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Behavior Related to Taxation System: Example of Bi-Criteria Linear Program for Animal Diet Formulation

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Abstract

The Pareto set of a mathematical model is a useful tool for a decision maker in case of conflicting criteria. This Pareto set contains all the information on the behavior resulting from an optimal decision. For a linear model, this Pareto set shows abrupt (discrete) changes in the behavior depending on the weight, or the tax, associated to the criteria. Moreover, for a bi-criteria linear model, the Pareto curve in the criteria space is a simple polygonal curve. We illustrate those facts by solving the pig diet formulation problem which considers not only the cost of the diet but also its environmental impact based here on the phosphorus excretion related to the diet.

Keywords: Behavior; Taxation System; Bi-Criteria Linear Program; Pareto Set; Weighted-Sum; Diet Formulation

Introduction

Animal diet formulation is a very important problem from an economic and environmental point of view, so it is an interesting example in the field of operations research. Many modern animal diet formulation methods tend to consider not only the cost of the diet but also excretions that are detrimental from an environmental point of view. Following [5], it is appropriate to apply a tax on excretions to change the behavior of the producers in the swine industry. These changes in behavior are studied using a formulation of the problem as a bi-criteria model and are obtained by the determination of its Pareto set. For linear models, the changes in behavior of the producer are abrupt (discrete) and correspond to specific values of the tax. In other words, even in increasing the tax it can happen that there is no change in the behavior of the producer. Behavior changes happened only at very specific values of the tax. We will see that these behaviors correspond to efficient extreme points of the Pareto set, and to every extreme point corresponds a tax interval so that any value of the tax in this interval leads to the behavior given by that same extreme point.

The outline of the paper is the following. In Section 2, we present the diet formulation problem considering the phosphorus excretion. The general form of the bi-criteria problem is presented, the geometric structure of its Pareto set is described, and we indicate methods to compute this set in Section 3. Finally, in Section 4, we present the pareto set for our original diet formulation problem and shows the effect of the taxation system on the behavior of a producer. The results presented in this paper, the behavior of the decision maker, can be applied to any linear bi-criteria problem.

Pig Diet Formulation

To illustrate the effect of a tax on the criteria, we consider the pig diet formulation problem considering not only the cost of the diet but also an environmental consideration such as the reduction phosphorus excretion [1]. One way to analyze this problem is to rewrite the problem as bicriteria problem. Hence the Pareto set indicates the effect of the reduction of phosphorus excretion on the cost of the diet. It also presents different behaviors for the producers associated to levels of taxation. This information is certainly useful for a decision maker which must choose a diet which decreases the excretion without being too expensive.

Classical Model

The least cost diet problem, introduced in [2], is a classical linear programming problem [3-5]. A decision variable x_j is assigned to each ingredient and represents the amount (in kg) of the j^{th} ingredient per unit weight (1 kg) of the feed. Together, they form the decision vector $x = (x_j)^n{}_{j=1}$ in our model. The model's objective function is the diet cost. A vector of unit costs $c = (c_j)^n{}_{j=1}$ is used, where each c_j represents the unit cost of the ingredient (euro/kg or $/kg). Thus, the total cost of a unit of weight (1 kg) of diet $x = (x_j)^n_{j=1}$ is $z = cx = \sum_{j=1}^n c_j x_j$ which must be minimized over the set of feasible diets denoted by S. The classic least cost animal diet formulation model is:

$$(P_{diet})\begin{cases}\min z=cx\\ \text{subject to } x\in S=\left\{x\in R^n \mid Ax \lesseqqgtr b \text{ et } x\geq 0\right\}\end{cases}$$

The constraints impose some bounds on the quantity of the different ingredients in the diet. For example, a unit of feed is produced (a 1 kg mix), expressed by the constraint $\sum_{j=1}^{n} x_j = 1$. Some ingredients, or combinations of ingredients, can be imposed on the diet. These restrictions give rise to equality constraints (=) or inequality constraints (≥ or ≤). More specifically, to satisfy protein requirements, the following constraints are introduced for the L groups of amino acids contained in the ingredients. We set

$$\sum_{j=1}^{n} aa_{lj}^{dig} x_j \geq b_l^* \quad (l = 1,\ \ldots,\ \text{L})$$

where aa_{lj}^{dig} represents the amount of digestible amino acid l contained in a unit of ingredient j and b_l^* is the minimum amount of digestible amino acid l required. Finally, the diet must satisfy the digestible phosphorus requirements b_{ph}^* given by

$$\sum_{j=1}^{n} ph_j^{dig} x_j \geq b_{ph}^*$$

where ph_j^{dig} is the amount of digestible phosphorus contained in a unit of ingredient j.

