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Dynamics of Mechanical Systems in the Field of Relative Inertial Forces

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Abstract

This paper considers a mechanical system consisting of two bodies with masses and interacting via a rigid rod mounted pivotally to each of these bodies, allowing for their relative rotational motion. It is shown that the complex translational relative motion of bodies and the mechanical system generate the central dynamic gravitational field of the two-mass mechanical system, which exerts an effect on this system equivalent to an external force. Based on D'Alembert's principle and the constraint axiom for relative inertial forces, equations of motion are derived for each of the bodies and. It is shown that time in the proper frames of reference of these bodies and in their center-of-mass reference frame flows differently compared to time in a fixed laboratory frame of reference and depends on the mass ratio of these bodies.

Keywords: Two-mass problem; Equations of Motion; Complex Translational Relative Motion; Dynamic Gravitational Field; Relativity of time; Momentum Transfer rate

Introduction

In theoretical mechanics, inertial forces are viewed from different perspectives [1]. According to one of these, the inertial force is a fictitious force [2] that must be applied to the interacting bodies m_i and m_j to transform Newton's second law (taking into account the constraint axiom) into the more convenient equilibrium form known as D'Alembert's principle $\vec{\Phi}_i + \vec{R}_{ij} + \vec{F}_i = 0, i, j = 1, 2; i \neq j$ (1)

where $\vec{\Phi}_i = m_i \vec{a}_i$ is the inertial force of the *i*th body in a fixed laboratory frame of reference K ; $\vec{a}_i$ is its absolute acceleration in this frame; $\vec{R}_{ij} = -\vec{R}_{ij}$ are the constraint reactions R between bodies m_i and m_j ; and F_i is the external force applied to the *i*th body in the laboratory frame K .

In D'Alembert's principle (1), the inertial force $\vec{\Phi}_i$ accounts for the non-inertial nature of the proper frame of reference K_i of the body m_i , for which this principle is written so that it remains invariant relative to both this frame of reference K_i and the laboratory frame K .

Consequently, the inertial force $\vec{\phi}_i$ in D'Alembert's principle (1) assumes the status of an active force, which contradicts the viewpoint that it is fictitious [2].

According to another perspective, the inertial force is considered to be applied in parts to the accelerating body and is directed along the inertial force of the accelerated (working) body [1]. To justify this, the following reasoning is provided. A material body moves with acceleration a_i because other bodies act on it with a force $\vec{F}_i + \vec{R}_{ij}$. According to Newton's third law, the material point must counteract these bodies with a force equal magnitude but opposite direction $-\vec{F}_i - \vec{R}_{ij}$. According to D'Alembert's principle (1), this force is equal to the inertial force $\vec{\phi}_i = -\vec{F}_i - \vec{R}_{ij}$.

According to the third point of view, the inertial force is considered to be applied to a moving body opposite to its acceleration [1].

Based on a review of studies [3–16], this paper introduces three types of inertial forces for a closed ($\vec{F}_i = 0$) mechanical system (MS2) consisting of two interacting bodies m_i and m_j:

- The intrinsic inertial force $\vec{\Phi}_i = -m_i \frac{d}{dt}\vec{\upsilon}_i = m_i a_i; i = 1, 2$ of the reference (accelerating) body m_i included in the D'Alembert principle (1), which is applied to the reference body m_i as it undergoes absolute motion together with its proper frame K_i in the laboratory frame. Here $\vec{\upsilon}_i$ and $\vec{a}_i$ are the absolute velocity and absolute acceleration of body m_i in the laboratory frame K. This inertial force is applied to the ith body, chosen as the reference, and is directed opposite to its absolute acceleration $\vec{a}_i$. It characterizes the physical property of the reference body m_i to resist the bending of its uniform rectilinear trajectory in the laboratory frame K. When two bodies of a MS2 have equal masses ($m_i = m_j$), either one can be treated as the reference body. If $m_i > m_j$, the heavier body should be chosen as the reference body.

- The translational inertial force $\vec{\Phi}_{je} = -\frac{d}{dt} m_j \vec{\upsilon}_{je} = \frac{d}{dt} m_i \vec{a}_{je}; i, j = 1, 2; i \neq j$ of the test body m_j, where $\vec{\upsilon}_{je} = \vec{\upsilon}_i$ and $\vec{a}_{je} = \vec{a}_i$ are its translational velocity and acceleration together with the proper frame K_i of the reference body m_i. It is applied to the test body m_j, which moves with translational velocity $\vec{\upsilon}_{je} = \vec{\upsilon}_i$ in the laboratory frame K.

- The relative inertial force $\vec{\Phi}_{ji} = -\frac{d}{dt} m_j \vec{\upsilon}_{ji} = \frac{d}{dt} m_i \vec{a}_{ji}; i, j = 1, 2; i \neq j$ of the test body m_j. It is applied to this body in the proper frame K_i of the reference body m_i and, in accordance with the constraint axiom for relative inertial forces, affects the motion of the reference body m_i in the laboratory frame K.

Furthermore, the field of the relative inertial forces $\vec{F}_{iC,O}$ and $\vec{F}_{jC,O}$ (a central dynamic gravitational field of attraction in the form of an inertial domain (ID) with a center of force O) is introduced into consideration. This field is formed by the interacting bodies m_i and m_j due to their complex translational relative motion in the laboratory frame K [5–9].

Depending on the initial momenta $\vec{P}_{i(0)}$ and $\vec{P}_{j(0)}$ of bodies m_i and m_j, two cases of distribution of their masses are possible:

- For the initial momenta $\vec{P}_{i(0)} = -\vec{P}_{j(0)}$ of bodies m_i and m_j of equal masses $m_i = m_j$, their center of mass (CM C) coincides with the center of force O of the ID and the distribution of the masses of bodies m_i and m_j relative to the center O is diametrical. In this case, in accordance with the constraint axiom, the relative inertial force $\vec{F}_{iC}$ of body m_i acts on body m_j, while the relative inertial force $\vec{F}_{jC}$ of body m_j acts on body m_i.

