

Review of Wear Model Optimization for Ceramic and Metal Total Hip Arthroplasty with Medical Physics Applications



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Abstract

In a series of previous contributions, two types of bioengineering studies of total hip arthroplasty (THA) modelling optimization were presented. Namely, computational modelling THA in vitro wear modelling parameter optimization for ceramic-on-ceramic (CoC), and metal-on-metal THA (MoM) one. The objective of the research was to obtain the optimal-principal model parameters for in vitro tribological predictions. These optimal are an adimensional K constant parameter, hardness, and hip-biomechanical load over the THA implant. In those contributions, the aim was carry out both dual and multi objective optimization. In ceramic, dual optimization was presented for Alumina (Al_2O_3), and Zirconium (ZrO_2) CoC, and secondly a multi objective one for Alumina, Zirconium, ZTA BioloX, and ZTA BioloX-Delta. In metal, dual optimization was obtained for Titanium and Co-Cr-Mo materials. This article presents a review of those most important results in Numerical, 2D Graphical, and 3D Interior Optimization. Applications obtained are useful/efficacious for in vitro tribology predictions in the area of clinical medical physics and bioengineering—with the new 4D Interior Optimization included. A second innovation related to the reviewed papers group is the GNU-Octave software design/comparison with Matlab for 4D Interior Optimization graphics. Most important of these usages in ceramic and metal are an adimensional K for wear modelling, and a optimal model parameter group for THA erosion/durability tribopredictions. Clinical orthopedics, bioengineering, and medical physics applications emerge from all the results.

Keywords: Total hip arthroplasty (THA); Orthopedics; Mathematical models; Nonlinear optimization; Dual optimization; Multiobjective optimization; 2D graphical optimization; 3D Interior optimization; Model parameters; Orthopedics applications

Introduction

Total hip arthroplasty (THA) is defined as the surgical implantation of a medical device whose main parts are cup (for natural acetabular substitution), head (for natural femur head substitution), and leg (the mechanical support inserted along the femur metaphysis) [1-4]. THA has a number of important properties, usually interdependent, directly and inversely. Among them, are biomechanical functionality, resistivity, material durability, lifetime, and histocompatibility. For instance, functionality is function of the lifetime, and lifetime is function of material durability. Instead, implant lifetime is not totally conditioned by the material durability [1-22]. Currently, there are three material types/groups widely used in total hip metal arthroplasty (THA). Namely, ceramic, metal, and polyethylene. Head-cup material combinations could be even (CoC, MoM) or uneven (PoM, CoM, PoC). When a polyethylene/polymer cup forms at least one component of the THA, the bearing is considered soft, [1-4], otherwise the bearing is hard. The wear of the THA implant occurs in-between the head and cup, specifically

as an erosion and abrasion bio tribological phenomena. Ceramic prostheses are subject of wear. However, metal prostheses/materials experiment also corrosion. When erosion and corrosion are synergic in metallic THA, tribocorrosion occurs. Erosion is a physical phenomenon; corrosion is a chemical one. Free radicals in plasma are the principal cause of corrosion. Erosive cracks or micro erosive damage can open the path for free radicals initiate the metal corrosion [1-22]. Other types of complications are caused by debris particles of the eccentric wear in the acetabular cup or the cup dislocation [7] when bone hardness is weak and screws cannot hold sufficiently. As an example, in previous spinal surgery modelling contributions, optimization for screw-insertion was mathematically presented [25]. THA biotribological wear-interface is based on complex biomechanical forces distribution and was presented in previous publications [1-4]. After the surgical setting of the THA, the biodynamics and biomechanics cannot result as equal to natural hip biomechanics. Therefore, the THA is subject of wear, mainly erosion and abrasion of the implant

[1-22]. Statistically, [1-4], the pathogenesis of the hip fracture and/or hip articulation malfunction is caused mainly by the high incidence/prevalence of femur neck fracture due to osteoporosis. Equation 1 and (Figure 1) prove the clinical path biomechanics. This happens usually in elderly patients, with higher incidence/prevalence in women [1-22]. This incidence/prevalence increases in developed countries in correlation with the increment of the average population age and lifetime expectancy. Other factor is the lack of physical activity at late lifetime that causes both ligaments and muscles weakness and osteoporosis-together with the sitting-down time increase in modern life. According to statistics in Europe, Germany and Switzerland are the countries where a higher number of THA are surgically implanted/fixed. Given a femur neck with osteoporosis, [1], the typical fracture happens because the neck unites the head and trochanter, which is the weakest and thinnest bone zone. In addition, the hip load over the femur head causes a mechanical torque, whose arm is along the neck and fixed on trochanter as follows,

$$\vec{\tau} = \vec{F} \times \vec{r} \quad (1)$$

Where τ is torque (vector resulting of vectorial product), F (vector) is load force (thorax+ abdomen weight) and r is mechanical arm (vector). Equation 1 is a simplification because there are mechanical reactions at trochanter caused by floor and at the hip articulation for load, and regional muscle and ligaments forces. The extremes of this physical arm are the femur head and the trochanter. The hip biomechanical-load that makes the torque-magnitude is exerted over the head. There is a synergism

factor, statistically in an elderly patient: osteoporosis, muscles and ligaments weakness, unexpected movements, or accidental falling down. When the patient moves abruptly or falls for any reason, the fracture could occur. Accordingly, the mathematical models optimization/design constitutes an efficacious method for sorting THA clinical difficulties and perform in vitro and in vivo wear predictions. The continuous evolutions/improvements in modelling, both analytical, numerical, hybrid and in finite elements, is performed with the mathematical computational bioengineering optimized for experimental data. A THA Taxonomy of THA models was published in a contribution of this review [3]. In addition, along these previous contributions [1-4], a number of optimization models were developed with computational mathematical methods. In consequence, the principal objective of this study was to obtain advances in an analytical model. This result is the numerical determination of the K adimensional constant parameter of the model for THA ceramic and metal. There are individual patient THA clinical factors, and others external to patients. In [1,3], a classification of clinical factors related to THA surgery was developed. The PF-TCF Hip Arthroplasty Functional Treatment Classification [1,3]. PF are defined as factors depending on the patient, and TCF are defined as technical-clinical factors of the hospital and/or traumatology-orthopedics service. Therefore, the contribution novelty of this review is to sum up THA model optimization to obtain/sort the orthopaedics and surgical clinical difficulties. Numerical, 2D Graphical, 3D Interior and new 4D Interior Optimization is presented sharing principal results [1-4].

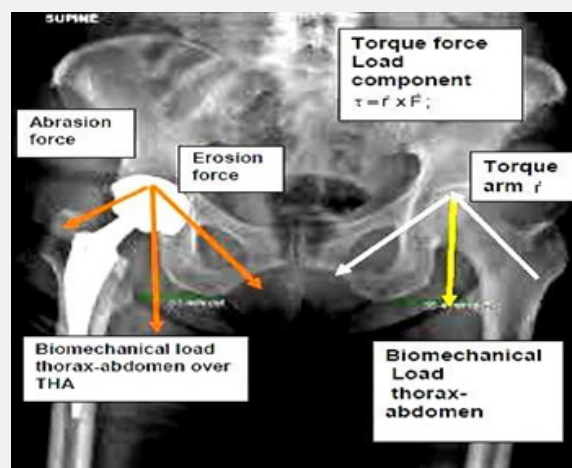


Figure 1: Pictured inset right, the mechanical torque sketch that could cause a femur neck fracture-provided the neck bone has osteoporosis or other pathological structure weakening cause. On the left, inset, the biomechanical forces distribution that make wear and abrasion in the THA device [Google free images labeled and drawn by author.