Modelling of phosphorus excretion

Phosphorus excretion is directly related to the excess of phosphorus in the diet. Hence, we must establish the phosphorus content of the diet and consider the parts that is assimilated. The phosphorus content of a unit weight diet $x = (x_j)_{j=1}^{n}$ is $q_{ph}x = \sum_{j=1}^{n} \text{ph}_j x_j$ is the amount of phosphorus per unit of ingredient j. The amount b_{ph}^* is the the amount of phosphorus which is digested. In this way the phosphorus excretion $r_{ph}(x)$ is given by the phosphorus content of the diet from which we remove the amount of phosphorus which is digested

$$r_{ph}(x) = q_{ph}x - b_{ph}^*$$

Hence, decreasing the phosphorus excretion $r_{ph}(x)$ is equivalent to decreasing the phosphorus content $q_{pr}x$ of the diet while maintained fixed the needs b_{ph}^* in phosphorus.

Bi-Criteria Problem: Cost and Phosphorus Excretion

Since we look for least cost diet while considering the phosphorus excretion, we have two conflicting criteria. The bi-criteria linear model is then formulated as follows

$$(P_{c,ph})\begin{cases}\min z_1=cx\\ \min z_2=q_{ph}x\\ \text{subject to } x\in S\end{cases}$$

For this problem, the Pareto curve will indicate the diet cost increase caused by a phosphorus excretion decrease. It will give us the taxation levels producing changes in the behavior of the producer.

Data

To illustrate the problem and the method we consider data that represent real situation [1]. The ingredients and their corresponding variables are described in (Table 1, Table 2) contains the entire model together with the values of the technical coefficients of the model.

Table 1: List of available ingredients.

Type	Ingredient	Variable
Cereals	oats	x_1
	hard wheat	x_2
	corn	x_3
	barley	x_4
Oleaginous	soybean meal	x_5
	colza meal	x_6
Animal byproducts	meat and bones meal	x_7
	animal fat	x_8
Minerals	dicalcique phosphate	x_9
	calcium carbonate	x_{10}
	sodium chloride	x_{11}
Synthetic amino acids	L-lysine	x_{12}
	DL-methione	x_{13}
	L-threonine	x_{14}
	L-tryptophane	x_{15}
Premix	fixed quantity 5g/kg	x_{16}

Table 2: Technical coefficients of the model: $b_{ph}^* = 2,84$ g/kg.

	Ingredients																	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}		
	MATRIX A																type	b
Total Weight	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	=	1
Energy	2820	3310	3390	3070	3520	2760	2695	8281.25	0	0	0	4780	5640	4120	6570	0	≥	3400 kcal/kg
Sodium	0,2	0,1	0,04	0,1	0,3	0,4	8	0	1,8	0,8	395	0	0	0	0	0	≥	1,29 g/kg
	0,2	0,1	0,04	0,1	0,3	0,4	8	0	1,8	0,8	395	0	0	0	0	0	≤	2,5 g/kg
Calcium	0,9	0,8	0,4	0,7	3,4	8,3	76	0	220	385	3	0	0	0	0	0	≥	6,66 g/kg
Fixed Ingredient	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0,005 kg/kg