- If the initial momenta of these bodies are $\vec{P}_{i(0)} = \vec{P}_{j(0)} \neq 0$, the CM C of the MS2 does not coincide with the center of force O of the ID and moves relative to this center uniformly along a closed circular geodesic path s_C under the action of the field of the relative inertial forces $\vec{F}_{iO}$ and $\vec{F}_{jO}$ with a radial distribution of the masses of bodies m_i and m_j relative to the center.

For these two cases, equations of motion are derived for bodies and of the closed MS2 and its CM in the field of its ID are written.

Dynamic equation of a two-mass mechanical system with a diametric distribution of its masses

The derivation of the dynamic equation for a closed MS2 in the field of its ID with a diametric distribution of its masses is illustrated by the kinematic diagram [3–9] shown in Figure 1(a).

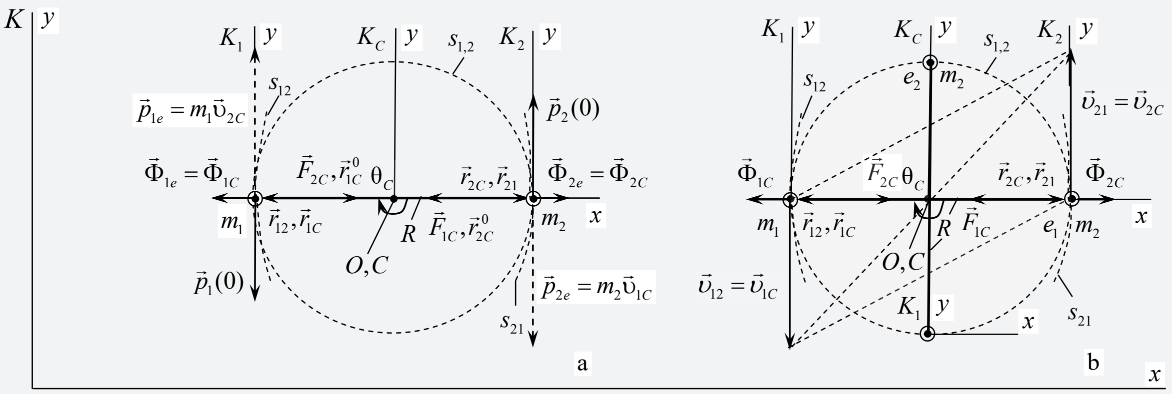

Figure 1: A two-mass mechanical system with a diametrical distribution $\theta_C = -\pi$ of the masses of bodies m_1 and m_2 relative to their center of mass C, which are hinged together by a rectilinear rigid rod with the possibility of rotation of these bodies relative to each other (a) and an event map e_1 and e_2, characterizing that for an invariant rotation of a body m_2 in the frames K_1 and K_C by an angle $\Delta\varphi = \{0; \pi/2\}$, this body will be at the same point of the frame K (b)

Its bodies m_1 and m_2 are interconnected by a rigid rectilinear rod with a length $R = r_{12} = r_{21}$ mounted to them via ideal hinges. This ensures the planar relative translational motion of these bodies along circular trajectories S_{12} and S_{21} (shown partially in Figure 1(a)) in the proper frames of reference and of the diametrically opposite bodies K_1 and K_2. Simultaneously, the bodies undergo complex planar translational relative motion along trajectory $S_{1,2}$ (which is common for the bodies when their masses $m_1 = m_2$) in the frame of reference K_C attached to their CM C. Throughout these motions, the axes of the frames K_C, K_1, K_2, and the fixed laboratory frame

K always remain parallel.

The trajectories $S_{1,2}$ of bodies m_1 and m_2 in the frame K_C are defined by radius vectors $\vec{r}_{1C}$ and $\vec{r}_{2C}$, while the trajectories s_{12} and s_{21} of their motion in the proper frames K_2 and K_1 of the diametrically opposite bodies m_2 and m_1 are defined by radius vectors $\vec{r}_{12}$ and $\vec{r}_{21}$.

The number of independent coordinates of the MS2 in plane motion with a constant length of the rod $R = r_{12} = r_{21} = \text{const}$ is determined by two polar coordinates r_{1C} and φ_{1C} of body m_1 in the frame K_C of the CM C and an additional polar coordinate φ_{21} of body m_2 in the proper frame K_1 of body m_1, or it is determined by two polar coordinates r_{2C} and $\ddot{o}_{2C}$ of body m_2 in the frame K_C of the CM C and an additional polar coordinate φ_{12} of body m_1 in the proper frame K_2 of body m_2. The polar coordinates $\varphi_{12} = -\varphi_{21}$ are dependent.

Thus, the number of independent coordinates required to unambiguously describe the planar motion of the MS2 in reference frame K is $N = 3$ out of the total number of its degrees of freedom $s = n + m - l = 2 + 2 - 1 = 5$, where $n = 3$ and $m = 3$ are the numbers of degrees of freedom of bodies m_1 and m_2 in the reference frame K; $l = 1$ is the single constraint imposed by the rod R. The total number of degrees of freedom $s = 5$ determines the total energy E of the MS2, which includes both the kinetic energy T of its bodies m_1 and m_2 and the potential energy U of these bodies in the field of their ID.

Indeed, in the frame K, bodies m_1 and m_2 of the MS2 with $s = 5$ perform complex planar translational relative motion with absolute velocities

$$\vec{\upsilon}_i = \vec{\upsilon}_{ie} + \vec{\upsilon}_{ij}; i, j = 1, 2; i \neq j, \quad (2)$$

where $\vec{\upsilon}_{ie}$ and $\vec{\upsilon}_{ij}$ are the translational velocity of the *i*th body in the laboratory frame K and its relative velocity in the proper frame K_j of the *j*th body.