Materials and Optimization-Computational Methods

This section summarizes the most important material modelling selections along the recent contributions [1-4, 26]. First group is CoC materials and second one metals. Subsequently, the computational methods and algorithms used for both materials is presented. The difference between CoC and MoM algorithms

is given by the parameter constraints and the algebraic changes in the objective function to obtain better charts in 2D Graphical Optimization and 3D Interior Optimization. An innovation related to those papers is the 4D Interior Optimization Method shown to close the section. The 4D Interior Optimization graphs are included in Results Section. Other improvement is a computational-software comparison with GNU-Octave System. 4D

Interior Optimization graphics are presented and contrasted to Matlab images. Detailed data from ceramic/metal data selection is included in papers [1-4,26].

Materials

The necessary numerical data for ceramic optimization are hardness of CoC (hard bearings) materials, implant head standard diameter, experimental interval of erosion widely published in literature, units, and other complementary data which are mentioned but not implemented on this study model [14,5,7,17,18,21,22,27]. Their physical characteristics are detailed in Table 2. and unit's database. Ceramic histocompatibility is usually good. The parameters/intervals for dual optimization and multiobjective optimization are included. For dual optimization, selected materials are Alumina (Al_2O_3), and Zirconium (ZrO_2).

Multiobjective optimization comprises Alumina, Zirconium, ZTA BioloX, and ZTA BioloX-Delta. Materials selected for dual metal optimization are Cast Co-Cr and Titanium alloy [1-4,9,17-22]. Their physical characteristics are detailed in (Table 2). The material and corresponding experimental in vitro erosion data is taken from the literature [1-22,26]. In those studies, experimental metal wear intervals are implemented from [1-22]. In all cases, units were adapted on kg, mm, and mm^3 criteria. Table 2 shows the material selection data within computational intervals. The reason is that actually there are variants of Titanium alloys, and Co-Cr [1-4]. Provided the units are set in mm, mm^3 , kg, and s, the standard K parameter of the model results adimensional [1]. These intervals are very important when setting numerical data into the simulation software.

Table 1: PF-TCF classification for hip arthroplasty surgical treatment [Casesnoves, 2021]. It is improved from a previous publication [1-4].

PF TCF Hip Arthroplasty Functional Treatment Classification [Casesnoves, 2020]		
Type	PF Patient Factors	TCF (Technical-Clinical Factors)
Factor		
General Factor Type	Patient anatomy Osteoporosis Age, Individual patient, kinetics/dynamics Optimal THA individualized Immunology, special severe patient metal/ceramic alegria Histocompatibility, Associated Pathologies, Infection (resistance to radiation for eliminating infection) Risk Factors: Head size <50 mm Acetabular inclination >50 degrees Severe pain, Bilateral implants HR (head radius) <44 mm head Any accident, injury or unexpected circumstance in patient life	Imaging equipment available Surgical staff available Operation time available at surgical theater technical equipment Instrumentation functionality Hospital functionality Type of THA Economical factors Country technology in Industrial Medical Technology Precision in pre-operative and evaluation Rehabilitation programs Possibility of individual prostheses In perfectly designed THA devices
Female Factor Type	Immunosuppression drugs (any type) Renal insufficiency Sub-optimal device alignment Suspected metal sensitivity Severely overweight in women High levels of physical activity Low levels of physical activity	

Table 2: Materials CoC data implemented in dual and multiobjective optimization models with complementary details [1-4,29].

CoC Numerical Data [Dual]				
Material	Hardness (GPa)	Density (g/cm ³)	Histocompatibility	Standard Head Diameter (mm) and interval
Alumina (Al ₃ O ₂)	22	3.98	good	28 [22-28]
Zirconium (ZrO ₂)	12.2	5.56	good	28 [22-28]
CoC Numerical Data [Multiobjective]				
Material	Hardness (GPa) and		Density (g/cm ³) and Head Diameter (mm)	
Alumina (Al ₃ O ₂)	22		3.98 28 [22-28]	
Zirconium (ZrO ₂)	12.2		5.56 28 [22-28]	
ZTA BioloX	[15.2, 16.3]		4.37 (approx) 28 [22-28]	
ZTA Bio loX-Delta	[15.2, 16.3]		4.37 (approx) 28 [22-28]	
Complementary Data	Elasticity Modulus and Fracture Toughness are useful for other type of calculations. Density varies slightly in literature. The standard femoral head used diameter is 28mm. Hardness also varies in literature. For BioloX and BioloX-Delta (ZTA) hardness magnitude varies in the literature.			

Optimization-computational methods and algorithms

This section describes the general method and algorithm used for CoC and MoM [1-4,26]. Subsequently, specific mathematical constraints for CoC and MoM are presented. The method for dual and multiobjective optimization for ceramic THA and dual metal was previously published [1-4,29]. Algorithms implemented are based on classical Archard's model [1,2,15,16,20,28], but with vector-matrix and units' modifications [Casesnoves Algorithm, 2020-1]. A variant from this model with evolved algorithms was developed in previous contributions [26]. In the literature, a linear type of the classical equation for wear optimization of hip implants without hardness reads,

$$W = k_1 \times \left| \vec{F}_\perp \right| \times D \quad (2)$$

Where K_1 is a constant that depends on hardness, F is load force (vector, and it is taken the normal component norm), and D is the sliding distance that depends on the velocity. There are several versions for this linear type of equation published studies [1-22,26]. However, it is considered the non-linear classical formula for these study objectives. The criterion is that hardness must be, for optimization precision, a separated parameter such as,

$$W = k \frac{L \times X}{H} \quad (3)$$

Where K is wear constant specific for each material, L biomechanical load (N, passed here to kg and mm), X sliding distance of the acetabular semi-sphere of the implant (mm), W is wear (mm^3), and H is the hardness of the implant material (MPa, here it is used always kg and mm). In the literature [19], this K was set dimensional. In the reviewed publications, an adimensional K constitutes the practical innovation of the algorithms [1-4,26]. Hence, setting vector calculus, for Inverse Least Squares optimization, minimize,

$$\left\| \vec{W} - k \frac{\vec{L} \times \vec{X}}{H} \right\|_2^2 \cong 0 \quad (4)$$

Subject to (generically and with vector-matrix calculus),

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \leq \begin{bmatrix} \left| \vec{k}_i \right| \\ \left| \vec{L}_i \right| \\ \left| \vec{X}_i \right| \\ \left| \vec{H}_i \right| \\ \left| \vec{W}_i \right| \end{bmatrix} \leq \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \end{bmatrix} \quad (5)$$

Where K is wear constant specific for each material, L biomechanical load (vector, N, passed here to kg and mm), X sliding distance of the acetabular semi-sphere of the implant

