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An Investigation into College Students' Learning about Logarithmic Functions: A Thorny problem

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Abstract

There is only a small handful of studies that investigate students' learning of logarithmic functions. Therefore, there is little evidence to help educators improve the quality of instruction in this topic. This study investigated college students' learning of logarithmic functions of a college Pre-Calculus course (MATH 115) offered at the University of Delaware. By identifying specific skills that students find challenging may give insight to how educators better promote students' fluency. All students in MATH 115 were invited to participate in the initial stage of the project, a Pre-test, and the final stage of the project, a Post-test. A subset of the students that took the Pre-test were randomly selected to participate in a series of three one-on-one individual interviews. Out of 22 students that were asked to participate in the interviews, 7 students accepted. Results suggest that students may significantly improve their fluency in logarithms after receiving the instruction presented by the professors who taught MATH 115. In addition, students' developed fluency when they could read a logarithm, understand the notation, and use terminology. Future research should collect data from larger samples of students learning logarithms. This will allow researchers and educators to learn how to promote students' learning of logarithms.

Keywords: Math Education; Logarithms; Logarithmic Functions; Teaching Logarithms.

Introduction

I worked for the University of Delaware as a tutor and classroom assistant. I was a classroom assistant for six sections of MATH 115, and I observed that one of the most difficult units was logarithms. I know logarithms are applicable to various areas, such as banking and psychology, however the students in this course are unaware of the various applications. Therefore, I wanted to help students learn the importance of logarithms, understand what the issues students have and how could I improve my teaching to increase students' fluency. I wanted to contribute to a field that has little research for educators to learn from. The goal of the study was to collect information about students' perceptions of developing fluency in logarithms and help instructors develop more effective lesson plans that promote students' learning.

Literature Review

Berezovski conducted a study in two British Columbia high school classrooms. She had a sample size of 27 students enrolled in a twelfth-grade classroom. Her goal was to investigate students' difficulties of learning logarithmic functions. The students that participated in her study were "very motivated and could be considered 'the cream of the crop'" Berezovski [1]. She collected data through written questionnaires (one quiz, one-unit test, and 6 open-ended questions). The goal of each questionnaire was to assess students' learning about logarithmic properties and applications. Berevzoski conducted interviews and participated in classroom discussions. The goal of including interviews and classroom discussion was to get an insight to how students were learning logarithms in the moment. She found that students had difficulties expression logarithms as numbers and applying logarithms in real-life situations.

Williams [2] conducted a study at a United States western university mathematics education course. Her goal was to create a "research-based framework of what it means to understand logarithms" (p. 3). She administered a survey to assess students "wiliness to participate, knowledge of logarithms, and ability to explain their thinking" (p. 26). After she analyzed the surveys, she invited a subset of students to participate in a series of one-on-one interviews. The first interview assessed the "object and process conceptions of logarithms" that tied back to her initial framework (p. 26). The second interviewed examined students' learning of logarithmic functions and their applications.

The third interview was similar to the first interview; the final interview assessed similar topics as the past three interviews.

Williams concluded that:

a. There was a difference between "the practice of switching forms… and the process of meaning for logarithms" and

b. Her framework was successful in identifying students' misunderstandings of logarithmic functions (p. 71).

She suggested researchers and educators to help build a more effective framework to teach logarithms effectively and to continue to investigate students' learning of logarithms.

Overview of Methodology

The goal of my study was to investigate students' fluency in logarithms and to follow a subset of students to better understand how they perceived learning logarithms. To achieve my goal, I invited all MATH 115 students to take a Pre-test. The goal of the Pre-test was to obtain information about students' incoming knowledge of logarithms. Then I randomly selected 22 students to participate in three one-on-one interviews. 7 students accepted the invitation. The interviews were strategically conducted during the first week of learning logarithms, at the end of learning logarithms, and a few weeks after the logarithm lessons ended.

This allowed to follow students' individual journeys to developing fluency. After the interviews were conducted, I invited all MATH 115 students again to participate in a Post-test. The goal of the Post-test was to examine students' exiting knowledge of logarithms. Next, I conducted a series of statistical tests to examine if the scores of the Pre-test were statistically significant to the scores on the Post-test. I performed a t-test for each question on the tests. Both tests were identical. I also examined the 7 interview participants to gain insight into similar characteristics of learning logarithms.

Results

A Cross-Analysis of the Pre- and Post-Tests

I compared students' performance on each item from the Pre-test (N = 249) to the Post-test (N = 306). I found the difference of the mean scores and then divided that by the score on the Pre-test to determine the percent increase from the Pre- to Post-test. This revealed that students' mean scores at least doubled on nine items: Questions 4b (160%), 5c (150%), 6a (160%), 6b (170%), 6c (160%), 7a (150%), 7b (170%), 7c (160%), and 8c (160%). The questions can be seen in Table 1.

Another similarity across both assessments was three of the five lowest-scoring items pertained to students performing multiple steps to a problem. This indicated that student had a more difficult time solving problems that required this skill. This aligned with what previous researchers, Kenney [3] and Liang and Wood [4] concluded as well.