For the initial momenta of bodies m_1 and m_2 equal to $\vec{P}_{i(0)} = -\vec{P}_{j(0)}$, the motion of these bodies in the frame K_C is characterized by a diametric distribution

$$\theta_C = \varphi_{21} - \varphi_{1C} = -\text{arcoss}(p_i(0)/p_j(0)) = -\pi, \quad (3)$$

where θ_C is the lag angle of the phase $\varphi_{1C} = \varphi_{21} + \theta_O = \varphi_{21} - \pi$ of the translational motion of body m_1 in the frame K_C relative to the phase φ_{21} of the translational motion of body m_2 in the proper frame K_1 of body m_1.

We assume that each of the bodies of the closed MS2 satisfies D'Alembert's principle (1).

Then, in accordance with the constraint axiom, the test body m_j can be conventionally discarded and its constraint reaction $\vec{R}_{ij} = \vec{\Phi}_{ij}$ can be replaced by the relative inertial force $\vec{\Phi}_{ij} = m_j \vec{a}_{ij}$ of this body. As a result, we obtain the dynamic equation for the remaining reference body m_i of the closed MS2 system in the form of D'Alembert's principle or Newton's second law in the field of the relative inertial force of the discarded test body m_j:

$$\vec{\Phi}_i + \vec{\Phi}_{ij} = 0; m_j \vec{a}_i = \vec{\Phi}_{ij}; i, j = 1, 2; i \neq j \quad (4)$$

where $\vec{a}_i$ and $\vec{a}_{ij}$ are the absolute acceleration of the reference body m_i in the laboratory frame K and the relative acceleration of the test body m_j in the proper frame K_i of the reference body m_i.

We now determine the relative inertial forces $\vec{\Phi}_{12}$ and $\vec{\Phi}_{21}$ (4) of the interacting bodies m_1 and m_2 undergoing complex planar translational motion in the reference frame K. For this, taking into account the absolute velocities $\vec{\upsilon}_i$ (2) of these bodies, we express the kinetic energy of their translational motion in the form [8]

$$T_{er} = \frac{1}{2}\sum_i m_i \vec{\upsilon}_i^{\,2} = \frac{1}{2} m_1 \vec{\upsilon}_{2e}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{1e}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{12}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{21}^{\,2} + U, \quad (5)$$

where U is the generalized kinetic potential

$$U = m_1 \upsilon_{1e} \upsilon_{12} \cos\theta_C + m_2 \upsilon_{2e} \upsilon_{21} \cos\theta_C = (m_1 \upsilon_{1e} \upsilon_{12} + m_2 \upsilon_{2e} \upsilon_{21}), \quad (6)$$

which characterizes the contribution of the relative $\vec{\upsilon}_{12r}, \vec{\upsilon}_{21r}$ motion of bodies m_1 and m_2 to their translational motion $\vec{\upsilon}_{1e} = \vec{\upsilon}_{2C}, \vec{\upsilon}_{2e} = \vec{\upsilon}_{1C}$; here $\theta_C = \widehat{\vec{\upsilon}_{2e} \vec{\upsilon}_{21}} = -\pi$ or $\theta_C = \widehat{\vec{\upsilon}_{1e} \vec{\upsilon}_{12}} = \pi$ is the angle between the vectors of the translational $\vec{\upsilon}_{1e}, \vec{\upsilon}_{2e}$ and relative $\vec{\upsilon}_{12}, \vec{\upsilon}_{21}$ velocities (see Figure 1a).

The sum of the first and second terms of T_{er} (5) is the kinetic energy of the translational motion of bodies m_1 and m_2

$$T_{Ce} = \frac{1}{2} m_1 \vec{\upsilon}_{1e}^{\,2} = \frac{1}{2} m_2 \vec{\upsilon}_{2e}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{2C}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{1C}^{\,2} + \frac{1}{2} m \vec{\upsilon}_{C}^{\,2}, \quad (7)$$

which is characterized by their translational momenta $p_{1e} = m_1 \vec{\upsilon}_{2C}$ and $p_{2e} = m_2 \vec{\upsilon}_{1C}$ along with the proper frames K_2 and K_1 of the diametrically opposite bodies m_2 and m_1; this defines the physical meaning of the sum T_{Ce}.

The sum of the third and fourth terms in the expression of the kinetic energy T_{er} (5) is the kinetic energy of the relative motion of body $T_r = \frac{1}{2} m_1 \upsilon_{12}^2 = \frac{1}{2} m_2 \upsilon_{21}^2$ in m_1 the frame K_2 and body m_2 in the frame K_1.

Potential U (6) ensures the determination of the relative inertial forces $\vec{\Phi}_{ij}$ (4) of bodies and for their complex translational-relative motion in the reference frame K_C in the form [8]

$$\begin{aligned} \vec{F}_{1C} &= -\frac{\partial U}{\partial r_{2C}} . \vec{r}_{2C}^{\,0} = m_1 \omega_{2C} \omega_{12} r_{12} . \vec{r}_{2C}^{\,0}; \\ \vec{F}_{2C} &= -\frac{\partial U}{\partial r_{1C}} . \vec{r}_{1C}^{\,0} = m_2 \omega_{1C} \omega_{21} r_{21} . \vec{r}_{1C}^{\,0}; \end{aligned} \quad (8)$$

where for the diametrical distribution $\theta_C = -\pi$ (3) the force center O formed by them ID coincides with the CM C of the MC2

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(see Figure 1(a)); $\vec{r}_1^{\,0} = -\vec{r}_{1C} / r_{1C}$ and $\vec{r}_2^{\,0} = -\vec{r}_{2C} / r_{2C}$ are unit vectors opposing the radius vectors $\vec{r}_{1C}$ and $\vec{r}_{2C}$ and determining the direction of the relative inertial forces $\vec{F}_{2C}$ and $\vec{F}_{1C}$ toward the CM C (see Figure 1(a), where the choice of directions for $\vec{r}_{1C}^{\,0}$ and $\vec{r}_{2C}^{\,0}$ is subject to further justification); $\omega_{1C} = \omega_{2C} = \omega_C$, ω_{12}, and ω_{21} are the angular velocities of bodies m_1 and m_2 in the reference frame K_C and their angular velocities in the proper frames K_2 and K_1 of the diametrically opposite bodies m_2 and m_1, which in [3–5, 8] are expressed as:

$$\omega_C = \omega_{1C} = \omega_{2C} =, \omega_{12} + \omega_{21};$$

$$\omega_{12} = \frac{m_2 \omega_C}{m_1 + m_2}; \omega_{21} = \frac{m_1 \omega_C}{m_1 + m_2}. \quad (9)$$