(vector, mm), W is wear (vector, mm^3), and H is the hardness of the implant material (vector, MPa, here it is always passed on to kg and mm). X is measured as the number of rotations of the implant multiplied by about half distance of its circular-spherical length. This formula was set both for CoC and MoM optimization [1-4,26]. In the paper-reviewed group, X is approximated according to human biomechanics and kinesiology. The average rotation of femur head cannot reach 180° at any biomechanical movement in common patients. This is valid for flexion, extension, flexion-rotation, extension-rotation, abduction, adduction, external/internal rotation, or combinations of these [1-4]. For the program settings in [1-4], one cycle was taken as the length corresponding to the maximum kinesiological rotation angle. Abduction, flexion and extension angles could be higher than average in special patients such as athletes or sporting individuals. The maximum femur rotation angle value is 145° in flexion. In the software, this magnitude was implemented. That is, Mc (million cycles) of rotation length is calculated: circumference implant-head radius R by π for a factor of angle of 145° and by 106. Therefore, the erosion in vitro data resulted from this optimization always has to be considered as the maximum angle possible. Number of rotations depend on the individual patient factors, [1-22]. The approximations for load for vector L for nonlinear optimization were a load-magnitude of around 200% of body weight (200%BW) [1-4]. Constraints for load are set from a 50kg patient till a 80kg patient. 50 kg corresponds, for example, to the body weight of a lightweight patient, or any elderly women who present a high incidence/prevalence of femur head fractures. It is recommended to read more mathematical details in all open access papers [1-4]. The OF with L2 Norm that was used, [Casesnoves Algorithm, 2020-1], without fixed constraints reads,

$$\text{minimize,} \quad \left\| F \left(\vec{W}, \vec{K}, \vec{H}, \vec{L}, \vec{X} \right) \right\|_2^2 \cong \dots \quad (6)$$

$$\dots \cong \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \dots \quad (7)$$

$$\dots \sum_{i=1}^{i=N} \left(F_{ijk} \left(W_{ijk}, K_{ijk}, H_{ijk}, L_{ijk}, X_{ijk} \right)^2 + \dots \right. \\ \left. \dots + F_N \left(W_{N,N,N}, K_{N,N,N}, H_{N,N,N}, L_{N,N,N}, X_{N,N,N} \right)^2 \right);$$

subject generically to,

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \leq \begin{bmatrix} \left| \vec{k}_i \right| \\ \left| \vec{L}_i \right| \\ \left| \vec{X}_i \right| \\ \left| \vec{H}_i \right| \\ \left| \vec{W}_i \right| \end{bmatrix} \leq \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \end{bmatrix}; \quad (8)$$

where the constraints values (a-e), are detailed in Table 4 for CoC and MoM. K is wear constant specific for each material, L biomechanical load (vector, N, passed here to kg and mm), X sliding distance of the acetabular semi-sphere of the implant (vector, mm), W is wear (vector, mm³), and H is the hardness of the implant material (vector, MPa, it was always passed on to kg and mm). Fortran 90, [24], GNU-Octave and Freemat were used to check/validate the numerical precision of the numerical results and all graphics. 2D Graphical Optimization and 3D Interior Optimization methods in Matlab and GNU-Octave Systems are different programs compared to the numerical optimization ones. In every case, programming involves different structure, subroutines, and patterns. K is the principal variable for optimization. The reason is that with a multiobjective K parameter it is possible to carry out in vitro simulations in the materials selection process. The hardness for simulations in vitro, within the optimization hardness interval, could be therefore different from the ceramic/

metal types of Tables 2,3. Therefore, the Inverse Least-Squares OF inverse algorithm [Casesnoves,2021,1,2,24] general formula implemented for constraints [1-4] (Table 4),

is minimize OF, subject to,

$$\begin{aligned} N &= 2 \times 10^6 \\ \forall |W| \in \vec{W}, \vec{H}, \in \vec{H}, \vec{L}, \in \vec{L}, \vec{X}, \in \vec{X}, \\ e &\leq |W| \leq e_1 \text{ mm}^3 \\ d &\leq |H| \leq d_1 [kg, mm] \\ b &\leq |L| \leq b_1 [200\%, BW] \end{aligned} \quad (9)$$

$$\|\vec{X}\| = \pi \times 28 \times (145 \times 10^6) / 180 \text{ (1 million cycles); (10)}$$

Table 3: MoM material parameters, intervals, and programming numerical settings. Dual Optimization exclusively [1-4].

MoM Numerical Data Intervals [Dual]			
Material	Hardness (Hv) and Histocompatibility	Density Intervals (g/cm ³)	Cup Head Diameter (mm)
Cast Co-Cr alloy	300 average	[7.80,9.15]	28 [22-28]
Titanium alloy	362 (approx.) good	[4.43,4.66]	28 [22-28]
MoM Optimization Data Intervals [Dual]			
Hardness (GPA)	[2.7, 4.0]		
Experimental Erosion (mm ³ / Mc)	[0.01,1.8]		
Complementary Data	Elasticity Modulus and Fracture Toughness are useful for other type of calculations. The standard femoral head used diameter is 28mm. Cast Co-Cr alloy hardness varies in literature. There are a large number of Titanium alloys available with closely hardness.		

Table 4: All numerical data for programming constraints in the articles reviewed [1-4]. In [4], intervals are something different as it was a first stage of MoM optimization.

Constraints Parameters CoC (Dual and Multi objective) and MoM (Dual)							
Material Method	e	e1	d	d1	b	b1	Comments
CoC Dual	0.02	0.1	12 x10 ⁶	23 x10 ⁶	7.5 x 10 ⁴ x 9.8066	2.0 x 10 ⁵ x 9.8066	
CoC Multi- Objective	0.02	0.1	12 x10 ⁶	23 x10 ⁶	7.5 x 10 ⁴ x 9.8066	2.0 x 10 ⁵ x 9.8066	Note that wear interval is the same as ZTA Biolox and ZTA Biolox Delta are within CoC Dual interval
MoM Dual	0.01	1.8	2.7 x10 ⁶	4.0 x10 ⁶	7.5 x 10 ⁴ x 9.8066	2.0 x 10 ⁵ x 9.8066	Note the magnitude order difference with CoC

Equation 6 specific numerical numbers for constraints are detailed in (Table 4) for CoC and MoM. Provided this OF and constraints, running program time resulted be between 2-7 minutes-with a standard current microprocessor and pc memory. 3D Interior Optimization Matlab plots take longer time because there are 3D volume matrices, a number of nested arrays and

patters compared to 2D Graphical Optimization. Scale factors are essential in both types of imaging subroutines and codes for sharp visualization [1-4]. Image processing and computer vision program-sentences and commands are essential to obtain good graphics. 3D Interior Optimization GNU Octave plots take longer time than MATLAB and provide with good computer vision

imaging. GNU Octave Image Processing programs are set with different subroutines compared to Matlab but similar software, arrays, nested loops, and patterns than MATLAB [1-4,26].

Review of Results

The presentation of the results review is mainly graphical and

numerical, divided into several subsections. The difference with papers [1-4] is the new development of 4D Interior Optimization with Matlab and GNU-Octave. All numerical data is included in Tables 5-8 and Figures 2-13. Further details can be found in [1-4,26].

