

On The Contact Model Under Static Friction in Contact Problems of Elasticity Theory

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Introduction

It is well known that there are three main contact models for classical contact problems. In the first it is assumed that the contact surfaces are smooth, as a result of which the shear contact stresses are equal to zero. The second model assumes that the shear contact stresses are related to the normal contact pressure by Coulomb's law. Finally, the third method takes into account the presence of complete adhesion between the contact surfaces [1-4]. It is also known that the second model is used only in the case when, in addition to the normal, compressive load, the stamp is also subject to a shear load. Since it is not correct to consider the model without taking into account shear load Using the third model, oscillating singularities arise near the ends of the stamps, as a result of which, near the endpoints of the contact zone, the modulus of shear stresses becomes greater than the contact pressure, thereby violating the adhesion condition [3].

To eliminate this defect, in 1945 Galin L.A. proposed a new contact model, which assumes that in the middle part of the contact zone there is complete adhesion, and at the ends of the stamp there is sliding, and the shear stresses are related to the contact pressure by Coulomb's law [5]. The Galin's contact model is actually a combination of the second and third contact models and more accurately describes the contact process. Based on this model, efficient solutions to many contact problems for classical foundations were obtained [6-10]. However, the use of this contact model, especially in cases of non-classical foundations, often leads to mathematical and computational complexities.

Based on the above, we have proposed a new contact model taking into account static friction. It is assumed that the shear stresses acting in the contact zone are related to the normal contact

pressure under the stamp according to the formula of dry friction in which the friction coefficient depends on the coordinates of the points of the contacting surfaces and is directly proportional to them. It should be noted that the proposed model, like the Galin contact model, is close to the model of complete adhesion in the central part of the contact zone and to the model of Coulomb friction at the end points of the stamp. A characteristic feature of this model is that solutions of contact problems for classical foundations within the framework of this model are reduced to solving singular integral equations with variable coefficients. However, for a number of classical bases, simple exact solutions can be obtained.

In [11], the indicated contact model is justified and an exact solution for the problem of pressing a stamp with a flat base into an elastic half-plane is constructed. In [12], the proposed model is generalized for the case where, in addition to the normal compressive load, a shear load also acts on the stamp, and an exact solution to the problem in quadratures is obtained. This model is also generalized for a rigid interphase inclusion located in a piecewise homogeneous plane [13].

Based on the proposed model, closed and efficient solutions were also obtained for a number of plane and axisymmetric contact problems for classical foundations such as a half-plane [14], a half-space [15], a homogeneous and composite plane with a crack [16-18], and a homogeneous space with a disc-shaped crack [19]. Namely, the papers [20-22] discuss the exact solutions for plane and axisymmetric contact problems of pressing a stamp of arbitrary shape into an elastic half-plane and half-spaces with a variable contact zone. In most of the cited works, numerical

calculations were carried out and the obtained results were compared with the results obtained for similar problems within the framework of the Galin's model. Comparisons showed that the obtained results are very close. Note that the proposed model is not difficult to apply to any non-classical foundations, since the contact conditions are the same throughout the entire contact zone.

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