I performed a t-test using SAS to determine if the students' scores on the Post-test were statistical significantly higher than the scores on the Pre-test (alpha = 0.05, N = 155). The t-test concluded that all but one question was statistically significant. The one question that was not statistically significant was a True/False/I'm not sure question, $\log(1)=1$. Due to time constraints, an analysis to figure out why this question was not statistically significant was not performed [5-8]. The findings on the Pre-and Post-test indicated that students significantly improved their fluency in logarithms through the course instruction.

Table 1: A summary of pre- and post-test items that had large increase in mean scores.

Number on Test	Question
4b (True, False, I'm not sure)	$2\ln(3x-4)=\ln(6x-8)$
5c (Open Ended: Solving for x, Basic)	$\ln(x)=2$
6a (Open Ended: Simplify)	$\log_5(2x^3-x^2)-\log_5(x)$
6b (Open Ended: Simplify)	$8\log_2(x-2)+\log_2(x-5)$
6c (Open Ended: Simplify)	$\log(xy^2)-\log(z)$
7a (Open Ended: Expansion)	$\ln(\frac{2x-5}{y})$
7b (Open Ended: Expansion)	$\log(x)^3$
7c (Open Ended: Expansion)	$\ln(\frac{xy}{z^2})$
8c (Open Ended: Solve for x, Advanced)	$\log_4(x^2)-\log_4(x-1)=1$

Table 2: Students use of logarithmic terminology and relationship to test performance.

Participant	Interview #		Pre-test Score	Post-test Score	Percent Difference in Students' Scores
	#2	#3			
Fiona	2	2	14	36	157%
Jerry	2	2	20	35	75%
Renee	2	2	27	36	33%
Natalie	1	2	25	36	50%
Ellen	0	1	15	33	120%
Nancy	0	1	10	24	140%
Emma	1	1	21	29	38%

A Correlation between Students' use of Logarithmic Terminology and the Impact on Developing Fluency with Logarithms. From the interviews I found that a student's use of logarithmic terminology may impact their knowledge of logarithms. There may be a possible correlation between logarithmic terminology use and test performance. I rated the students' use of logarithmic vocabulary using a two-point scale. A student that received a zero did not use any logarithmic terminology. A student that received a one sometimes used logarithmic terminology in their verbal explanations [9]. A student that received a two correctly and consistently used logarithmic terminology. The results of students' terminology frequency are presented in Table 2.

Fiona, Jerry, and Renee quickly learned logarithmic terminology and incorporate it in their responses. Renee, who

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scored high on the Pre-test, increased her score by the maximum amount, earning a 36 out of 36 points (33%). Fiona also earned a 36 out of 36 on the Post-test and increased her score (157%). Jerry earned a 35 out of 36 points on the Post-test and increased his score (75%). Natalie who sometimes incorporated logarithmic terminology increased her score by 50%. In contrast to students, Ellen and Nancy, who used no logarithmic vocabulary to some increased their scores by 120% and 140%, respectively [10].

Emma who was consisted using some logarithmic terminology only increased her score by 38%. Her Post-test score was the second lowest compared to the rest of the interview participants. Besides Renee, she had the lowest score increase [11]. This could be a result in no change of incorporating logarithmic terminology in her explanations in the interviews. Therefore, this data hints at the possibility of a correlation between logarithmic terminology use and fluency with manipulation and solving logarithmic equations (Table 3).

Table 3: The calculated statistical significance of each question.

Question	P-value	Significance Rating
4a	< 0.0001	***
4b	< 0.0001	***
4c	< 0.0001	***
4d	= 0.1338	
4e	= 0.0001	**
4f	< 0.0001	***
5a	< 0.0001	***
5b	< 0.0001	***
5c	< 0.0001	***
6a	< 0.0001	***
6b	< 0.0001	***
6c	< 0.0001	***
7a	< 0.0001	***
7b	< 0.0001	***
7c	< 0.0001	***
8a	< 0.0001	***
8b	< 0.0001	***
8c	< 0.0001	***

Discussion

Implications for Pre-Calculus results impact students taking Pre-Calculus at other Educational Institutions

Even though my results are limited to the University of Delaware's MATH 115 course, mathematics educators at other schools, can learn from my study. My data suggested that students may develop fluency in logarithms if they learn to read logarithms and incorporate logarithmic terminology into their explanations [12]. This could be emphasized during lecture. An educator can constantly point out the different parts of a logarithm and explain the relationship between the different parts. For example, "What happens if I manipulate the argument? How does that impact my expression?" By incorporating logarithmic terminology may promote students' fluency in learning logarithms [13].

Next Steps: Where do researchers and educators go from here?

I suggest further investigation is needed to improve students' fluency of learning logarithms. My study suggests that using a mixed-methods model may gain more insight into students' difficulties learning about logarithmic functions rather than a quantitative or qualitative approach [14]. By using mixed-methods, I was able to gather numerical information and students' perceptions of learning logarithms. Collecting information from a larger sample is needed to help gain insight and promoting a more effective curriculum. There are many strategies to assess students' learning, and I believe the best way to learn about students' difficulties in learning logarithms is by using both written assessments and individual or group interviews [15].

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