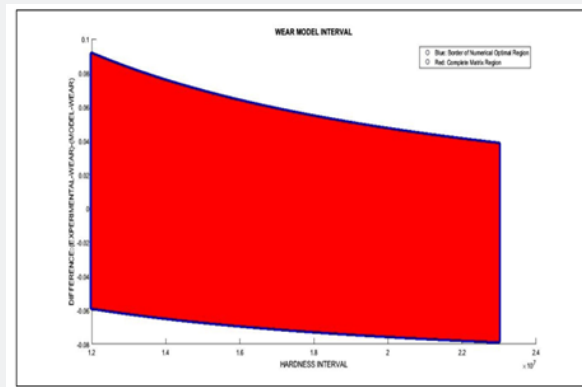


Figure 2: 2D CoC Dual Graphical Optimization of model for K. All parameters are in Kg and mm (one million cycles). The matrix for all evaluated parameters in optimization program covers a 2D region. The numerical result of the difference between model and experimental wear axis Y. Matrix array has all possible combinations of parameters, namely, load, hardness, and experimental wear. The initial Volume Matrix, that is, a 3D matrix with 3 variables, hardness, load, and experimental magnitudes was converted with programming arrays to a 2D matrix of 2×10^6 functions. Optimal K is 9.587464×10^{-9} . Residual is 1.76697×10^3

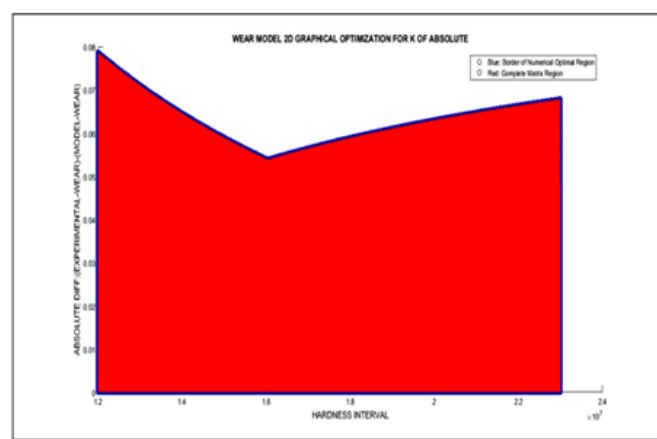


Figure 3: 2D graphical optimization of model for K. Absolute value of of at Y axis. All parameters are in kg and mm (10^6 cycles). . The matrix for all evaluated parameters in optimization program covers a 2D region. The numerical result of the difference between model and experimental wear is at Y axis. Matrix has all possible combinations of parameters, namely, load, hardness, and experimental wear. Optimal hardness can be observed at peak-concavity approximately at 1.5×10^7 (units at Table 2, kg and mm^3). With Graphical Optimization is exactly 1.526×10^7 . OF absolute value is within interval $[0,0.08]$, which is an acceptable result for the experimental data implemented, $[0.02, 0.1]$.

Dual and multiobjective CoC numerical results

Numerical results are detailed in Tables 5,6. The dual and multiobjective CoC nonlinear optimization for Alumina and Zirconium numerical results can also be obtained from graphics [1-4] with Matlab graphics cursor and Image Processing Tools.

DUAL MoM numerical results

The Numerical Optimization for MoM is included in Table 7. The numerical results, which are presented in Table 7, correspond to the subroutine optimization output. They can also be read/validated from graphics with Matlab. The optimal

K value obtained is 28.9295×10^{-9} with residual 660.4426×10^3 . The optimal hardness obtained is 3.054×10^6 . Nonlinear Dual 2D Optimization matrix was set with 2×10^6 functions. Running time was about 2–7 min to obtain local minima and graphics. As it occurred for THA ceramic modelling optimization [1-4], the exclusive existence of local minima is demonstrated. The proof that several local minima exist is got after multiple tentative

optimizations with different initial search vectors along interval parameters. Among all of them, it was selected the best one with minimum residual. Residuals are low considering the 2×10^6 OFs of the optimization matrix. Freemat [1-4,26] was used to verify 3D graphics and Fortran [1-4,26] for all numerical results. Note that the a dimensional K of MoM is one magnitude order higher than CoC K.

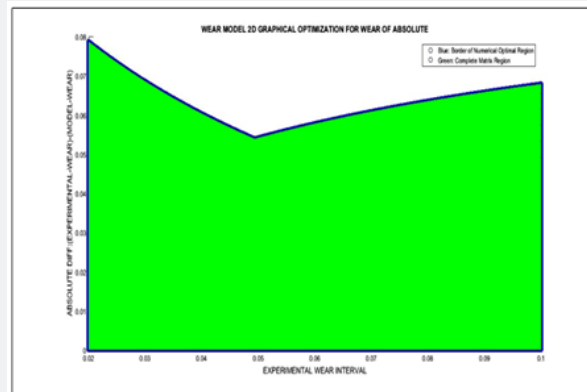


Figure 4: 2D graphical optimization of model for experimental wear versus model of absolute value. All parameters are in kg and mm (10^6 cycles). The optimal value of that gives the minimum difference between model and experimental is seen as the peak around the middle of graphics. The matrix for all evaluated parameters in optimization program covers a 2D region. The numerical result of the difference between model and experimental wear is at axis Y. Matrix has all possible combinations of parameters, namely, load, hardness, and experimental wear. Optimal wear can be observed at peak-concavity exactly at 0.0437 mm^3 . It is a local minimum [1-4].

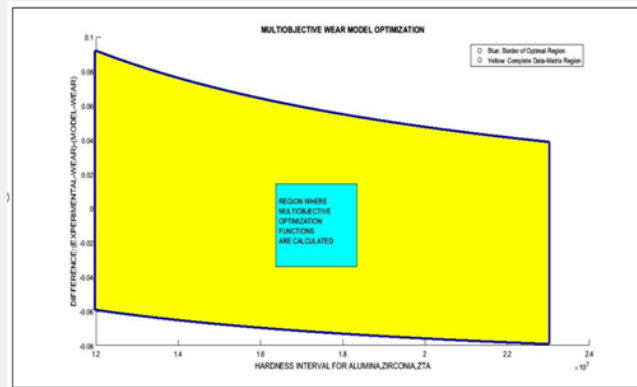


Figure 5: 2D multi objective CoC graphical optimization of model for K. All parameters are in kg and mm (10^6 cycles). The matrix for all evaluated parameters in optimization program covers a 2D region. The numerical result of the difference between model and experimental wear is at axis Y. Matrix has all possible combinations of parameters, namely, load, hardness, and experimental wear. The initial 3D volume-matrix, that is, a 3D matrix with 3 variables, hardness, load, and experimental magnitudes was converted with programming arrays to a 2D matrix of 2×10^6 functions. Optimal a dimensional K is 9.587464×10^{-9} . Residual is 1.76697×10^3 .

DUAL CoC imaging results

The CoC Dual Graphical Optimization results are presented a series of in graphics (Figures 2-7) show the model graphical optimization. The curves and areas correspond to model objective function (Y-axis) related to parameter values (X-axis). Nonlinear optimization matrix was set with 2×10^6 functions. Running time was about 2-7 minutes to obtain local minima and graphics. The 2D surfaces obtained have all the OF values for these functions with

all model parameters combined. Residuals are low considering the 2×10^6 OFs of the optimization matrix.