Based on the kinematic feasibility of the motion of bodies m_1 and m_2 the MS2 without breaking the constraint R, the angular displacements of these bodies in the frames K, K_C, K_1, and K_2 must be invariant:

$$\vartriangle\varphi = \omega_C \vartriangle t_C = \omega_{12} \vartriangle t_2 = \omega_{21} \vartriangle t_1 \quad (10)$$

where $\vartriangle t_1 = t_1 - t_0$, $\vartriangle t_2 = t_2 - t_0$, and $\vartriangle t_C = t_C - t_0$ are time increments; t_0 is the common initial time in all frames of reference.

The invariant (10) characterizes the fact that for any angular displacement $\vartriangle\varphi = \{0; \pi / 2\}$ of a body m_2 in the frames K_1 and K_C (which is also true for a body m_1 in the frames K_2 and K_C), this body will be at the same point of the trajectory $s_{1,2}$ of the frame K, as, for example, for this body shown in Fig. 1, b by events e_1 and e_2.

The transformations (9) together with the invariant $\vartriangle\varphi$ (10) require the introduction of proper time in the frames K_1 and K_2:

$$t_1 = \frac{m_1 + m_2}{m_1} t; t_2 = \frac{m_1 + m_2}{m_2} t \quad (11)$$

Time in the frames K and K_C is absolute: $t_C = t$.

In view of the relative inertial forces $\vec{F}_{1C}$ and $\vec{F}_{2C}$ (8) that form the ID, the dynamic equation for the closed MS2 in the form of D'Alembert's principle and Newton's second law (5) becomes

$$\vec{\Phi}_i + \vec{F}_{jC} = 0; m_j \vec{a}_i = \vec{F}_{jC}; i, j = 1, 2; i \neq j \quad (12)$$

where the subscript C corresponds to the diametric distribution $\theta_C = -\pi$ (3) of bodies m_i and m_j in the frame K_C.

The first equation (12) characterizes the equilibrium of body m_i in the laboratory frame K (geodetic motion), and the second its dynamics in the same reference frame K.

For MS2 bodies of equal masses $m_1 = m_2$, the translational kinetic energy T_{Ce} (7) can be reduced by the permutation $m_1 \Leftrightarrow m_2$ to the kinetic energy T_C of these bodies in the frame K_C

$$T_{Ce} = T_C = \frac{1}{2} m_1 \vec{\upsilon}_{1C}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{2C}^{\,2} = \frac{1}{2} m \vec{\upsilon}_C^{\,2} \quad (13)$$

Substituting T_C (13) for $T_{Ce} = \frac{1}{2} m_1 \vec{\upsilon}_{2e}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{1e}^{\,2}$

in (5) yields the Lagrangian L of the MS2

$$L = \frac{1}{2} m_1 \vec{\upsilon}_{1C}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{2C}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{12}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{21}^{\,2} + U =$$

$$= \frac{1}{2} m_1 \omega_{1C}^2 r_{1C}^2 + \frac{1}{2} m_2 \omega_{2C}^2 r_{2C}^2 + \frac{1}{2} m_1 \omega_{12}^2 r_{12}^2 + \frac{1}{2} m_2 \omega_{21}^2 r_{21}^2 -$$

$$-(m_1 \omega_{2C} r_{2C} \omega_{12} r_{12} + m_2 \omega_{1C} r_{1C} \omega_{21} r_{21}) \qquad (14)$$

for its absolute motion in the frame K at $m_1 = m_2$.

The partial derivatives $-\frac{\partial L}{r_{1C}} = 0$ and $-\frac{\partial L}{r_{2C}} = 0$ of the Lagrangian L (14) give the generalized forces

$$\begin{aligned} &-m_1 \omega_{1O}^2 \vec{r}_{1C} + -m_2 \omega_{1C} \omega_{21} r_{21} = 0; \\ &-m_2 \omega_{2C}^2 \vec{r}_{2C} + -m_1 \omega_{2C} \omega_{12} r_{12} = 0 \end{aligned} \qquad (15)$$

which define equations (12).

Indeed, the first terms in (15) define the intrinsic inertial forces $\vec{\Phi}_1 = \vec{\Phi}_{1C} = -m_2 \omega_{1C}^2 \vec{r}_{1C}$ and $\vec{\Phi}_2 = \vec{\Phi}_{2C} = -m_2 \omega_{1C}^2 \vec{r}_{2C}$ of bodies m_1 and m_2 (see Figure 1(a)). These forces are directed away from the center of force O, which coincides with the CM C.

The second terms define the relative inertial forces $\vec{F}_{1C} = m_1 \omega_{2C} \omega_{12} r_{12} . r_{2C}^0$ and $\vec{F}_{2C} = m_2 \omega_{1C} \omega_{21} r_{21} . r_{1C}^0$ (8), where the choice of the direction of the unit vectors $r_{1C}^0 = -\vec{r}_{1c} / r_{1C}$ and $r_{2C}^0 = -\vec{r}_{2c} / r_{2C}$ toward the center of force O (see Figure 1(a)) is due to the fact that the relative inertial forces $\vec{F}_{1C}$ and $\vec{F}_{2C}$ are counteracted by the intrinsic inertial forces $\vec{\Phi}_1$ and $\vec{\Phi}_2$ (15) of bodies m_1 and m_2.

