarrow some(g,D-u)$ (i.e. $AMI-1$, by [9] and Definition D3)

[11] $\vdash all(r,D-u) \wedge most(g,r) \rightarrow some(D-u,g)$ (i.e. $MAI-4$, by [10] and Fact (3.1))

[12] $\vdash all\neg(u,r) \wedge fewer$ than $half\ of\ the\neg(g,r) \rightarrow not\ all(g,u$ (by [2], Fact (1.2) and (1.5))

[13] $\vdash all(u,D-r) \wedge fewer$ than $half\ of\ the\neg(g,D-r) \rightarrow not\ all(g,u)$ (i.e. $AFO-2$, by [12] and Definition D3)

[14] $\vdash \neg not\ all(g,u) \wedge all(u,D-r) \rightarrow \neg fewer$ than $half\ of\ the(g,D-r)$ (by [13] and Rule 2)

[15] $\vdash all(g,u) \wedge all(u,D-r) \rightarrow at\ least\ half\ of\ the(g,D-r)$ (i.e. $AAS-1$, by [14], Fact (2.2) and (2.7))

[16] $\vdash \neg not\ all(g,u) \wedge fewer$ than $half\ of\ the(g,D-r) \rightarrow \neg all(u,D-r)$ (by [13] and Rule 3)

[17] $\vdash all(g,u) \wedge fewer$ than $half\ of\ the(g,D-r) \rightarrow \neg not\ all(u,D-r)$ (i.e. $FAO-3$, by [16], Fact (2.1) and (2.2))

[18] $\vdash no\neg(g,u) \wedge all(r,u) \rightarrow at\ most\ half\ of\ the(g,r)$ (by [4], Fact (1.1) and (1.2))

[19] $\vdash no(g,D-u) \wedge all(r,D-u) \rightarrow at\ most\ half\ of\ the(g,r)$ (i.e. $AEH-2$, by [18] and Definition D3)

[20] $\vdash no(D-u,g) \wedge all(r,D-u) \rightarrow at\ most\ half\ of\ the(g,r)$ (i.e. $AEH-4$, by [19] and Fact (3.1))

[21] $\vdash no\neg(g,u) \wedge most(g,r) \rightarrow not\ all\neg(r,u)$ (by [7], Fact (1.1) and (1.3))

[22] $\vdash no(g,D-u) \wedge most(g,r) \rightarrow not\ all(r,D-u)$ (i.e. $EMO-3$, by [21] and Definition D3)

[23] $\vdash no(D-u,g) \wedge most(g,r) \rightarrow not\ all(r,D-u)$ (i.e. $EMO-4$, by [22] and Fact (3.2))

So far, through the above 23 steps of reduction operations, the above 14 valid generalized syllogisms in Theorem 2 have been inferred from the valid syllogism $EMO-1$.

Conclusion and Future Work

All in all, making much of reduction operations, this paper firstly uses relevant definitions, rules and facts to prove the validity of the generalized syllogism $EMO-1$, and then infers the other 14 valid generalized ones from $EMO-1$. That is to say that these syllogisms are reducible. The reason why these syllogisms have reducibility is that the quantifiers in Square{all} can be mutually defined, and that so can the quantifiers in Square{most}. All proofs in this paper are deductive reasoning, therefore this work is consistent.

In the same way, one can derive more valid syllogisms by continuing to use reduction operations. Moreover, their validity can be proven similar to Theorem 1. As a matter of fact, there are (8´8´8´4-4´4´4´4=) 1972 non-trivial generalized syllogisms merely involving the quantifiers in Square {all} and Square{most}. How can one effectively filter out all the valid ones among them? This question deserves further discussion.

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