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Restricted Ingredients	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	≤	0,400 kg/kg
	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	≤	0,030 kg/kg
	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	≤	0,050 kg/kg
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	≤	0,600 kg/kg
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	≤	0,600 kg/kg
	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	≤	0,250 kg/kg
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	≤	0,050 kg/kg
Digestible Amino Acids																		
Lysine	7	2,5	1,9	2,8	24,9	13,6	25,5	0	0	0	0	798	0	0	0	0	≥	9,73 g/kg
Threonine	5,7	2,7	2,5	2,6	15,3	10,8	16,2	0	0	0	0	0	0	990	0	0	≥	6,07 g/kg
Methionine	2,5	1,5	1,5	1,4	5,9	6	7	0	0	0	0	0	990	0	0	0	≥	2,63 g/kg
Methionine+-Cystine	5,3	3,7	3,3	3,4	11,6	12,6	10,4	0	0	0	0	0	990	0	0	0	≥	5,53 g/kg
Tryptophane	2,2	1,1	0,4	1	5,2	3,3	2,8	0	0	0	0	0	0	0	985	0	≥	1,77 g/kg
Isoleucine	5,8	3,4	2,7	2,9	18,7	10,6	13,4	0	0	0	0	0	0	0	0	0	≥	5,31 g/kg
Valine	8,7	4	3,6	4,1	19,3	13,1	21,3	0	0	0	0	0	0	0	0	0	≥	6,61 g/kg
Leucine	10,2	6,4	9,5	5,7	29,8	18,6	31,9	0	0	0	0	0	0	0	0	0	≥	9,84 g/kg
Phenylalanine	7	4,5	3,7	4,1	20,6	10,9	18,4	0	0	0	0	0	0	0	0	0	≥	5,81 g/kg
Phenylalanine+Tyrosine	12,1	7,1	6,8	6,5	34,5	18,8	29,6	0	0	0	0	0	0	0	0	0	≥	9,12 g/kg
Histidine	4,3	2,2	2,1	1,8	10,9	7,4	9,5	0	0	0	0	0	0	0	0	0	≥	3,11 g/kg
Arginine	10,7	4,7	3,5	4	31,6	17,6	31	0	0	0	0	0	0	0	0	0	≥	4,55 g/kg
Non-Essential Amino Acids	69	54,7	36,3	46,6	206,1	144,9	254,4	0	0	0	0	0	0	0	0	0	≥	72,4 g/kg
Digestible Phosphorus	3,44	1,7	0,364	1,02	1,922	2,394	31,4	0	180,4	0,2	0	0	0	0	0	0	≥	2,84 g/kg
	Technical Coefficients for Ingredients for each Criterion																	
Prices ($/Kg)	0,242	0,321	0,32	0,288	0,455	0,317	0,532	1,236	0,906	0,08	0,198	2,808	5,75	3,75	57	5,269		
Phosphorus Excretions (G/Kg)	8,4	3,4	2,6	3,4	6,2	11,4	38,8	0	185	0,2	0	0	0	0	0	0		

General Bi-Criteria Linear Program and its Pareto Set

In this section we present the general formulation of a bi-criteria linear problem, and the main results on its Pareto set. We use the link with the parametric analysis to get information on the system of taxation and behavior of the decision maker.

Bi-Criteria linear programming problem

Let us consider the standard form of the bi-criteria linear programming problem [6].

$$(P)\begin{cases} \min\ z_1(x)=c_1x \\ \min\ z_2(x)=c_2x \\ \text{subject to } Ax=b, x\geq 0 \end{cases}$$

where x is a column vector in $\mathbb{R}^n$, and they c_k's$(k=1,2)$ are row vectors in $\mathbb{R}^n$. The *feasible set* S in $\mathbb{R}^n$ is defined by $S=\{x\in\mathbb{R}^n \mid Ax=b \text{ and } x\geq 0\}$, where A is a *(m, n)*-matrix, and *b* are a column vector in $\mathbb{R}^n$. Let C be the (2, *n*)-matrix given by

$$c=\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}.$$

The feasible set in the criterion space $\mathbb{R}^2$ is then $S_C=\{z\in\mathbb{R}^2 \mid z=C_X \text{ for } x\in S\}=CS$. It is well-known that S and S_C are polyhedral sets in $\mathbb{R}^n$ and $\mathbb{R}^2$ respectively. Throughout this paper we will suppose that the two criteria are lower bounded on S which means that for i = 1, 2 we have

$$z_i^{\min}=\min\{z_i(x)=c_ix \mid x\in s\}>-\infty.$$

Pareto set

A feasible solution $x\in s$ is an *efficient solution* if and only if it does not exist any other feasible solution $\bar{x}\in S$ such that (a) $z_i(\bar{x})\leq z_i(x)$ for *i* = 1, 2, and (b) $z_j(\bar{x})\leq z_j(x)$ for at least one

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$j \in \{1,2\}$. The set of all efficient solutions is called the *efficiency* set noted $\mathcal{E}$, also called Pareto set. The corresponding set in the criterion space is the set $\mathcal{E}_c = c\mathcal{E}$.