For the diametric distribution $\theta_C = -\pi$ (3), the relative inertial forces $\vec{F}_{1C}$ and $\vec{F}_{2C}$ of bodies m_1 and m_2 (8) that form the ID act on the diametrically opposite bodies m_2 and m_1, respectively, as a balanced system of forces $\vec{F}_{1C} + \vec{F}_{1C} = 0$. Consequently, in this case, the ID does not affect the motion of the closed MS2 in the frame K, which, in view of the substitution (14), is absolute, since the momentum of its CM C is conserved

($\vec{\mathrm{P}}_{\mathrm{C}} = \overrightarrow{\mathrm{Const}}$) in the case of the diametric distribution $\theta_C = -\pi$ (3).

If the masses of the MS2 bodies are unequal ($m_1 \neq m_2$), it must be reduced to a single body with mass $m = m_1 + m_2$ with equations of its motion of the form [10–16]:

$$\begin{aligned} &m \frac{d^2 \vec{r}_1}{dt^2} = \vec{\Phi}_{21} + \vec{F}_1; \vec{\Phi}_1 + \vec{\Phi}_{21} + \vec{F}_1 = 0 \\ &or \\ &m \frac{d^2 \vec{r}_2}{dt^2} = \vec{\Phi}_{12} + \vec{F}_2; \vec{\Phi}_2 + \vec{\Phi}_{12} + \vec{F}_2 = 0 \end{aligned} \qquad (16)$$

When $m_1 > m_2$, body m_1 should be chosen as the reference body, with the polar coordinates r_{1C}, φ_{1C}, and φ_{21} (the first two equations in (16)) taken as independent coordinates. Conversely, when $m_1 > m_2$, body m_2 should be chosen as the reference body, with the polar coordinates φ_{2C} and φ_{21} (the last two equations in (16)) taken as independent coordinates r_{2C}.

Solutions to equations (4) and (16) are discussed in detail in [3–16].

Dynamic Equation of a Two-Mass Mechanical System with a Radial Mass Distribution

The derivation of the dynamic equation for a closed MS2 with a radial mass distribution in the field of its ID [8, 9] is illustrated in Figure 2.

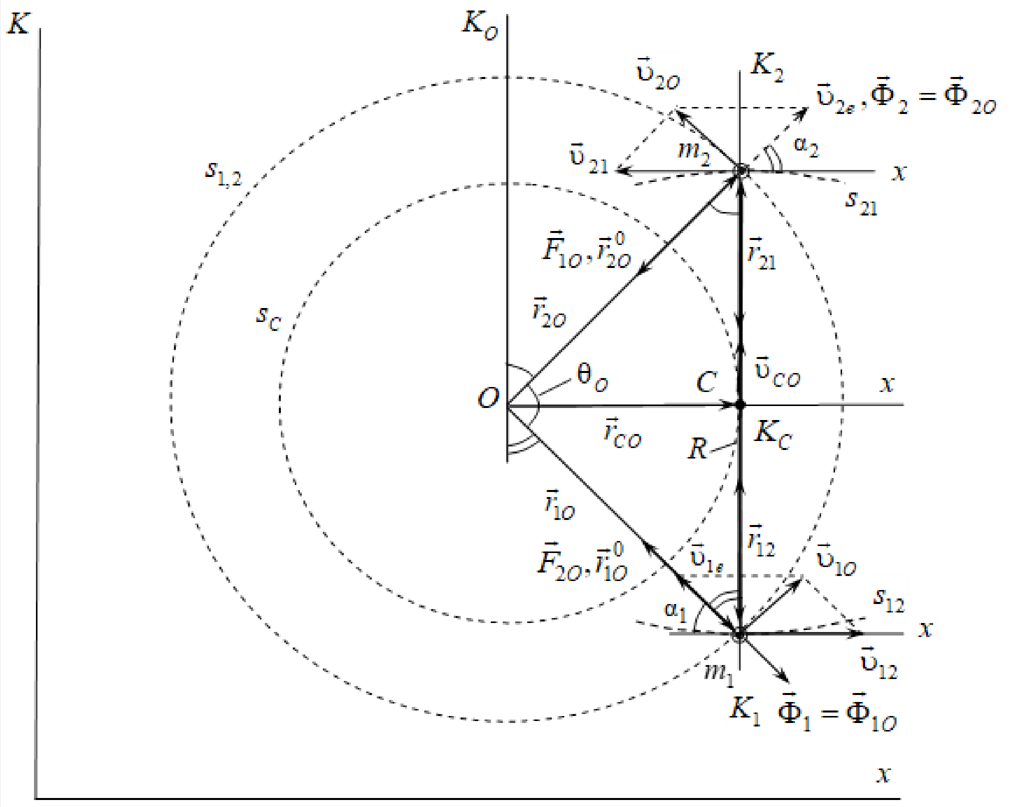

Figure 2: A two-mass mechanical system with a radial distribution $\theta_0 = \pi / 2$ of the masses of the bodies m_1 and m_2 relative to the force center O, which are pivotally connected by a straight rigid rod R with the possibility of rotation of these bodies relative to each other.

The bodies m_1 and m_2 are connected to each other by a rigid rectilinear rod with a length $R = r_{12} = r_{21}$, which is mounted to them using ideal hinges. This ensures their relative planar translational motion in the proper frames K_2 and K_1 of the diametrically opposite bodies m_2 and m_1 along circular trajectories s_{12} and s_{21} (partially shown in Figure 1a). Simultaneously, the bodies undergo complex planar translational relative motion in reference frame K_O attached to the center of force O of the ID along trajectory $s_{1,2}$ (which is common for the bodies when their masses $m_1 = m_2$). Throughout these motions, the coordinate axes of frames K_O, K_1, K_2, and the fixed laboratory reference frame K always remain parallel.

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For the radial distribution $\theta_C = \pi / 2$ (17), the trajectories $s_{1,2}$ of bodies m_1 and m_2 in the reference frame K_O attached to the center of force O of the ID are defined by the radius vectors $\vec{r}_{1O}$ and $\vec{r}_{2O}$, and the trajectories s_{12} and s_{21} of their motion in the proper reference frames K_2 and K_1 of the radially opposite bodies m_2 and m_1 are defined by the radius vectors $\vec{r}_{12}$ and $\vec{r}_{21}$.