Multiobjective CoC imaging results

in graphics Figures 2-7 show CoC graphical optimization of model for K. All parameters are in kg and mm (10^6 million cycles). As was made in CoC Dual Optimization, the matrix for all evaluated parameters in optimization program covers the 2D region along the graphics. The numerical result of the difference between model

and experimental wear is at axis Y. For 2D Graphical Optimization, the matrix has all possible combinations of parameters, namely, load, hardness, and experimental wear. In the programs, the initial 3D volume-matrix, that is, a 3D matrix with 3 variables, hardness, load, and experimental magnitudes was converted with programming arrays to a 2D matrix of 2 million functions. Optimal

K is 9.587464×10^{-9} . Residual is 1.76697×10^3 (Figure 7) shows 3D Interior Optimization to check the optimal K value. It illustrates also the inversely proportional relation between hardness and erosion magnitude. Volume-matrix array has 10^5 elements.

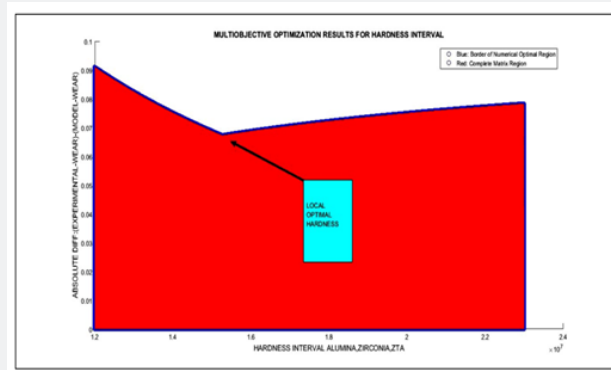


Figure 6: 2D Graphical Optimization of model for hardness. Absolute value of [1-4]. All parameters are in kg and mm (10^6 cycles). The matrix for all evaluated parameters in optimization program covers a 2D region. The numerical result of the difference between model and experimental wear axis Y. Matrix has all possible combinations of parameters, namely, load, hardness, and experimental wear. Optimal hardness can be observed at peak-concavity approximately at 1.5×10^7 (units at Table 2, kg and mm^3). With 2D Graphical Optimization is exactly 1.526×10^7 . OF absolute value is within interval [0,0.08], which is an acceptable result for the experimental data implemented, [0.02, 0.1].

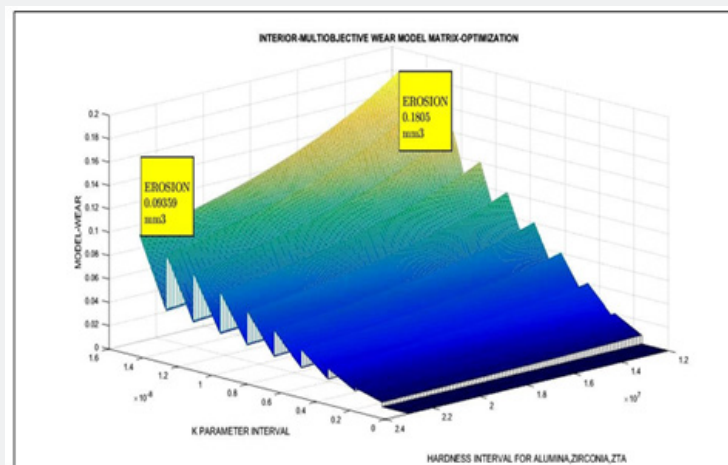


Figure 7: 3D Interior CoC Multi objective Optimization to prove with numbers in mm^3 that erosion is higher (0.1805 mm^3) when hardness is lower (0.09359 mm^3) and load is higher. Optimal K value obtained in numerical optimization is verified [1-4].

Dual MoM imaging results

These 2D and 3D optimization program(s) are based on nested arrays and a 3D volume-matrix with 10^5 and 10^6 elements, Figures 8-13, second program. For 2D Graphical Optimization, X-axis shows hardness, and Y-axis shows differences between experimental and model values [1-4]. For 3D Interior Optimization, X-axis shows K interval around optimal value obtained with optimization program, and Y-axis shows the hardness interval [1-4]. All numerical values are expressed in mm, mm^3 , and kg. The

software design was rather complicated [2,5,6,24].

4D CoC interior optimization

This new development does not correspond to the series review. In Figure 12, a sketch showing 4D Interior Optimization is proven. Based on the same programming method than (Figures 10,11), a 4D graph for multiobjective ceramic optimization is presented, Figure 12. 1D is the model wear, 2D is hardness interval, 3D is the adimensional K interval, and 4D is the model

load. 4D corresponds to the vertical increase of the load in the graph block. Lower data means lower load, and upper higher one. To develop the 4D load image, it is necessary to include load interval within the 3D volume-matrix of the program pattern. In this way, it is clearly proven the tribology fundamentals: model-

erosion is directly proportional to load and K, and inversely proportional to hardness. 4D Interior Optimization development in image processing depends of the particular model and the programming method.

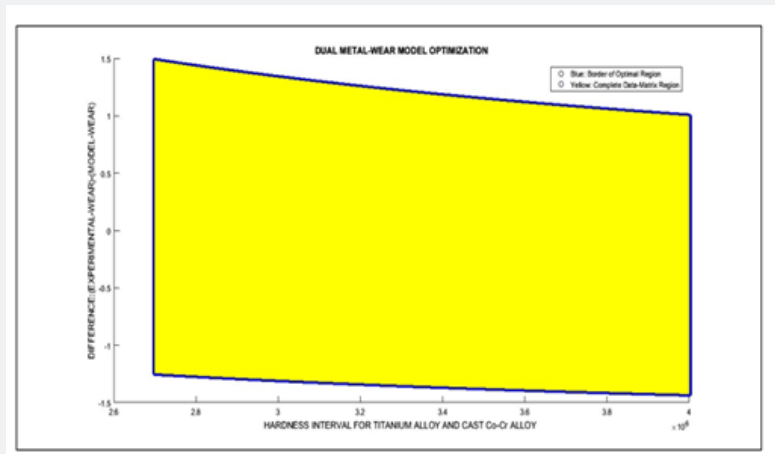


Figure 8: Optimization region and the decrease of MoM erosion when hardness increases, and the difference between experimental values and model figures [1-4].

4D CoC GNU-octave comparative interior optimization

Just the same, programming method was used from Figure 12 for GNU-Octave System. Recently, GNU Octave software shows a number of new imaging processing tools. These computer vision programming tools give a good quality images/graphs. Programming code in GNU Octave is similar to Matlab but different. It gives sharp images. Time for getting an image is about 30 seconds longer than Matlab. GNU Octave imaging tools are simpler than Matlab, but work correctly and fast. For setting a 3D image in GNU-Octave, there are several image-processing subroutines/options.