Geometric structure of the pareto set

Under the assumption that the two cost vectors c_1 and c_2 are linearly independent, and using weighted sums, we can replace the bicriteria linear programming problem by a single criterion linear programming problem. We consider $\lambda \in [0,1]$ and the weighted-sum function is

$$z(x;\lambda) = (1-\lambda)z_1(x) + \lambda z_2(x) = [(1-\lambda)c_1 + \lambda c_2]x,$$

and we consider the single criteria problem for $\lambda \in [0,1]$

$$(\mathrm{P}(\lambda)) \begin{cases} \min\ z(x;\lambda) = (1-\lambda)z_1(x) + \lambda z_2(x) = [(1-\lambda)c_1 + \lambda c_2]x, \\ \text{subject to } x \in s. \end{cases}$$

The *value function* $\varphi(\lambda)$ of $(\mathrm{P}(\lambda))$ is defined by

$$\varphi(\lambda) = \min\ \{z(x;\lambda) \mid x \in s\}.$$

From [8] we have

$$\varepsilon = \bigcup_{\lambda \in (0,1)} \arg\min_{x \in s} z(x;\lambda).$$

Hence the efficiency set ε in the decision space is a connected set and is the union of faces, edges and vertices of S. This set may be quite complex due to the high dimension of the decision space. On the other side ε_c, which is the image in $\mathbb{R}^2$ of $\mathcal{E}$ by a linear transform, is a much simpler set.

Since we have assumed that both criteria are lower bounded on S, it follows that ε_c is a simple compact polygonal line. Indeed, in that case ε_c is the union of a finite number L of segments $[Q_{l-1}, Q_l]$

$$\varepsilon_C = \bigcup_{l=1}^{L} \left[Q_{l-1}, Q_l\right]$$

where

$$[Q_{l-1}, Q_l] = \{Q \in \mathbb{R}^2 \mid Q = (1-\sigma)Q_{l-1} + \sigma Q_l \ for\ \sigma \in [0,1]\},$$

and such that

$$(Q_{l-1}, Q_l) \cap (Q_{i-1}, Q_i) = \varnothing \ if\ l \neq i,$$

with

$$[Q_{l-1}, Q_l] = \{Q \in \mathbb{R}^2 \mid Q = (1-\sigma)Q_{l-1} + \sigma Q_l \ for\ \sigma \in (0,1)\},$$

To each segment is associated a weight $\lambda_{l-1,l}$ such that the vector $(1-\lambda_{l-1,l}, \lambda_{l-1,l})^t$ is orthogonal to the segment $[Q_{l-1}, Q_l]$ in $\mathbb{R}^2$. To each point Q of ε_c is associated an interval $\Lambda(Q)$ defined by

$$\Lambda(Q) = \begin{cases} [\underline{\lambda}_l, \overline{\lambda}_l] & \text{if } Q = Q_l & (l=0, \ldots, L), \\ [\lambda_{l-1,l}, \lambda_{l-1,l}] & \text{if } Q \in (Q_{l-1}, Q_l) & (l=0, \ldots, L), \end{cases}$$

where

$$\begin{cases} \underline{\lambda}_0 = 0, \\ \overline{\lambda}_{l-1} = \underline{\lambda}_l = \lambda_{l-1,l} \quad \text{for } l = 1, \ldots, L, \\ \overline{\lambda}_L = 1, \end{cases}$$

With $\overline{\lambda}l - \underline{\lambda}_l > 0\ for\ l = 0, \ldots L.$ More mathematical details are given in [7].

