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Further Developments on the T-Transmuted X Family of Distributions II

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Abstract

We review the exponentiated generalized (EG) T-X family of distributions and propose some further developments of this class of distributions [1].

Keywords: T-X(W) family of distributions; Transmuted family of distributions; Exponentiated

Abbreviations: EG: exponentiated generalized; QRTM: Quadratic Rank Transmutation Map;

Introduction

Transmuted family of distributions

According to the quadratic rank transmutation map (QRTM) approach in Shaw W, et al. [2], the CDF of the transmuted family of distributions is given by

$$F(x) := (1+\lambda)G(x) - \lambda G(x)^2$$

Where, $-1 \le \lambda \le 1$ and $G(x)$ is the CDF of the base distribution. When $\lambda = 0$ we get the CDF of the base distribution

Remark 1.1. The PDF of the transmuted family of distributions is obtained by differentiating

the CDF above.

A plethora of results discussing properties and applications of this class of distributions have appeared in the literature, and for examples see Faton Merovci, et al. [3] and Muhammad Shuaib Khan, et al [4].

T-X(W) family of distributions

This family of distributions is a generalization of the beta-generated family of distributions first proposed by Eugene et al. [5]. In particular, let $r(t)$ be the PDF of the random variable $T \in [a,b]$, $-\infty \le a < b \le \infty$, and let $W(F(x))$ be a monotonic and absolutely continuous function of the CDF $F(x)$ of any random variable X. The CDF of a new family of distributions defined by Alzaatreh et al. [6] is given by

$$G(x) = \int_a^{W(F(x))} r(t)\,dt = R\{W(F(x))\}$$

Where $R(\cdot)$ is the CDF of the random variable T and $a \ge 0$

Remark 1.2. The PDF of the $T-X(W)$ family of distributions is obtained by differentiating the CDF above

Remark 1.3. When we set $W(F(x)) := -\ln(1-F(x))$, then we use the term "T-X Family of Distributions" to describe all sub-classes of the $T-X(W)$ family of distributions induced by the weight function $W(x) = -\ln(1-x)$. A description of different weight functions that are appropriate given the support of the random variable T is discussed in Alzaatreh A, et al. [6]

A plethora of results studying properties and application of the $T-X(W)$ family of distributions have appeared in the literature, and the research papers, assuming open access, can be easily obtained on the web via common search engines, like Google, etc.

T-Transmuted X family of distributions

This class of distributions appeared in Jayakumar K, et al. [7]. In particular the CDF admits the following integral representation for $a \ge 0$

$$J(x) = \int_a^{-\ln(1-F(x))} r(t)dt$$

Where $r(t)$ is the PDF of the random variable T and $F(x)$ is the transmuted CDF of the random variable X, that is,

$$F(x) := (1+\lambda)G(x) - \lambda G(x)^2$$

Where $-1 \le \lambda \le 1$ and $G(x)$ is the CDF of the base distribution.

Remark 1.4. The PDF of the T-Transmuted X family of distributions is obtained by differentiating the CDF above.

The exponentiated generalized (EG) T-X family of distributions

This class of distributions appeared in Suleman Nasiru, et al. [1]. In particular the CDF admits the following integral representation

$$G(x)=\int_{0}^{-\log\left[1-\left(1-\bar{F}(x)^{d}\right)^{c}\right]} r(t)dt$$

Where, $c,d>0$ and $\bar{F}(x)=1-F(x)$

Remark 1.5. Note that if we set $L(x):=\left(1-\bar{F}(x)^{d}\right)^{c}$, where $c,d>0$ and $\bar{F}(x)=1-F(x)$, then $L(x)$ gives the CDF of the exponentiated generalized class of distributions [8]

Further developments

In this section, inspired by quantile generated probability distributions and the T transmuted X family of distributions [6,9], we propose some new extensions of the exponentiated generalized (EG) T-X family of distributions. We give the CDF of these new class of distributions, only in integral form. However, the CDF and PDF can be obtained explicitly by applying Theorem 2.2 and Theorem 2.3, respectively.

The q_T-X family of distributions

Definition 2.1. Let V be any function such that the following holds:

a) $F(x)\in\left[V(a),V(b)\right]$

b) $F(x)$ is differentiable and strictly increasing

c) $\lim_{x\to-\infty}F(x)=V(a)$ and $\lim_{x\to-\infty}F(x)=V(b)$

then the CDF of the q_T-X family induced by V is given by

$$K(x)=\int_{a}^{V(F(x))}\frac{1}{r(Q(t))}dt$$

Where $\frac{1}{r(Q(t))}$ is the quantile density function of random variable $T\in[a,b]$, for $-\infty\le a<b\le\infty$, and $F(x)$ is the CDF of any random variable X.