Numerical general-formula for results demonstration

The numerical results verification can be checked by two ways [1,4,26]. The first one is the 2D, 3D, and 4D graphics parameters and intervals that provide with numerical data distribution. The second is to check whether the model optimal values are within the experimental interval. To verify the numerical results the optimal values are implemented in the model, the general formula reads,

$$\left| k(\text{optimal}) \times \frac{\text{Load}(\text{Average}) \times \text{Mc}}{\text{Hardness}(\text{Optimal})} \right| = A \in [B, C]; \quad (11)$$

Where A is the numerical result of erosion of fitted model in mm^3 . B, C are the lower and upper limits of the experimental interval in mm^3 (Table 8) shows parameters A, B, C for CoC Dual,

CoC Multiobjective and MoM Dual Optimization. Accuracy of the model is good. Note that CoC Dual and Multiobjective Optimization have the same values because optimization interval comprises ZTA Biolox and ZTA Biolox Delta parameters [1-4].

Orthopedics and medical physics applications

Review of applications in traumatology surgery and orthopedics are presented along a series of reviewed contributions [1-4]. Hip articulation is rather a complex anatomical, physiological, and biomechanical system. The hip articulation biomechanical system is essential for locomotion and its biomechanical structure is significantly complex for kinematics and kinetics. In brief, the optimal hip biodynamics depends on two factors. First, one is the powerful muscles that perform the movement, walk and run mainly. Second is the strong ligaments that set the supporting forces to control correctly the muscles contraction/relaxation and create constraints to guide and support/prevent/resist any biased movement. Main ventral muscles for walk and run that create hip rotation cycles are Psoas Major and Minor, Iliacus, Abductor Longus and Adductors system. At dorsal, Gluteus group is essential for locomotion. Most important ligaments are Iliofemoral (supporting), Pub femoral (resisting), Iliolumbar (limiting), and Ischiofemora (resisting). In plain language, hip joint resembles a sophisticated-precise locomotion human machine with a wide range of movements/biodynamics. The energy-power is provided by the muscle system, while the ligaments and bones constitute the biomechanical frame structure. This biodynamical system has been genetically optimized during the

human evolution along million years. Therefore, it is difficult to synopsise the applications in a simple taxonomy-format. Here is intended to summarize the principal findings and set a new classification/perspective for the optimized model usage-that can be extrapolated to other articulation models. There are two main groups of applications. The first is the biotribotesting in vitro itself/exclusively, Table 9. The second comprises two strands: applications for THA improvements, and applications to avoid/minimize THA complications, Table 10. Literature for pre-surgery and post-surgery THA pathology is profuse [1-17]. Laboratory applications are tribotesting predictions for wear rates, no matter what type of apparatus/experimental-system is used. For every experimental data obtained, model can be improved. The possibility to design new models based on the initial one is other chance Table 9 summarizes all these perspectives. For practical medical industrial applications, the advantage of the demonstrated model is its simple non-linearity and adimensional K. That is, in engineering medical devices production the common method is to set as simpler as possible for manufacturing, avoiding complex equations. As a cautious and tentative surgical pre-hypothesis, THA normality restoration is a multifunctional

clinical subject, depending on a number of synergic-interactive factors. Therefore, the second group of applications is more difficult to abbreviate, because both THA improvements and THA complications prevention depend on multifunctional factors [1-4]. Besides, these factors are synergic one another. For example, post-operation increase of THA wear causes a post-operation biomechanical problem and viceversa [1-4]. Other factor, for instance, could be a post-operation deformation (cause) by osteoporosis, that could result (effect) in wear increase or THA malfunction [1-17]. Therefore, there are two main surgical-pathology concepts: cause and effect. Namely, if cause were aware post-operation increase for defective THA device lab design, the effect would be a biomechanical partial/total failure. Conversely, if a biomechanical post-operation complication cause occurs (increase of osteoporosis, osteonecrosis, biomechanical injury, etc.), the effect would be an increase of THA wear symmetric, non-symmetric, with/without cracks etc. Table 10 sums up all these possibilities/ ideas. It shows causality and multi-effect links for improvements and complications prevention. Literature and clinical case reviews are profuse in this matter [1-17].

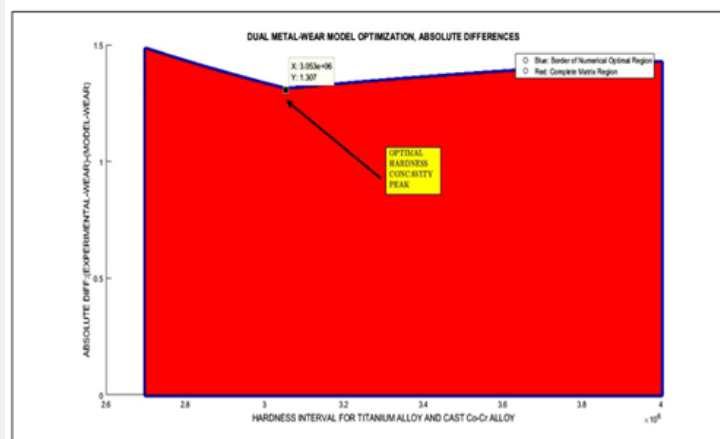


Figure 9: Optimal hardness obtained verification with 2D MoM Graphical Optimization. Numerical value can be obtained both with software and graphics [1-4].

Table 5: CoC Dual Optimization numerical results [1-4].

CoC Dual Optimization Numerical Results					
Material	Optimal K a dimensional	Optimal hardness (kg, mm)	Optimal Erosion (mm ³)	Optimal Load (kg, mm)	Residual
Alumina	9.587464 x 10 ⁻⁹	1.526 x 10 ⁷	0.0489	1.099 x 10 ³	1.76697 x 10 ³
Zirconum					
Additional Data	All units used in optimization are passed in Kg and mm. Number of nonlinear functions for program is 2 million. The initial Vol- ume-Matrix, that is, a 3D matrix with 3 variables, hardness, load, and experimental magnitudes was converted with programming arrays to a 2D matrix of 2 million functions. Absolute difference between (experimental wear interval) -(model wear interval) ∈ [0.02, 0.1].				

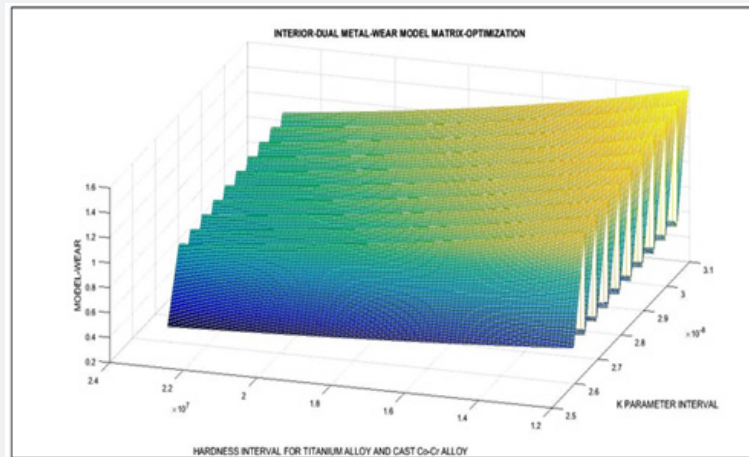


Figure 10: Lateral view of the MoM 3D Interior Optimization matrix image with 10^5 elements. K optimal value is verified. It is visualized clearer that erosion is higher when hardness is lower, and load is higher [1-4].