The number of independent coordinates of the MS2 in planar motion $R = \vec{r}_{12} = \vec{r}_{21} = const$ is determined by two polar coordinates r_{1O} and φ_{1O} of body m_1 in the frame K_O and one additional polar coordinate φ_{21} of body m_2 in the proper frame K_1 of body m_1 , or it is determined by two polar coordinates r_{2O} and φ_{2O} of body m_2 in the frame K_O and one additional polar coordinate φ_{12} of body m_1 in the proper frame K_2 of body m_2 . The polar coordinates $\varphi_{12} = -\varphi_{21}$ are dependent. Choosing m_1 or m_2 as

the reference body is equivalent.

Thus, the number of independent coordinates of the MS2 for planar motion is $N = 3$ out of the total number of its degrees of freedom $s = n + m - l = 2 + 2 - 1 = 5$, where $n = 3$ and $m = 3$ are the numbers of degrees of freedom of bodies m_1 and m_2 in the frame K ; $l = 1$ is one constraint by the rod R . In this case, the total number of degrees of freedom $s = 5$ determines the total energy E of the MS2, which includes both the kinetic energy T of bodies m_1 and m_2 their potential energy U in the ID field; for the radial distribution of bodies m_1 and m_2 in the frame K_O considering the substitution (15), the ID field has a significant effect on the absolute motion of the MS2 in the frame K .

Indeed, when $s = 5$, bodies and undergo complex translational relative motion in the frame K with absolute velocities $\vec{\upsilon}_i = \vec{\upsilon}_{ie} + \vec{\upsilon}_{ij}; i, j = 1, 2$ (2).

When the initial momenta of bodies m_1 and m_2 are $\vec{p}_1(0) = 0$ and $\vec{p}_2(0) = 0$, the motion of these bodies in the frame K_O attached to the center of force O of the ID is characterized by a diametric distribution $\theta_O = \arccos(p_1(0) / p_2(0)) = \pi / 2$ (17)

where θ_O is the lag angle of the phase $\theta_O = \varphi_{2O} - \varphi_{1O} = \pi / 2$ of the translational motion of body m_1 relative to the phase φ_{2O} of the translational motion of body m_2 in the frame K_O in the stationary state of the MS2, as shown in Figure 2.

For the radial distribution $\theta_O = \pi / 2$ (17), the dynamic equation for the closed MS2 (12) can be represented as:

$$\vec{\Phi}_i + \vec{F}_{jO} = 0; m_i \vec{a}_i = \vec{F}_{jO}; i, j - 1, 2; i \neq j \quad (18)$$

where $\vec{\Phi}_i = -m_i \vec{a}_i$ is the intrinsic inertial force of the *i*th body together with its proper frame of reference K_i in the frame K and $\vec{F}_{jO}$ is the relative inertial force of the *j*th body during its complex translational relative motion.

The first equation (18) also characterizes the equilibrium of the body m_i in the laboratory frame K (geodetic motion), and the second – its dynamics in the same reference frame K . The subscript O indicates that for the radial distribution $\theta_O = \pi / 2$ (17), the trajectories $s_{1,2}$ of bodies m_1 and m_2 are defined by the radius vectors $\vec{r}_{1O}$ and $\vec{r}_{2O}$ in the frame K_O attached to the center of force O of the ID.

We now determine the relative inertial forces $\vec{F}_{1O}$ and $\vec{F}_{2O}$ of bodies m_2 and m_1 for their complex planar translational relative motion in the frame K for the radial distribution $\theta_O = \pi / 2$ (17).

Using the absolute velocities $\vec{\upsilon}_i$ (2) of bodies m_i, the kinetic energy of the relative motion of the MS2 can be expressed as [8, 9]

$$T_{er} = \frac{1}{2}\sum_i m_i \vec{\upsilon}_i^{\,2} = \frac{1}{2} m_1 \vec{\upsilon}_{2e}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{1e}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{12}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{21}^{\,2} + U \quad (19)$$

where U is the generalized kinetic potential

$$\begin{aligned} U &= m_1\upsilon_{1e}\upsilon_{12}\cos\varphi_1 + m_2\upsilon_{2e}\upsilon_{21}\cos\varphi_2 = \\ &-(m_1\upsilon_{2c}\upsilon_{12}\cos\alpha_1 + m_2\upsilon_{1c}\upsilon_{21}\cos\alpha_2) = \\ &-(m_1\omega_{2O}r_{2O}\omega_{12}r_{12}\cos\alpha_1 + m_2\omega_{1O}r_{1O}\omega_{21}r_{21}\cos\alpha_{2}), \end{aligned} \quad (20)$$

which characterizes the contribution of the relative $\vec{\upsilon}_{12r}, \vec{\upsilon}_{21r}$ motion of bodies m_1 and m_2 to their translational motion $\vec{\upsilon}_{1e} = \vec{\upsilon}_{2C}, \vec{\upsilon}_{2e} = \vec{\upsilon}_{1C}$. Here-and $\varphi_1 = \widehat{\vec{\upsilon}_{1e}\vec{\upsilon}_{12}} = \pi - \alpha_2$ are the acute angles between the translational $\vec{\upsilon}_{1e}, \vec{\upsilon}_{2e}$ and relative $\vec{\upsilon}_{12}$, $\vec{\upsilon}_{21}$ velocity vectors; α_1 and α_2 are the angles of rotation of the translational velocities $\vec{\upsilon}_{1e}$ and $\vec{\upsilon}_{2e}$ relative to their orientation for the diametric distribution $\theta_C = -\pi$ (see Figure 1(a)), which for $\alpha_1 + \alpha_2 = \theta_O = \pi/2$ are defined as [8, 9]:

$$\alpha_1 = \operatorname{arcctg}(\xi_1); \alpha_2 \operatorname{arcctg}(\xi_2), \quad (21)$$

where $\xi_1 = m_2/m_1; \xi_2 = \xi_1^{-1} = m_1/m_2$ is the mass ratio of bodies m_1 and m_2, and the angles α_1 and α_2 are indicated in Figure 2 by one ark and two arcs, respectively.