Link to parametric analysis

The parametric analysis is based on the weighted sum given by

$$\tilde{z}(x;\mu) = z_1(x) + \mu z_2(x)$$

for $\mu \in [0, +\infty)$, which can represent a tax on the second criteria. The value function in this case is defined by

$$\tilde{\varphi}(\mu) = \min\ \{\tilde{z}(x;\mu) \mid x \in s\}.$$

We could consider the single criteria problem for $\mu \geq 0$

$$(\mathrm{P}(\mu)) \begin{cases} \min \tilde{z}(x;\mu) = z_1(x) + \mu z_2(x) = (c_1 + \mu c_2)x \\ \text{subject to } x \in s. \end{cases}$$

Since λ and μ are related by the formulae

$$\lambda = \frac{\mu}{1+\mu} \ and\ \mu = \frac{\lambda}{1+\lambda},$$

to the efficient extreme points $\{Q_l\}_{l=0}^{L}$ on the efficiency set ε_C we associate to these extreme points the following intervals for the parameter μ

$$\tilde{\Lambda}(Q) = \begin{cases} [\underline{\mu}_l, \overline{\mu}_l] & \text{if } Q = Q_l & (l=0, \ldots, L), \\ [\mu_{l-1,l}, \mu_{l-1,l}] & \text{if } Q \in (Q_{l-1}, Q_l) & (l=0, \ldots, L), \end{cases}$$

where

$$\begin{cases} \underline{\mu}_0 = 0 \\ \overline{\mu}_{l-1} = \underline{\mu}_l = \mu_{l-1,l} \quad \text{for } l = 1, \ldots, L, \\ \mu_L = +\infty. \end{cases}$$

In many applications, the parameter μ is in fact a taxe over the the second criteria (for a minimization problem). Interesting enough is to observe that the behavior (extreme point) change only for the critical values $\mu_{l-1,l}$ of the parameter μ. Indeed when μ increases and its value passes through $\mu_{l-1,l}$, the behavior moves from the extreme point Q_{l-1} to the extreme point Q_l. Moreover, any level of taxes μ strictly between the values $\mu_{l-1,l} = \underline{\mu}_l$ and $\mu_{l,l+1} = \overline{\mu}_l$ gives the same behavior described by the extreme point Q_l.

Software

Several methods exist for computing the Pareto set of a bi-criteria linear program, for example [7,8]. We have developed our own method which requires only elementary results from a linear program solver [9]. It has been programmed in MATLAB and uses Linprog as the linear program solver.

Back to Pig Diet Formulation: Cost and phosphorus excretion

Let us come back to our bi-criteria linear model

$$(\mathrm{P}_{c,ph}) \begin{cases} \min\ z_1 = cx \\ \min\ z_2 = q_{ph}x \\ \text{subject to } x \in S \end{cases}$$

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Its two associated parametric models are

$$(P_{c,ph}(\lambda))\begin{cases}\min z(x;\lambda)=(1-\lambda)z_1(x)+\lambda z_2(x)=[(1-\lambda)c+\lambda q_{ph}]x,\\ \text{subject to } x\in S.\end{cases}$$

and

$$(P_{c,ph}(\mu))\begin{cases}\min \tilde{z}(x;\mu)=z_1(x)+\mu z_2(x)=(c_1+\mu q_{ph})x\\ \text{subject to } x\in S.\end{cases}$$

Its Pareto curve contains all the information for optimal decision considering the level of taxation. (Table 3) presents the efficient extreme points in the criterion space while the Pareto curve is sketched in (Figure 1). For this problem, the algorithm detects L = 22 segments and 23 extreme points. A total of 45 calls to a linear program software was required [9].

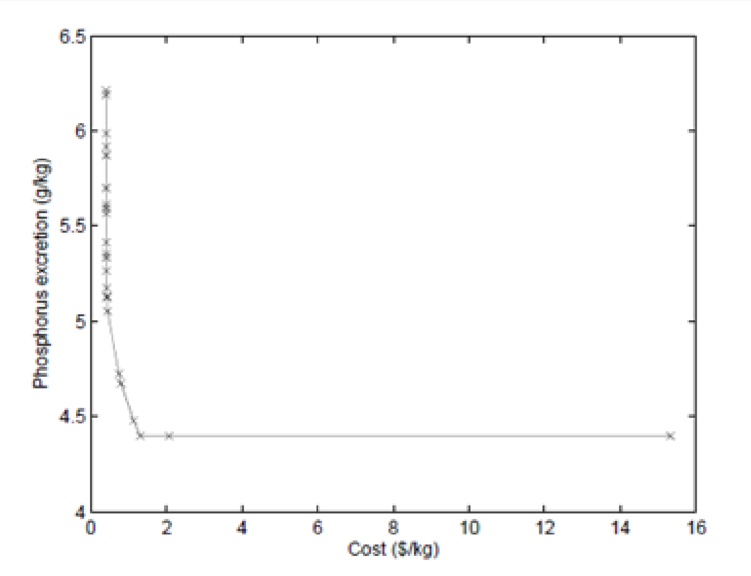

Figure 1: Pareto curve: phosphorus excretion vs diet cost.