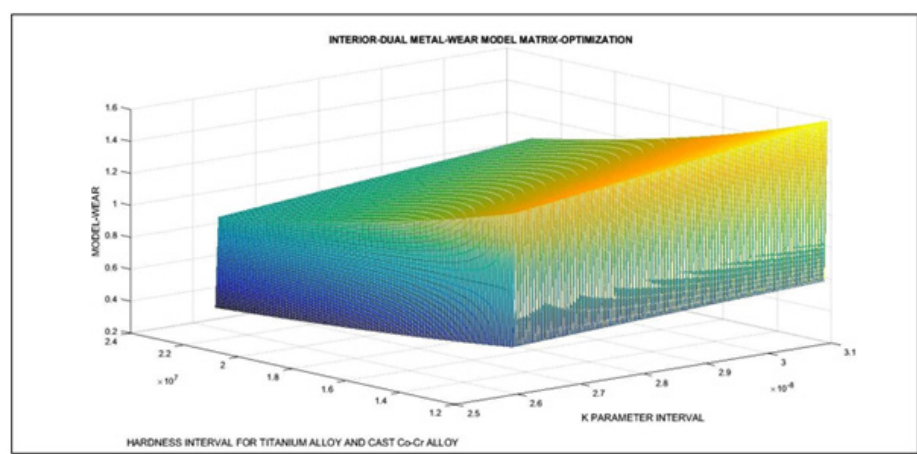


Figure 11: The 3D MoM Interior Optimization matrix image with 10^6 elements. It proves even better that erosion is higher when hardness is lower, and load is higher. The running time of this program is longer as the matrix has 10^6 elements. K optimal value is verified [1-4].

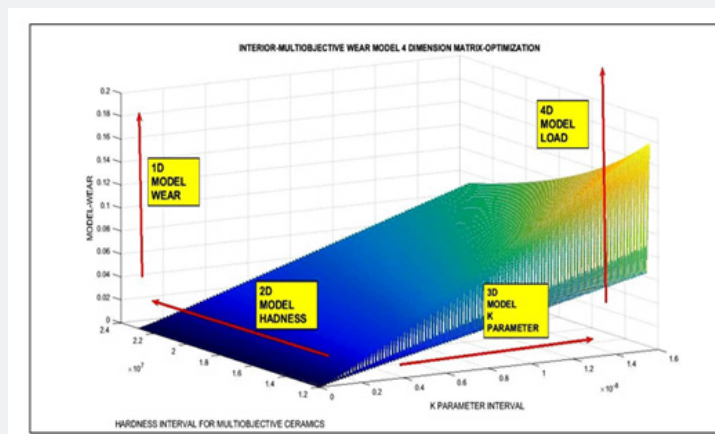


Figure 12: Matlab demonstration of 4D CoC Interior Optimization, model accuracy, and numerical K value validation. The 4D Interior Optimization volume-matrix image with 10^6 elements but setting at Z axis the model wear. It proves that erosion is higher when hardness is lower, and load is higher. Matrix has 10^6 elements and was set with K optimal interval that was obtained with 2D optimization algorithm. 4D Load at any level can be obtained setting at model Eq.2, 1D, 2D, 3D parameters and making Load explicit at Eq., 2, [1-4].

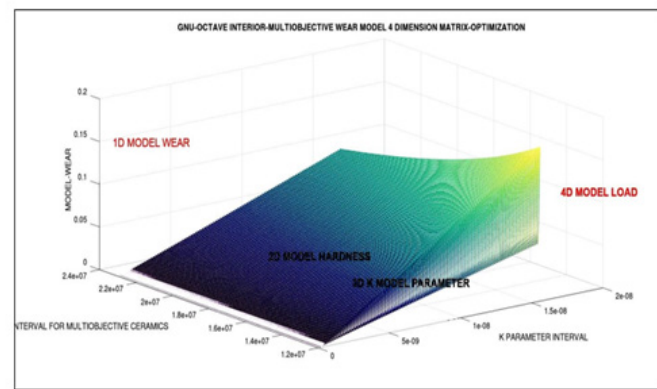


Figure 13: GNU-Octave demonstration of 4D CoC Interior Optimization, model accuracy, and numerical K value validation. GNU-Octave Image Processing subroutines give good quality graphics. The 4D Interior Optimization volume-matrix image has 10^6 elements but setting at Z axis the model wear. It proves that erosion is higher when hardness is lower, and load is higher. Matrix has 10^6 elements and was set with K optimal interval that was obtained with 2D optimization algorithm. GNU-Octave running time is higher than Matlab, and Image Processing Tools are fewer. Dimension labels are red and black. The 4D load increases from the lower part of the block to its higher part. It proves that erosion is higher when hardness and load are lower, and load is higher. Matrix has 10^6 elements and was set with K optimal interval that was obtained with 2D Numerical and 2D Graphical Optimization algorithm. GNU-Octave Imaging Processing commands, simpler than Matlab, were used to obtain this picture.

Table 6: CoC Multi objective Optimization numerical results [1-4].

CoC MULTIOBJECTIVE OPTIMIZATION RESULTS		
Material	Optimal K a dimensional	Optimal hardness (kg, mm)
Alumina	9.6 x 10 ⁻⁹ [truncated]	1.5 x 10 ⁷ [truncated]
Zirconum		
ZTA BioloX		
ZTA BioloXdelta		
Material	Optimal Erosion (mm ³)	Optimal Load (kg, mm)
Alumina	0.05 [truncated]	1.01 x 10 ³ [truncated]
Zirconum		
ZTA BioloX		
ZTA BioloXdelta		
Residual For Optimal Hardness and Additional Data	1.8 x 10 ³ [truncated]	
3D Interior Multi objective Optimization Results	All units used in optimization are passed in Kg and mm. Number of nonlinear functions for program is 2 million. The initial Volume-Matrix, that is, a 3D matrix with 3 variables, hardness, load, and experimental magnitudes was converted with programming arrays to a 2D matrix of 2 million functions. Absolute difference between (experimental wear interval) -(model wear interval) ∈ [0.02, 0.1].	
3D matrix Program	Validation of K optimal parameter in chart. Validation of erosion rises when Hardness decreases.	

Table 7: Dual Optimization numerical results [1-4].

MoM Dual 2D Optimization Results		
Material	Optimal K adimensional	Optimal hardness (kg, mm)
Cast Co-Cr alloy	28.93 x 10 ⁻⁹ [truncated]	3.05 x 10 ⁶ [truncated]
Titanium		
Residual for optimal k	660.44 x 10 ³ [truncated]	
MoM 3D Interior Optimization Results		
3D matrix Program	Validation of K optimal parameter. In chart. Validation of erosion rises when Hardness decreases	

Table 8: Numerical validation model of parameters. A, B and C are parameters for Eq 6 [1-4].

Numerical Validation Model Parameters CoC (Dual and Multi Objective) and MoM (Dual)			
Parameter	A	B	C
CoC Dual	0.0489	0.02	0.1
CoC Multiobjective	0.0489	0.02	0.1
MoM Dual	0.7393	0.01	1.8

Table 9: First group of studies extended/improved applications [1-22].