Potential U (20) ensures the determination of the relative inertial forces $\vec{F}_{jO}$ (18) of bodies m_1 and m_2 for their complex translational-relative motion in the reference frame K_O in the form:

$$\begin{aligned} \vec{F}_{1O} &= -\frac{\partial U}{\partial r_{2O}} . \vec{r}_{2O}^{\,0} = m_1\omega_{2O}\omega_{12}r_{12} . \vec{r}_{2C}^{\,0}; \\ \vec{F}_{2O} &= -\frac{\partial U}{\partial r_{1O}} . \vec{r}_{1O}^{\,0} = m_2\omega_{1O}\omega_{21}r_{21} . \vec{r}_{1O}^{\,0} \end{aligned} \quad (22)$$

where for the radial distribution $\theta_O = \pi/2$ (17) the force center O formed by these forces ID does not coincide with the CM C of the MS2 (see Figure 2); $\vec{r}_{1O}^{\,0} = -\vec{r}_{1O}/r_{1O}$ and $\vec{r}_{2O}^{\,0} = -\vec{r}_{2O}/r_{2O}$ are unit vectors opposing the radius vectors $\vec{r}_{1O}$ and $\vec{r}_{2O}$ and determining the direction of the relative inertial forces $\vec{F}_{2O}$ and $\vec{F}_{1O}$ to this center O (see Figure 2, where the choice of the directions of $\vec{r}_{1\hat{\imath}}^{\,0}$ and $\vec{r}_{2\hat{\imath}}^{\,0}$ is subject to further justification); $\omega_{1C} = \omega_{2C} = \omega_C$, ω_{12}, and ω_{21} are the angular velocities of m_1 bodies m_2 and in the frame K_C and in the proper frames K_2 and K_1 of the radially opposite bodies m_2 and m_1, which in [8, 9] are defined as:

$$\begin{aligned} &\omega_C = \omega_{1C} = \omega_{2C} =, \omega_{12} + \omega_{21} = \omega_{CO}\frac{m_1^2 + m_2^2}{m_1 m_2}; \\ &\omega_{12} = \omega_{CO}; \omega_{21} = \omega_{CO}\frac{m_1}{m_2}; \end{aligned} \quad (23)$$

$\omega_{CO} = \omega_{1O} = \omega_{2O}$ is the angular velocity of the CM C and bodies m_2 and m_1 in reference frame K_O.

For the radial distribution $\theta_O = \pi / 2$ (17), the relative inertial forces $\vec{F}_{1O}$ and $\vec{F}_{2O}$ of bodies m_1 and m_2 (22) that form the ID act on the radially opposite bodies m_2 and m_1, respectively, so that their principal vector is nonzero: $\vec{F}_{1O} + \vec{F}_{2O} \neq 0$.

Based on the kinematic feasibility of the motion of bodies m_1 and m_2 MS2 without breaking the constraint R, the angular displacements of these bodies in the frames K, K_C, K_1, and K_2 must be invariant

$$\Delta\varphi = \omega_{CO}\Delta t_{CO} = \omega_C \Delta t_C = \omega_{12}\Delta t_2 = \omega_{21}\Delta t_1 \quad (24)$$

where $\Delta t_1 = t_1 - t_0$, $\Delta t_2 = t_2 - t_0$, and $\Delta t_C = t_C - t_0$ are time increments; t_0 is the common initial time in all frames of reference.

The invariant $\Delta\varphi$ (24) determines that for any angular displacement of body m_1 in the frames K_O, K_C, and K_2, as well as body m_2 in the frames K_O, K_C, and K_1, these bodies are located at the same point of the trajectory $s_{1,2}$ in the frame K ,as, for example, for a body m_2 with a diametrical distribution $\theta_C = -\pi$ (3) and its angle of rotation $\Delta\varphi = \{0; \pi / 2\}$ is shown in Figure 1(b) by events e_1 and e_2.

The transformations (23) together with the invariant $\Delta\varphi$ (24) require the introduction of proper time in the frames K_1, K_2, and K_C:

$$t_1 = \frac{m_1}{m_2} t; t_2 = \frac{m_1}{m_2} t; t_c = \frac{m_1}{m_2} t \quad t_{CO} = t \quad (25)$$

Furthermore, for $m_1 = m_2$, time in the frame K_1, K_2, K_O, and K is absolute: $t_1 = t$, $t_2 = t$,and $t_{CO} = t$.

Substituting T_C (13) for $T_{Ce} = \frac{1}{2} m_1 \vec{\upsilon}_{2e}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{1e}^{\,2}$ in (19) yields the Lagrangian L of the MS2

$$\begin{aligned} L = \frac{1}{2} m_1 \vec{\upsilon}_{1O}^{\,2} + \frac{1}{2} m_2 \vec{\upsilon}_{2O}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{12}^{\,2} + \frac{1}{2} m_1 \vec{\upsilon}_{21}^{\,2} + U = \\ \frac{1}{2} m_1 \omega_{1O}^2 r_{1O}^2 + \frac{1}{2} m_2 \omega_{2O}^2 r_{2O}^2 + \frac{1}{2} m_1 \omega_{12}^2 r_{12}^2 + \frac{1}{2} m_1 \omega_{21}^2 r_{21}^2 - \quad (26) \\ -(m_1 \omega_{2O} r_{2O} \omega_{12} r_{12} \cos\alpha_1 + m_2 \omega_{1O} r_{1O} \omega_{21} r_{21} \cos\alpha_2) \end{aligned}$$

for its absolute motion in the frame K at $m_1 = m_2$.

The partial derivative $-\frac{\partial L}{r_{1O}} = 0$ and $-\frac{\partial L}{r_{2O}} = 0$ of the Lagrangian L (26) give the generalized forces:

$$\begin{aligned} -m_1 \omega_{1O}^2 \vec{r}_{1O} + -m_2 \omega_{1O} \omega_{21} r_{21} \cos\alpha_2 = 0; \\ -m_2 \omega_{2O}^2 \vec{r}_{2O} + -m_1 \omega_{2O} \omega_{12} r_{12} \cos\alpha_1 = 0 \end{aligned} \quad (27)$$

which define equations (18).