Table 3: Efficient extreme points in the criterion space $\mathbb{R}^2$ for $(P(c, ph))$, and the corresponding taxes.

Pareto Set					Taxation system	
l	$\underline{\lambda}_l$	$\overline{\lambda}_l$	$z_{l,cost}$	$z_{l,phosphorus\ excerction}$	$\underline{\mu}_l$	$\overline{\mu}_l$
			$/kg	g/kg		
0	0	0.00428	0.40062	6.21226	0	0.0043
1	0.00428	0.00452	0.40072	6.18977	0.0043	0.00454
2	0.00452	0.00456	0.40164	5.98711	0.00454	0.00458
3	0.00456	0.005	0.40196	5.91713	0.00458	0.00502
4	0.005	0.00528	0.40219	5.87162	0.00502	0.00531
5	0.00528	0.00628	0.4031	5.69979	0.00531	0.00632
6	0.00628	0.00708	0.40365	5.61223	0.00632	0.00713
7	0.00708	0.00783	0.40379	5.59297	0.00713	0.00789
8	0.00783	0.00919	0.404	5.56609	0.00789	0.00927
9	0.00919	0.01003	0.40541	5.41416	0.00927	0.01013
10	0.01003	0.01458	0.40601	5.35505	0.01013	0.01479
11	0.01458	0.02357	0.40633	5.33336	0.01479	0.02414
12	0.02357	0.09694	0.40798	5.26498	0.02414	0.10734
13	0.09694	0.11478	0.41768	5.17458	0.10734	0.12967
14	0.11478	0.12931	0.42351	5.12967	0.12967	0.14852
15	0.12931	0.14182	0.42429	5.1244	0.14852	0.16526

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16	0.14182	0.4861	0.43631	5.05165	0.16526	0.94589
17	0.4861	0.49168	0.74777	4.72237	0.94589	0.96727
18	0.49168	0.62773	0.79624	4.67226	0.96727	1.68624
19	0.62773	0.69486	1.12394	4.47793	1.68624	2.27723
20	0.69486	0.99962	1.30843	4.39691	2.27723	2662.91
21	0.99962	0.99998	2.06125	4.39663	2662.91	59645.9
22	0.99998	1	15.32799	4.39641	59645.9	+∞

For each l, such that l = 0, . . . , 22, the extreme point Q_l is given by

$$Q_l = (z_{l,1}, z_{l,2}) = \left(z_{l,\text{cost}}, z_{l,\text{phosphorus execretion}}\right),$$

and corresponds to the optimal value of the criteria corresponding to any optimal solution of $(p_{c,ph}(\mu))$ for any values of μ in $(\underline{\mu}_l, \overline{\mu}_l)$. So, the value function, with tax, is

$$\tilde{\varphi}(\mu) = z_{l,\text{cost}} + \mu\, z_{l,\text{phosphorus execretion}}$$

as long as $\mu \in [\underline{\mu}_l, \overline{\mu}_l]$. So, we see that for any tax value in $[\underline{\mu}_l, \overline{\mu}_l]$ we will always have the same value function $\tilde{\varphi}(\mu)$, or the same behavior $(z_{l,\text{cost}}, z_{l,\text{phosphorus execretion}})$, and the change in the behavior will append only when the taxation level μ passes through the extremities $\underline{\mu}_l$ or $\overline{\mu}_l$ of this interval.

This is a nice example of abrupt (discrete) changes in behavior depending on the level of taxation of one criterion.

Conclusion

We have considered a diet formulation problem with two conflicting criteria, modelled as a bi-criteria linear model, to illustrate abrupt changes in behavior of a decision maker for a taxation system.

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