Computational Medical Physics and Bioengineering Applications	
Theoretical-Mathematical	
Modelling (1)	CT Scans data to measure with FE wear in post-operative, Analytical models can be set in FE modelling within elements
Modelling (2)	Prediction of wear after THA implant is set, short-term and long-term
Experimental/Clinical	
Laboratory Methods and Simulations	Improvements in simulation lab tasks getting more accuracy in modelling Design of new models Finite Elements method applications
Manufacturing THA	Simulations in design of materials
	Manufacturing tribo testing
	Medical device quality control
	New materials manufacturing
HIP Prostheses	Decrease of debris complications for wear increase
THA Orthopedics	Prevent dislocation metallosis and soft-tissue damage with material quality improvements
PREVENTIVE MEDICINE	Statistics for prevention of incidence/prevalence of hip joint diseases, statistic models for disease evolution. Lab statistics for THA lifetime, in vitro and/or in vivo
Extrapolated Modelling Applications	
Knee Arthroplasty Models	Erosion/wear modelling in knee arthroplasty similarity improvements
Other Articulations Modeling	Extrapolable methods for other articulation wear modelling Methodology extrapolation to other artificial articulations wear modelling
Sport Medicine	Modelling for high-performance sport-medicine prostheses with tougher, more flexible materials, or metastable materials

Table 10: Second group of studies applications [1-22]. Some of the clinical improvements are at the same time solutions to prevent complications and vice versa.

Clinical Biomedical-Bioengineering Applications			
THA Clinical Improvements			
Type	Cause	Effect [multiple]	Usage for Resolving/Ease
Erosion Prediction for Individual Patient	Patient individual Factors: Anatomy, Age Associated diseases etc, then erosion increases	THA does not fix those factors and THA fails in time or deteriorate soon	Clinical Individual THA selection/design computational-imaging pre-operation study
Research For Future THA	Simulations of new similar models in <i>vitro</i> and in <i>vivo</i>	Decrease of erosion surrounding complications and increase of durability	Making new THA models with better set-in conditions and durability
Research For Future THA	The composed materials wear whose hardness fall within the computed interval can also be simulated with these optimal results.	Possibility to manufacture new similar materials with better properties, and lower wear	Clinical THA improvements to sort post-operation problems and complications
THA Durability Prediction	Bio tribo testing faulty/inaccurate for optimal functionality	Durability prediction fails, re-operation, THA substitution and economic loss	Modelling improvements and bio tribo testing optimization
Model Parameter Optimization	optimal parameters for hardness, load, and wear efficacy		
Prevent/Avoid THA Complications			
Type	Cause	Effect [multiple]	Usage For Resolving/Ease
Pre-Operation Simulations	Computational and imaging simulations for optimal operation	post-operation success	THA pre-operation work but avoiding post-operation surgical complications problems/work
Research for Avoiding Zone Associated Complications	Wear postoperative increase for defective THA device lab design failure.	Effect would be a bio-mechanical partial/total failure. walking and biomechanics failure	Optimization of Surgery Department resolved cases

Discussion and Conclusion

In a series of articles [1-4,26], the objective to get a nonlinear optimized model for wear of THA CoC and MoM implants was developed. The nonlinear optimization was Numerical, 2D Graphical, 3D Interior, and 4D Interior Optimization. The innovation of this review contribution is the 4D Interior Optimization improvement—with a software comparison to Matlab System. The principal achievement is an adimensional K for the model that gives accuracy for THA tribotesting *in vitro* predictions. The K result can be considered acceptable and efficacious, provided the *in vitro* erosion intervals of the literature [refs]. The method was the classical Inverse Least-Squares technique. This method is efficient and accurate [1-4]. An inconvenient of the model is that has not global minimum, instead multiple local minima. The computational assessment between Matlab and GNU-Octave Systems for 4D Interior Optimization confirm the precision of the optimized model. The accuracy of the fitted model is almost equal for CoC and MoM parameter intervals. However, the optimization

for CoC has wider intervals as it was both Dual and Multiobjective. 3D and 4D imaging results show be satisfactory for Mat lab and GNU Octave in sharpness, contrast, and clarity. Both systems Image Processing Tools can be considered up to standards. Programming software for 1D-4D Computer Vision was rather difficult since large-dimension volume-matrices in arrays had to be built—with 10^5 and 10^6 elements. Therefore, the objectives of the studies [1-4] were found acceptable and useful. The results comprise two widely used THA materials, ceramic and metal. Numerical results, 2D Graphical, 3D Interior, and 4D Interior Optimization are adequate. The utility of the results is mainly focused on extrapolated simulations/predictions of erosion rates for *in vitro* THA studies. Different materials within the selected parameter-interval ranges can be set in the model whose optimal parameters are calculated along these studies. Improvements in algorithms, software, and model design are feasible from these findings. Improvements optimization method new 4D Interior Optimization Method emerge from the summary of the

paper series. From these findings, both numerical and graphical methods can be applied on similar models, not limited to clinical bio tribology.

The contribution of these articles [1-4,26] in predictive wear methods is focused on in vitro experimental computational erosion determinations. This means that the numerical adimensional K and optimal hardness intervals for this model could be used as an exact/approximated reference to obtain tentative data when planning a laboratory experiment. Setting K as an adimensional model-constant makes the tribotesting task easier with the selected units implemented within the model. Medical industrial applications comprise the advantage of the demonstrated model for its simple non-linearity and adimensional K. In other words, for clinical medical physics devices production the usual technique is to use as simpler as possible for manufacturing, avoiding complex physics equations. In brief, the reviewed series of contributions suppose an improvement for THA in vitro tribo testing with THA CoC and MoM clinical Medical Physics and Bioengineering useful applications.

Scientific Ethics Standards

This review article summarizes methods and results from a series of contributions [refs]. The difference is the new inclusion of 4D Interior Optimization innovative technique. Therefore, graphs and ideas from a group of articles are taken in. 2D/3D Graphical-Optimization Methods were created by Dr Francisco Casesnoves on December 2016 and Interior Optimization Methods in 2019. Author originally developed this software. This article has a few previous paper information, whose inclusion is essential to contribute understandable. One applications section paragraph related to biomechanics of hip is taken from an upcoming article, as it clarifies the clinical usage sharply. The nonlinear optimization software was improved from previous contributions in subroutines modifications, patters, loops, graphics and optimal visualization with GNU Octave new software. This study was carried out, and their contents are done according to the European Union Technology and Science Ethics. Reference, 'European Textbook on Ethics in Research'. European Commission, Directorate-General for Research. Unit L3. Governance and Ethics. European Research Area. Science and Society. EUR 24452 EN [29,30]. In addition, based on 'The European Code of Conduct for Research Integrity'. Revised Edition. ALLEA. 2017. This research was completely done by the author; the computational-software, calculations, images, mathematical propositions and statements, reference citations, and text is original for the author. When a mathematical statement, proposition or theorem is presented, demonstration is always included. The article is exclusively scientific, without any commercial, institutional, academic, political, or economic influence. When anything is taken from a source, it is adequately recognized. Ideas from previous publications were emphasized due to a clarification aim [29-30].

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