Theorem 2.2. The CDF of the q_T-X family induced by V is given by $K(x)=Q\left[V(F(x))\right]$

Proof. Follows from the previous definition and noting that $Q'=\frac{1}{roQ}$

Theorem 2.3. The PDF of the q_T-X family induced by V is given by

$$k(x)=\frac{f(x)}{r\left[Q\left(V(F(x))\right)\right]}V'\left[F(x)\right]$$

Proof. $k=K'$, $Q'=\frac{1}{roQ}$, $F'=f$ and K is given by Theorem 2.2

Remark 2.4. When the support of T is $[a,\infty)$, where $a\ge0$, we can take V as follows

I. $V(x)=1-e^{-x}$

II. $V(x)=\frac{x}{1+x}$

III. $V(x)=\left[1-e^{-x}\right]^{\frac{1}{\alpha}}$, where $\alpha>0$

IV. $V(x)=\left[\frac{x}{1+x}\right]^{\frac{1}{\alpha}}$, where $\alpha>0$

Remark 2.5. When the support of T is $(-\infty,\infty)$, we can take V as follows

i. $V(x)=1-e^{-ex}$

ii. $V(x)=\frac{e^{x}}{1+e^{x}}$

iii. $V(x)=\left[\frac{e^{-ex}}{1+e^{x}}\right]^{\frac{1}{\alpha}}$, where $\alpha>0$

iv. $V(x)=\left[\frac{e^{x}}{1+e^{x}}\right]^{\frac{1}{\alpha}}$, where $\alpha>0$

Definition 2.6. A random variable W (say) is said to be transmuted exponentiated generalized

distributed if the CDF is given by

$$(1+\lambda)L(x)-\lambda L(x)^{2}$$

Where, $-1\le\lambda\le1$, $L(x):=\left(1-\bar{F}(x)^{d}\right)^{c}$, c, $d>0$, $\bar{F}(x)=1-F(x)$, and $F(x)$ is a base distribution.

Some EG q_T transmuted X family of distributions

In what follows we assume the random variable T has PDF $r(t)$ and quantile function $Q(t)$. We also assume the random variable X has transmuted CDF

$$(1+\lambda)L(x)-\lambda L(x)^{2}$$

Where $-1\le\lambda\le1$, $L(x):=\left(1-\bar{F}(x)^{d}\right)^{c}$, c, $d>0$, $\bar{F}(x)=1-F(x)$, and $F(x)$ is a base distribution.

Families of EG q_T-transmuted X distributions of type I

The CDF has the following integral representation for $\alpha>0$ and $a\ge0$

$$K(x)=\int_{a}^{\left[1-e^{-\left\{(1+\lambda)L(x)^{2}\right\}}\right]^{\frac{1}{\alpha}}}\frac{1}{r(Q(t))}dt$$

Families of EG q_T transmuted X distributions of type II

The CDF has the following integral representation for $\alpha>0$ and $a\ge0$

$$K(x)=\int_{a}^{\left[\frac{\left\{(1+\lambda)L(x)-\lambda L(x)^{2}\right\}}{1+\left\{(1+\lambda)L(x)-\lambda L(x)^{2}\right\}}\right]^{\frac{1}{\alpha}}}\frac{1}{r(Q(t))}dt$$

Families of EG q_T transmuted X distributions of Type III

The CDF has the following integral representation for $\alpha>0$

$$K(x)=\int_{-\infty}^{\left[1-e^{-e^{\left\{(1+\lambda)L(x)-\lambda L(x)^{2}\right\}}}\right]^{\frac{1}{\alpha}}}\frac{1}{r(Q(t))}dt$$

Families of EG q_T-transmuted X distributions of type IV

The CDF has the following integral representation for $\alpha>0$

$$K(x)=\int_{-\infty}^{\left[\frac{e^{\left\{(1+\lambda)L(x)-\lambda L(x)^{2}\right\}}}{1+e^{\left\{(1+\lambda)L(x)-\lambda L(x)^{2}\right\}}}\right]^{\frac{1}{\alpha}}}\frac{1}{r(Q(t))}dt$$

Concluding Remarks

Our hope is that researchers will find these class of distributions practically significant in modeling biological data,

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health data, etc. We hope the researchers will further develop the properties and applications of these new class of distributions.

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