Indeed, the first terms in (27) define the intrinsic inertial forces $\vec{\Phi}_1 = \vec{\Phi}_{1O} = -m_1 \omega_{1O}^2 \vec{r}_{1O}$ and $\vec{\Phi}_2 = \vec{\Phi}_{2O} = -m_2 \omega_{2O}^2 \vec{r}_{2O}$ of bodies m_1 and m_2 (see Figure 2), which are directed away from the center of force O .The second terms define the relative inertial forces $\vec{F}_{1O} = m_1\omega_{2O}\omega_{12}r_{12}\cos\alpha_1.\overset{\rightarrow 0}{r}_{2O}$ and $\vec{F}_{2O} = m_2\omega_{1O}\omega_{21}r_{21}\cos\alpha_2.\overset{\rightarrow 0}{r}_{2O}$ (22), where the choice of the direction of the unit vectors $\overset{\rightarrow 0}{r}_{1O} = -\vec{r}_{1O} / r_{1O}$ and $\overset{\rightarrow 0}{r}_{2O} = -\vec{r}_{2O} / r_{2O}$ toward the center of force O (see Figure 2) is due to the fact that the relative inertial forces $\vec{F}_{1O}$ and $\vec{F}_{2O}$ (22) are counteracted by the intrinsic inertial forces $\vec{\Phi}_1$ and $\vec{\Phi}_2$ (27) of bodies m_1 and m_2 .

Thus, for the radial distribution $\theta_O = \pi / 2$ (17) of the equal masses $m_1 = m_2$ of MS2 bodies with $s = 5$ degrees of freedom, the product of the total mass of the MS2 and the acceleration $\vec{a}_C = \vec{a}_{CO}$ of its CM is equal to the action exerted on it by the field of the relative inertial forces $\vec{F}_{1O}$ and $\vec{F}_{2O}$ of its interacting bodies, so that the circulation of the momentum $\vec{p}_C$ of the MS2 along a closed loop $s_{\tilde{N}}$ is zero:

$$\overrightarrow{dp_C} / dt = \vec{F}_{1O} + \vec{F}_{2O} \neq \overrightarrow{const};\, p_C = const;\, \tilde{A} = \oint \vec{p}_C d\vec{l} = 0 \quad (28)$$

Here the last two equalities indirectly express the law of conservation of energy of the considered closed MS2 during its geodesic motion.

The solution of equations (18) and (28) taking into account the transient process occurring in the MS2 under the given initial conditions and the rate of momentum transfer between its bodies m_1 and m_2 are analyzed in detail in [8, 9].

For unequal masses $m_1 \neq m_2$ of MS2 bodies, it must be reduced to a single body with mass $m = m_1 + m_2$ with equations of motion of the form (16).

The equation of motion for the CM C $\overrightarrow{dp_C} / dt = \vec{F}_{1O} + \vec{F}_{2O}$ of the closed MS2 (28) allows for the possibility of its motion along a closed geodesic trajectory s_C in the ID field. In this context, the zero circulation ($\tilde{A} = 0$) of the momentum $\vec{P}_C$ indicates the impossibility of reactionless directional motion of mechanical systems with constant thrust, whereas propellers known as "inertioids" [17–20], providing pulsed movement in dissipative media via cyclic zero-velocity transitions, create dynamic loads that, coupled with low response rates, impose significant limitations on their practical application.

Conclusion

According to Einstein's principle of equivalence between gravity and inertia [21], the motion of bodies m_1 and m_2 along trajectories s_{21}, $s_{1,2}$,and s_{12} can be considered as motion along closed geodesic trajectories in the field of relative inertial forces (dynamic gravitational field [5–7]). This also holds true when these bodies interact via Newtonian gravity. When $m_i \gg m_j$, body m_i can be treated as being in rest, while dynamic gravitation and gravity are considered equivalent. For the case where R=const and $s \leq 3$, equation (16) can be used to solve the two-body problem in a dissipative medium.

The main results of the paper are based on D'Alembert's principle and the constraint axiom for relative inertial forces and can be summarized as follows:

1. Inertial forces are a fundamental property of material bodies to resist changes to their state of rest or uniform rectilinear motion during interactions of any nature (according to Newton's, Coulomb's, or Hooke's laws). For example, the interaction of material bodies via Newtonian gravity curves their trajectories. The direct primary cause of the curvature of the trajectories of material bodies is their interaction, rather than their masses (according to Einstein [21]).

2. The connection between two bodies in a mechanical system (whether mechanical or field-based) can be conditionally discarded, and its reaction can be replaced by the relative inertial force of the discarded body. This force is always counteracted by the inertial force of the remaining reference body, which provides resistance to the curvature of its trajectory [7].
3. The motion of a system of interacting material bodies in a field of relative inertial forces is geodesic, and, in the absence of external influences, it is closed. This allows for the possibility of reactionless motion only in the region of space locally curved by interaction, which casts doubt on the conclusions of [19, 20].
4. For a two-mass mechanical system with five degrees of freedom, time in the proper reference frames of the interacting material bodies (which are in complex translational relative motion) flows differently compared to time in the center-of-mass reference frame of these bodies and depends on the masses of these bodies [3–5, 8, 9]. For the diametric distribution of these bodies, time in their center-of-mass reference frame, as well as in any other inertial frame, is absolute. However, for the radial distribution of these bodies, time in their center-of-mass reference frame flows differently compared to time in the reference frame attached to the center of force of the inertial domain and depends on the masses of the bodies. Time in the reference frame of this center of force, as well as in any other inertial reference frame, is absolute [8, 9].

For a two-mass mechanical system with equal masses, radial mass distribution, and five degrees of freedom, the product of the total mass of this mechanical system and the acceleration of its center of mass is equal to the action exerted on it by the field of the relative inertial forces of its interacting bodies, so that the circulation of the momentum of its center of mass along a closed loop is zero [8, 9].

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